Marten Hillebrand, Van Hoang Khieu

Mathematical Methods, Part 1
Winter 2015/16

Problem Set 1

Problem 1.1. Consider the decision problem with finite time horizon $T > 0$ studied in class. Unless stated otherwise, all assumptions and the notation introduced there continue to hold. Specifically, assume that period utility u is of the form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{1}$$

where we assume $0 < \sigma < 1$.

(a) Set up the consumer's decision problem.

(b) Compute the solution to the decision problem

  (i) by Lagrangian methods

  (ii) by recursive methods.

  Show that both approaches yield the same solution.

(c) Interpret the form of optimal consumption-investment strategy economically.

(d) Explain why we need the restriction $\sigma < 1$ when setting up the decision problem. Describe how the setup could slightly be modified to handle cases $\sigma \geq 1$ as well (where $\sigma = 1$ yields $u(c) = \log c$). Argue that the solutions are of the same functional form derived in (b) for all $\sigma > 0$ (but, of course, depend on σ).

Problem 1.2. Consider the standard OLG model introduced in Section 1.6. Unless stated otherwise, all assumptions made there continue to hold. Assume that the intensive form production function is of the form

$$f(k) = Ak^{\alpha}, \quad A > 0, 0 < \alpha < 1 \tag{2}$$

and period utility is $u(c) = \log(c)$.

(a) State a young consumer's decision problem in period $t \geq 0$ and characterize the solution.

(b) State the firm's decision problem and derive the first order conditions.

(c) Determine the complete set of equilibrium equation and show that the equilibrium is completely determined by the sequence $(k_{t+1})_{t\geq 0}$.

(d) Show that this sequence evolves as

$$k_{t+1} = \mathcal{K}(k_t), \quad t \geq 0. \tag{3}$$

  Compute the function $\mathcal{K}$ and sketch it in a diagram.

Enjoy!

Please direct questions to: hkhieu@uni-mainz.de