Capital Income Risk and the Dynamics of the Wealth Distribution

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1. Introduction

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Strong public consensus that wealth distributions (at least for the US) are too unequal

- Indirect evidence from the success of Piketty's (2014) "Capital in the Twenty-First Century"
- Direct evidence from Norton and Ariely (2011) "Building a Better America"
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- The evolution of the wealth distribution over time

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- The determinants of the wealth distribution
- The evolution of the wealth distribution over time
- Focus and contribution here: quantitative understanding of evolution of the NLSY 79 wealth distribution from 1986 to 2008

1. Introduction

Our framework

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  - Idiosyncratic interest rate lies below and above the (growth-rate adjusted) time preference rate
  - We work with 'standard regime' and with 'exploding regime'
  - Individuals differ in their 'financial ability' (i.e. in transition rates between low and high returns)
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  - Individuals differ in their 'financial ability' (i.e. in transition rates between low and high returns)
  - Type and scale-dependence as in Gabaix et al. (2016)
- Quantitative analysis of the dynamics of the wealth distribution via Fokker-Planck equations
  - ... also known as Kolmogorov forward equations
  - FPEs are (two) partial differential equations (here)
  - We solve for optimal consumption paths via a shooting algorithm
  - FPEs can be solved by 'method of characteristics'

1. Introduction

Preview of findings (1)

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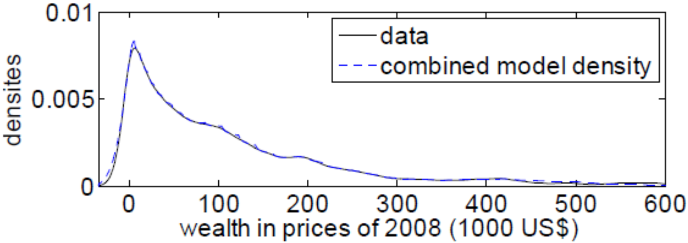
- Almost perfect model fit for wealth distribution in 2008
  - The model density in 2008 covers more than 96% of the empirical density

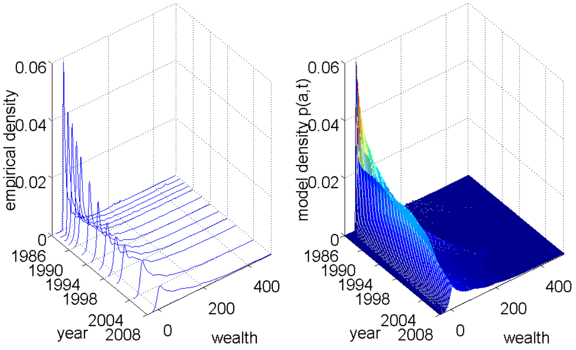


  - High realization of interest rate (at 4.5%) needs to lie above threshold level, yielding a non-stationary evolution of the wealth distribution aka the "exploding regime" (Benhabib and Bisin, 2017)

1. Introduction

Preview of findings (2)

- Very good fit for dynamics of wealth distribution



- When targeting all years/waves, the average fit is 88.9%

1. Introduction

Preview of findings (3)

- “Testing” calibration of our model

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- "Testing" calibration of our model
    - We "test" our calibration by comparing
        - the standard deviation of the idiosyncratic interest rate in our setup with
        - empirical standard deviations reported in the literature

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Preview of findings (3)

- "Testing" calibration of our model
  - We "test" our calibration by comparing
    - the standard deviation of the idiosyncratic interest rate in our setup with
    - empirical standard deviations reported in the literature
  - It seems that empirical standard deviations are one or two orders of magnitude *larger* than those needed in our model to match the dynamics of the wealth distribution
  - Interest rate uncertainty is therefore almost "too successful" in explaining wealth inequality

1. Introduction

Related literature

- Conventional determinants of the distribution of wealth
  - Idiosyncratic labour income risk (Bewley-Huggett-Aiyagari)
  - Castañeda et al. (2003) and many others successfully replicated empirical wealth distributions
  - Labour income state with empirically implausible high income level is employed (called "awesome state" by Benhabib and Bisin (2017) and Benhabib, Bisin and Luo (2017)

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- Search for alternative determinants
  - Benhabib, Bisin and Zhu (2011) suggest risky idiosyncratic returns
    - In an OLG framework, stationary wealth distribution has a Pareto distribution in the right tail
    - Thickness of the right tail increases in capital income risk
  - We see our paper in this tradition
    - Capital income risk is a quantitatively necessary ingredient to match the density of wealth over its entire range
    - An empirically convincing labour income process (i.e. without a 'superstar' or 'awesome' state) is employed

