Marten Hillebrand, Hoang Van Khieu **Mathematical Methods, Part 1**
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Problem Set 5

Problem 5.1. Suppose X_t is a (discrete) random variable following an $AR(1)$ process

$$X_t = aX_{t-1} + b\epsilon_t,\ -1 < a < 1, \text{ and } b \in \mathbb{R}, \tag{1}$$

where x_0 is a real number, and ϵ_t is normally distributed on $\mathbb{R}$ with mean μ_ϵ and variance σ_ϵ^2. Assume further that ϵ_t is i.i.d.

(i) Show that X_t is normally distributed on $\mathbb{R}$.

(ii) Compute the first moment and the second central moment of X_t.

(iii) Compute the limits of the moments and verify that they constitute a normal distribution.

Problem 5.2. Consider the stochastic OLG model introduced in Section 4.1. Unless stated otherwise, all assumptions made there continue to hold. Assume that the intensive form production function is of the form

$$f(k_t) = \theta_t k_t^\alpha,\ 0 < \alpha < 1 \tag{2}$$

where θ_t represents productivity shocks that consists of independent random variables with distribution μ and values in $\Theta = [\theta_{\min}, \theta_{\max}]$. Assume consumer's period utility is $u(c) = \log(c)$.

(i) State the consumer's decision problem and derive the first order conditions.

(ii) State the firm's decision problem and derive the first order conditions.

(iii) Determine the complete set of equilibrium equations and show that the equilibrium is completely determined by

$$k_{t+1} = \mathcal{K}(\theta_t, k_t),\ t \geq 0. \tag{3}$$

In particular, compute the function $\mathcal{K}$.

(iv) Define a stable set of (3) as an interval $[k_{\min}, k_{\max}] \subset \mathbb{R}_{++}$ such that

(a) $\mathcal{K}(k_{\min}, \theta_{\min}) = k_{\min}$

(b) $\mathcal{K}(k_{\max}, \theta_{\max}) = k_{\max}$

(c) $\mathcal{K}(k, \theta_{\min}) < k < \mathcal{K}(k, \theta_{\max})$ for all $k \in [k_{\min}, k_{\max}]$.

Determine all stable sets of (3).

Enjoy!

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