APPENDIX
to

A Model of Creative Destruction with Undiversifiable Risk and Optimising Households

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Klaus Wälde, University of Dortmund and The World Bank, kwaelde@worldbank.org
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(This is the appendix referred to on page C171 in the EJ paper)

Equation (3)

Profits in the consumption good sector are given by

$$\pi_y = \gamma^t \left(\alpha \gamma^t p^{-1}\right)^{\alpha/(1-\alpha)} - p\left(\alpha \gamma^t p^{-1}\right)^{1/(1-\alpha)}$$
$$= \left(\gamma^t\right)^{1/(1-\alpha)} \left(\alpha p^{-1}\right)^{\alpha/(1-\alpha)} - p^{-\alpha/(1-\alpha)} \left(\alpha \gamma^t\right)^{1/(1-\alpha)}$$
$$= \left(\gamma^t\right)^{1/(1-\alpha)} p^{-\alpha/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) = (1-\alpha)\left(\gamma^t \left(\alpha p^{-1}\right)^{\alpha}\right)^{1/(1-\alpha)}.$$

Rewriting the first order condition (2) as $x^{1-\alpha}\gamma^{-t} = \alpha p^{-1}$ and inserting gives

$$\pi_y = (1-\alpha)\left(\gamma^t \left(x^{1-\alpha}\gamma^{-t}\right)^{\alpha}\right)^{1/(1-\alpha)} = (1-\alpha)\left(\gamma^{t(1-\alpha)} x^{\alpha(1-\alpha)}\right)^{1/(1-\alpha)} = (1-\alpha)\gamma^t x^{\alpha}.$$

Equation (6)

Profits of the monopolists are by (5) and (2)

$$\pi_x = (1-\alpha) p x \qquad = (1-\alpha) p^{-\alpha/(1-\alpha)} \left(\gamma^t \alpha\right)^{1/(1-\alpha)} \qquad = (1-\alpha)\left(\gamma^t p^{-\alpha} \alpha^{\alpha} \alpha^{1-\alpha}\right)^{1/(1-\alpha)}$$
$$= \alpha(1-\alpha)\left(\gamma^t \left(\alpha p^{-1}\right)^{\alpha}\right)^{1/(1-\alpha)} = \alpha \pi_y,$$

where the last equality follows from the appendix to equation (3).

Equation (19)

Labour demand in the R&D sector is given from (18) and (8) by

$$n = (1-\delta)\left(1 + \frac{\pi_t}{w_t}\right) N .$$

Inserted into the labour market clearing condition gives

$$(1-\delta)\left(1+\frac{\pi_t}{w_t}\right)N+L=N\,.$$

Since from (27) and (28) $\frac{\pi_t\left(N^{-1}\right)}{w_t}=\frac{N^{-1}\left(1-\alpha^2\right)y_t}{\alpha^2 y_t L^{-1}}=\left(\alpha^{-2}-1\right)\frac{L}{N}$, we find

$$(1-\delta)\left(N+\left(\alpha^{-2}-1\right)L\right)+L=N \quad\Leftrightarrow\quad 1-\delta=\frac{N-L}{N+\left(\alpha^{-2}-1\right)L} \quad\Leftrightarrow\quad \delta=1-\frac{N-L}{N+\left(\alpha^{-2}-1\right)L}$$

$$=\frac{\alpha^{-2}L}{N+\left(\alpha^{-2}-1\right)L}=\frac{L}{\alpha^2 N+\left(1-\alpha^2\right)L}.$$

Equation (20)

Rewriting (17) gives

$$\delta^{-1}=\frac{\lambda(N\sigma\alpha)^{-1}}{\rho-\Gamma\left[\gamma^{\sigma}-1\right]}\left[\left(\alpha^2 N+\left(1-\alpha^2\right)L\right)\left(\gamma^{\sigma}-1\right)+\sigma\gamma^{\sigma}L(N-1)(1-\alpha)\right]$$

$$\Leftrightarrow\ \delta=\frac{N\sigma\alpha}{\lambda}\frac{\rho-\Gamma\left[\gamma^{\sigma}-1\right]}{\left(\alpha^2 N+\left(1-\alpha^2\right)L\right)\left(\gamma^{\sigma}-1\right)+\sigma\gamma^{\sigma}L(N-1)(1-\alpha)}$$

$$\Leftrightarrow\ \delta=\frac{N\sigma\alpha}{\lambda}\frac{\rho-\lambda(N-L)\left(\gamma^{\sigma}-1\right)}{\alpha^2\left[\gamma^{\sigma}-1\right]N+\left(\left(1-\alpha^2\right)\left(\gamma^{\sigma}-1\right)+\sigma\gamma^{\sigma}(N-1)(1-\alpha)\right)L}\equiv\frac{N\sigma\alpha}{\lambda}\frac{\rho-\lambda(N-L)\left(\gamma^{\sigma}-1\right)}{\alpha^2\left(\gamma^{\sigma}-1\right)N+\Psi L}.$$