1. Introduction

Related literature (cont'd)

- The dynamics of distributions (without capital income risk)
    - Studied much less so far
    - Gabaix et al. (2016) study the dynamics of *income* inequality (but see their appendix)
    - Kaymak and Poschke (2015) present how top 1%, 5% and 10% wealth shares evolve over time
    - We extend their work inter alia by looking at the entire density and thereby at all wealth shares
    - Parra-Alvarez et al. (2017) structurally estimate a heterogenous agent model. They focus on the identifiability of parameters and apply their method to the 2013 distribution of wealth in the SCF

1. Introduction

Related literature (cont'd)

- The dynamics of distributions (with capital income risk)
  - Benhabib, Bisin and Luo (2015) quantitatively explain wealth distributions and social mobility patterns
    - They emphasize the importance of capital income risk, persistent earnings inequality and bequests
    - They focus on stationary distributions – robustness checks evolution of wealth distributions over time
  - Capital income risk is also taken into account by Hubmer et al. (2016)
  - We extend the latter two by allowing for explicit stochastic labour income over time
    - Our numerical procedure does not require to assume perfect foresight or myopic behaviour with respect to all random events
    - We acknowledge that our partial equilibrium approach helps in this respect as aggregate changes do not affect private decision making

1. Introduction

Related literature (cont'd)

- Quantitative fits for the upper-tail wealth distribution
  - Nirei and Aoki (2016) construct a neoclassical growth model that yields a Pareto distribution for the upper tail
    - Closed-form solutions in the absence of labour income risk
    - With labour income risk, they focus on stationary economy
  - Aoki and Nirei (2017) describe dynamics of distributions employing Fokker-Planck equations
    - Closed-form solutions in the absence of labour income risk
    - Similar to Angeletos (2007)
  - Cao and Luo (2017) allow for stochastic returns and for ex-ante heterogeneity in labour productivity in a growth model
    - Closed-form solution for policy functions
    - Study transitional paths of the effects of policy reforms on top end wealth inequality and welfare

1. Introduction

Related literature (cont'd)

- Fokker-Planck equations (forward Kolmogorov equations)
  - Bayer and Wälde (2010a, sect. 5) showed how to derive them for relatively general cases (using a Bewley-Huggett-Aiyagari model as example)
  - More recently, FPEs became much more popular: Benhabib, Bisin and Zhu (2016), Kaplan et al. (2018), Jones and Kim (2017), Cao and Luo (2017) and Aoki and Nirei (2017)
  - We contribute to this literature by enquiring into the quantitative merits of FPEs
  - We use the method of characteristics to solve them (see Nagel, 2013, ch. 5, for an introduction)

1. Introduction

Related literature (cont'd)

- Empirical idiosyncratic interest rate distributions
  - For our "test", we employ findings on mean and standard deviation from e.g. Flavin and Yamashita (2002), Fagereng et al., 2016) and others
  - We show that this risky-return approach is quantitatively more than successful

1. Introduction

Related literature (cont'd)

- Labour income process
  - Inspired by the SaM literature (Diamond-Mortensen-Pissarides)
  - Labour income fluctuates between wage and unemployment benefits
    - Any realistic income process would need much more structure (see e.g. Blundell et al., 2015, and the references therein)
    - Empirically more convincing income process is the precautionary saving and on-the-job search model by Lise (2013)
    - Yet, he assumes a constant interest rate and focuses on one cross-section of wealth
  - An argument in favour of our simple income structure
    - Empirical skewness in the earnings distribution is not enough to generate sufficiently skewed and thick-tailed wealth distributions (Benhabib and Bisin, 2017, sect. 3.1)
    - Even with such a simple process, we can match the dynamics of the distribution of wealth

1. Introduction

Structure of the talk

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Structure of the talk

2. The model
3. Optimal consumption behaviour
4. Dynamics of Distributions

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Structure of the talk

2. The model
3. Optimal consumption behaviour
4. Dynamics of Distributions
5. The empirical fit
    - Data and quantitative phase diagram
    - Targeting wealth distributions and measuring the fit
    - Robustness checks
    - The distribution of idiosyncratic interest rates
6. Conclusion

2. The model

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2.1. Fundamentals

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Idiosyncratic labour income risk

