Problem Set 6

Problem 6.1. Consider the following RBC model. The representative household's utility is given by

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t(\log(c_t)+\upsilon(1-h_t))\Big],$$

where h is labor supply and υ is a strictly increasing and strictly concave function of leisure, $1-h$. Assume the production function is $F(k_t, A_t h_t) = k_t^{\alpha}(A_t h_t)^{1-\alpha},\ 0<\alpha<1$ and capital is fully depreciated, i.e. $\delta = 1$. A_t represents productivity shocks that consist of independent random variable with distribution μ and values in $\mathcal{A} = [\underline{A}, \overline{A}]$.

(a) State the consumer's problem and derive the FOCs.

(b) State the firm's problem and derive the FOCs.

(c) Saving rate (out of income) is defined as $s_t = k_{t+1}/F(k_t, A_t h_t)$. Show that a constant saving rate can be consistent with the Euler equation, and express the saving rate in terms of parameters.

(d) Given the constant saving rate, show that labor supply is also constant.

(e) Given the constant saving rate, compute the function $\mathcal{K}$ and $\mathcal{C}$ such that

$$k_{t+1} = \mathcal{K}(k_t, A_t), \tag{1}$$
$$c_t = \mathcal{C}(k_t, A_t). \tag{2}$$

Determine all the stable sets of (1) and (2).

Problem 6.2. Consider the economy in Problem 6.1 again. Unless otherwise stated, all assumptions there continue to hold. Assume further that $\upsilon(1-h_t) = \log(1-h_t)$. Solve this problem using the recursive methods by following steps below.

(a) State the Bellman equation.

(b) Compute the policy function for capital using the 'guess and verify' approach. (You are expected to be able to guess the functional form of the value function).

(c) Are labor supply and the saving rate constant? Compare with the ones obtained in Problem 6.1 when $\upsilon(1-h_t) = \log(1-h_t)$.

Enjoy!

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