The derivative is given by

$$\frac{\lambda}{N\sigma\alpha}\frac{\mathrm{d}}{\mathrm{d}L}\delta=\frac{\lambda\left(\gamma^{\sigma}-1\right)\left(\alpha^2\left(\gamma^{\sigma}-1\right)N+\Psi L\right)-\Psi\left(\rho-\lambda(N-L)\left(\gamma^{\sigma}-1\right)\right)}{\left(\alpha^2\left(\gamma^{\sigma}-1\right)N+\Psi L\right)^2}<0$$

$$\Leftrightarrow\ \lambda\left(\gamma^{\sigma}-1\right)^2\alpha^2 N+\lambda\left(\gamma^{\sigma}-1\right)\Psi L<\Psi\rho-\Psi\lambda(N-L)\left(\gamma^{\sigma}-1\right)$$

$$\Leftrightarrow\ \lambda\left(\gamma^{\sigma}-1\right)^2\alpha^2 N<\Psi\rho-\Psi\lambda N\left(\gamma^{\sigma}-1\right)\Leftrightarrow\lambda N\left[\gamma^{\sigma}-1\right]<\rho-\Psi^{-1}\lambda N\left(\gamma^{\sigma}-1\right)^2\alpha^2$$

Equation (23) and (24)

Given the derivative of the arrival rate $\Gamma=\lambda\left(N-\left(y\gamma^{-t}\right)^{1/\alpha}\right)$,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}y}=-\frac{\lambda}{\alpha}\left(y\gamma^{-t}\right)^{(1-\alpha)/\alpha}\gamma^{-t}=-\frac{\lambda}{\alpha}L^{1-\alpha}\gamma^{-t},$$

the instantaneous utility function (11) and the value function $V(t)=\vartheta\gamma^{t\sigma}$, the first order condition (22) reads

$$\sigma\left(\frac{y}{N}\right)^{\sigma-1}=\vartheta\gamma^{t\sigma}\left(\gamma^{\sigma}-1\right)\frac{\lambda}{\alpha}L^{1-\alpha}\gamma^{-t}$$

Inserting the production function (1) simplifies this expression to

$$\sigma N^{1-\sigma}\left(\gamma^t L^\alpha\right)^{\sigma-1} = \vartheta\gamma^{t\sigma}\left(\gamma^\sigma - 1\right)\frac{\lambda}{\alpha}L^{1-\alpha}\gamma^{-t} \Leftrightarrow \sigma N^{1-\sigma} = \vartheta\left(\gamma^\sigma - 1\right)\frac{\lambda}{\alpha}L^{1-\alpha\sigma} \tag{1}$$

which determines L as a function of constants and ϑ.

The Bellman equation reads

$$\rho\vartheta\gamma^{t\sigma} = N\left(\frac{y}{N}\right)^\sigma + \lambda n\vartheta\gamma^{t\sigma}\left(\gamma^\sigma - 1\right) \Leftrightarrow \rho\vartheta = N^{1-\sigma}L^{\alpha\sigma} + \lambda n\vartheta\left(\gamma^\sigma - 1\right)$$

$$\Leftrightarrow \vartheta = \frac{N^{1-\sigma}L^{\alpha\sigma}}{\rho - \lambda n\left(\gamma^\sigma - 1\right)}. \tag{2}$$

Inserting into the first order condition (40) gives equation (24) in the text,

$$\sigma N^{1-\sigma} = N^{1-\sigma}\frac{N-n}{\rho - \lambda n\left(\gamma^\sigma - 1\right)}\left(\gamma^\sigma - 1\right)\frac{\lambda}{\alpha} \Leftrightarrow \sigma\frac{\alpha}{\lambda} = (N-n)\frac{\gamma^\sigma - 1}{\rho - \lambda n\left(\gamma^\sigma - 1\right)}.$$

After some rearrangements, one obtains (23),

$$\sigma\frac{\alpha}{\lambda}\rho - \sigma\alpha n\left(\gamma^\sigma - 1\right) = (N-n)\left(\gamma^\sigma - 1\right) \quad \Leftrightarrow \quad n\left(\gamma^\sigma - 1\right)(1-\sigma\alpha) = N\left(\gamma^\sigma - 1\right) - \sigma\frac{\alpha}{\lambda}\rho \quad \Leftrightarrow$$

$$n = \frac{1}{1-\sigma\alpha}\left(N - \frac{\sigma\alpha}{\gamma^\sigma - 1}\frac{\rho}{\lambda}\right).$$

Equation (25)

Given the value function $V(t) = \vartheta\gamma^{t\sigma}$, we have $V(t+1) - V(t) = \vartheta\gamma^{t\sigma}\left(\gamma^\sigma - 1\right)$. Inserting (41) gives

$$V(t+1) - V(t) = \frac{\gamma^{t\sigma}L^{\alpha\sigma}}{\rho - \lambda n\left(\gamma^\sigma - 1\right)}\left(\gamma^\sigma - 1\right) = \frac{y^\sigma\left(\gamma^\sigma - 1\right)}{\rho - \lambda n\left(\gamma^\sigma - 1\right)}$$

which inserted in (22) immediately implies (25).

Equation (26)

We start from an individual's first order condition (15). From (32) and (37) in Appendix 1 we then have

$$u'(c) + \vartheta(\Phi y_t)^\sigma\left(\gamma^\sigma - 1\right)\frac{\mathrm{d}}{\mathrm{d}c}\Gamma + \Gamma\frac{\mathrm{d}}{\mathrm{d}c}V\left(iI^{-1}, t+1\right) = 0.$$

Inserting (39) and $\Phi = (\delta N)^{-1}$, which can be seen from (36) with (19), yields

$$u'(c) + \frac{c^\sigma}{\rho - \Gamma\left[\gamma^\sigma - 1\right]}\left(\gamma^\sigma - 1\right)\frac{\mathrm{d}\Gamma}{\mathrm{d}c} + \Gamma\frac{\mathrm{d}}{\mathrm{d}c}V\left(iI^{-1}, t+1\right) = 0.$$