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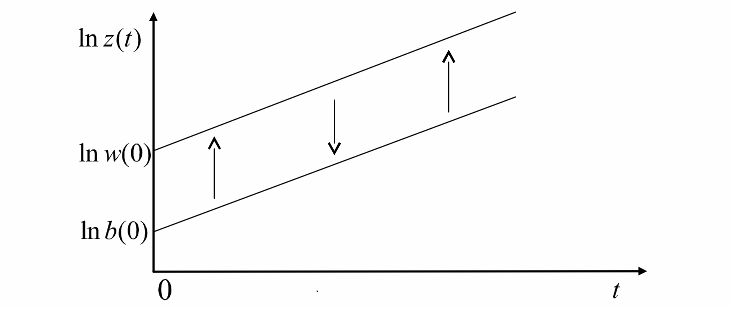
2.1. Fundamentals

Idiosyncratic labour income risk

- Labour income $z(t)$ stochastically jumps between two deterministically income levels growing at exogenous rate g



$$dz(t) = [w(t) - b(t)]\, dq_\mu(t) + [b(t) - w(t)]\, dq_s(t) + gz(t)\, dt$$

- Transition rates μ and s are exogenous and fixed

2. The model

2.1. Fundamentals

Idiosyncratic capital income risk ...

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2.1. Fundamentals

Idiosyncratic capital income risk ...

- Interest rate stochastically jumps back and forth between a "low" level and an "intermediate" level

$$dr(t) = \left[r^{\text{int}} - r^{\text{low}} \right] dq_{\lambda^{\text{low}}}(t) + \left[r^{\text{low}} - r^{\text{int}} \right] dq_{\lambda^{\text{int}}}(t)$$

- Transition rates λ^{low} and λ^{int} are exogenous and fixed

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- Transition rates λ^{low} and λ^{int} are exogenous and fixed

... with ex-ante heterogeneity in transition rates

- Individuals differ in their ability/ luck on the financial market
  - Some buy a house and sell it for a much higher price, some incur losses
  - Some are more lucky on the financial market than others
  - Some have better business ideas than others
  - Some have a higher *expected* number of periods with high returns than others

2. The model

2.1. Fundamentals

The individual

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2.1. Fundamentals

The individual

- Standard intertemporal and instantaneous (CRRA) utility functions

$$U(t) = E \int_h^{\infty} e^{-\rho[\tau-h]} u(c(\tau)) \, d\tau$$

$$u(c(\tau)) = \frac{c(\tau)^{1-\sigma} - 1}{1-\sigma}$$

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The individual

- Standard intertemporal and instantaneous (CRRA) utility functions

$$
\begin{aligned}
U(t) &= E\int_h^\infty e^{-\rho[\tau-h]} u(c(\tau))\, d\tau \\
u(c(\tau)) &= \frac{c(\tau)^{1-\sigma}-1}{1-\sigma}
\end{aligned}
$$

- Budget constraint for wealth $a(t)$

$$
da(t) = \{r(t)\, a(t) + z(t) - c(t)\}\, dt
$$

with fixed interest rate path $r(t)$ and natural borrowing limit

$$
a(t) \geq a^{\text{nat}} \equiv \frac{-(1-\xi)\, b(t)}{r-g}
$$

where ξ makes sure that the individual survives - $c^{\min}(t) = \xi b(t)$

2. The model

2.2. Optimal precautionary saving and detrending

Trade-off for maximization problem

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Trade-off for maximization problem

- Incomplete insurance via unemployment benefit system
- Consumption drops when losing a job
- Self-insurance (against labour income risk) via wealth accumulation
- Solution implies generalized Keynes-Ramsey rules for $c^z(a)$

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Detrending

- It is numerically simpler to "live in" a stationary environment
- Remove growth trend $\Gamma(t) \equiv \Gamma_0 e^{gt}$ from all endogenous variables $v(t)$

$$\hat{v}(t) = v(t) / \Gamma(t)$$

- [Empirical analysis works with variables in levels, i.e. with $v(t)$]

2. The model

2.3. Equilibrium

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Generalized (detrended) Keynes-Ramsey rule ...

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2.3. Equilibrium

Generalized (detrended) Keynes-Ramsey rule ...

- ... when employed

$$\frac{d\hat{c}^w(\hat{a}(t))}{d\hat{a}(t)} = \frac{\frac{r-\rho}{\sigma} - g + \frac{s}{\sigma}\left[\left(\frac{\hat{c}^w(\hat{a}(t))}{\hat{c}^b(\hat{a}(t))}\right)^{\sigma} - 1\right]}{(r-g)\,\hat{a}(t) + \hat{w} - \hat{c}^w(\hat{a}(t))}\hat{c}^w(\hat{a}(t))$$

- $s = 0$: deterministic world with $\dot{c}/c = (r-\rho)/\sigma$ (in case of CRRA)
- $s > 0$: consumption growth is faster for employed worker due to ...

2. The model

2.3. Equilibrium

Generalized (detrended) Keynes-Ramsey rule ...

- ... when employed

$$\frac{d\hat{c}^{w}\left(\hat{a}\left(t\right)\right)}{d\hat{a}\left(t\right)}=\frac{\frac{r-\rho}{\sigma}-g+\frac{s}{\sigma}\left[\left(\frac{\hat{c}^{w}\left(\hat{a}\left(t\right)\right)}{\hat{c}^{b}\left(\hat{a}\left(t\right)\right)}\right)^{\sigma}-1\right]}{\left(r-g\right)\hat{a}\left(t\right)+\hat{w}-\hat{c}^{w}\left(\hat{a}\left(t\right)\right)}\hat{c}^{w}\left(\hat{a}\left(t\right)\right)$$

- $s=0$: deterministic world with $\dot{c}/c=\left(r-\rho\right)/\sigma$ (in case of CRRA)
- $s>0$: consumption growth is faster for employed worker due to ...

Precautionary saving

- high growth of consumption if marginal utility in unemployment state is high relative to employment state
- consumption smoothing by accumulating wealth fast

Generalized (detrended) Keynes-Ramsey rule ...

- ... when unemployed

$$\frac{d\hat{c}_r^{\hat{b}}(\hat{a})}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g - \frac{\mu}{\sigma}\left[1 - \left(\frac{\hat{c}_r^{\hat{b}}(\hat{a})}{\hat{c}_r^{\hat{w}}(\hat{a})}\right)^{\sigma}\right]}{(r-g)\,\hat{a} + \hat{b} - \hat{c}_r^{\hat{b}}(\hat{a})}\hat{c}_r^{\hat{b}}(\hat{a})$$

Generalized (detrended) Keynes-Ramsey rule ...

- ... when unemployed

$$\frac{d\hat{c}_r^{\hat{b}}\left(\hat{a}\right)}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g - \frac{\mu}{\sigma}\left[1 - \left(\frac{\hat{c}_r^{\hat{b}}\left(\hat{a}\right)}{\hat{c}_r^{\hat{w}}\left(\hat{a}\right)}\right)^{\sigma}\right]}{\left(r-g\right)\hat{a} + \hat{b} - \hat{c}_r^{\hat{b}}\left(\hat{a}\right)}\hat{c}_r^{\hat{b}}\left(\hat{a}\right)$$

- ... implying "post-cautionary dis-saving"
- Compare $\mu = 0$ (unemployment is an absorbing state) with $\mu > 0$

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3.1. Interest rate regimes

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- Interest rate can take two levels, low and high

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- Interest rate can take two levels, low and high
- Low-interest-rate regime
  - Interest rate range (follow from Keynes-Ramsey rules)

$$r < \rho + \sigma g$$

  - Standard regime of precautionary savings literature for $g = 0$: $r < \rho$

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- High-interest-rate regime
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$$\rho + \sigma g < r < \rho + \sigma g + \mu$$

  - Upper bound empirically not binding (as job-finding rate $\mu \gg r$)

3. Optimal consumption behaviour

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$$\rho + \sigma g < r < \rho + \sigma g + \mu$$

  - Upper bound empirically not binding (as job-finding rate $\mu \gg r$)
- (Very-high-interest rate regime, $\rho + \sigma g + \mu < r$, not taken into account)

3. Optimal consumption behaviour

3.2. Illustration of consumption-wealth dynamics

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- Low-interest-rate regime (standard case, $r < \rho + \sigma g$)

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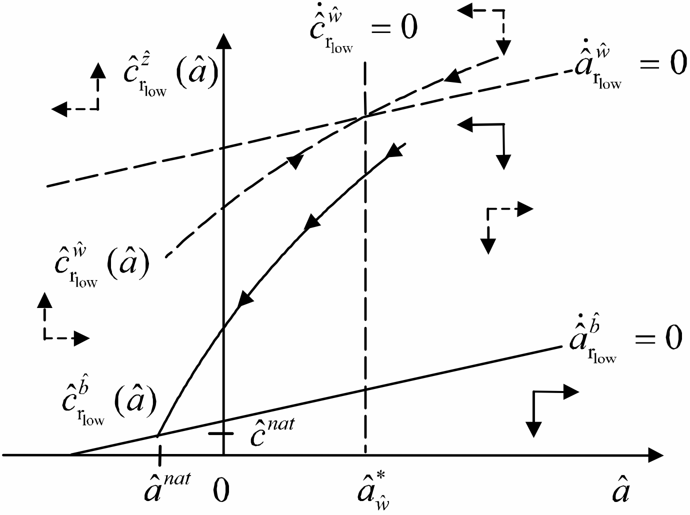

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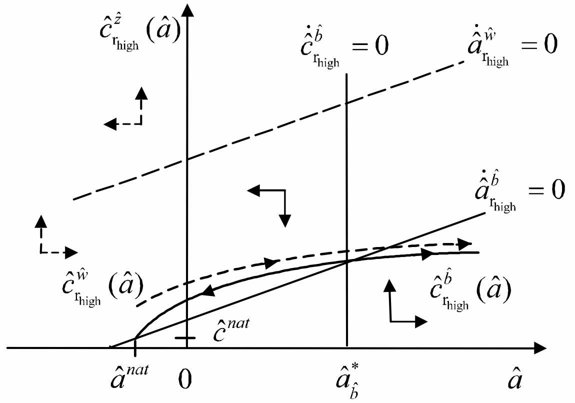

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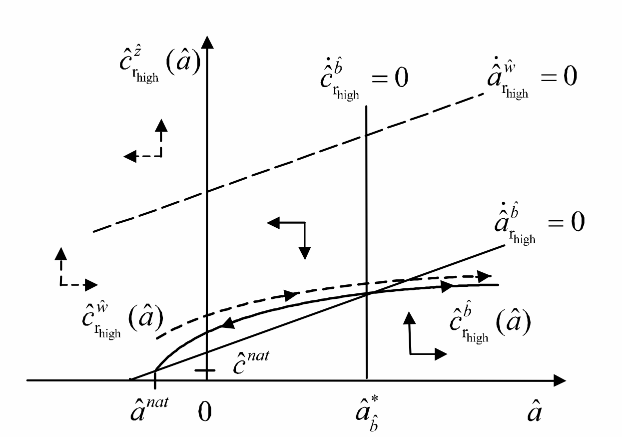

- Key to understanding right-skewness of wealth distribution (long tail on right-hand side)

4. Dynamics of distributions

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4.1. Formal analysis

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4.1. Formal analysis

Question

- Given an initial condition for $(a(t), z(t))$, what is the joint distribution of $(a(\tau), z(\tau))$ at $\tau \geq t$?

Approach

- Fokker-Planck equations describe the evolution of the joint density of $(a(\tau), z(\tau))$ - given laws of motion for $a(\tau)$ and $z(\tau)$ and parameters (including a given interest rate path)

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- Here: Partial differential equations

$$\frac{\partial}{\partial t}p^w(\hat{a},t) + \frac{\partial}{\partial \hat{a}}\{[(r-g)\hat{a} + w_0 - \hat{c}^w(\hat{a})]\,p^w(\hat{a},t)\}$$
$$= -sp^w(\hat{a},t) + \mu p^b(\hat{a},t)$$
$$\frac{\partial}{\partial t}p^b(\hat{a},t) + \frac{\partial}{\partial \hat{a}}\left\{\left[(r-g)\hat{a} + b_0 - \hat{c}^b(\hat{a})\right]p^b(\hat{a},t)\right\}$$
$$= sp^w(\hat{a},t) - \mu p^b(\hat{a},t)$$

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$$= -sp^w(\hat{a}, t) + \mu p^b(\hat{a}, t)$$
$$\frac{\partial}{\partial t} p^b(\hat{a}, t) + \frac{\partial}{\partial \hat{a}} \left\{ \left[(r-g)\hat{a} + b_0 - \hat{c}^b(\hat{a}) \right] p^b(\hat{a}, t) \right\}$$
$$= sp^w(\hat{a}, t) - \mu p^b(\hat{a}, t)$$

Economic aspects

- Optimal consumption $\hat{c}^w(\hat{a})$ and $\hat{c}^b(\hat{a})$ matters (all preferences "in there")
- Job finding and separation rates (s and μ) matter
- Interest rate r and growth rate g play a role

4. Dynamics of distributions

4.2. Variables with trend

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4.2. Variables with trend

- Transform findings for detrended variables back into *levels*
- Straightforward for "normal variables" $v(t)$ by $v(t) = \hat{v}(t)\,\Gamma(t)$
- Support for wealth $a(t)$ that evolves over time t,

$$a(t) \in \left[\hat{a}^{\text{nat}}\Gamma(t), \hat{a}^{\max}\Gamma(t)\right[.$$

- Densities $p^{\hat{z}}(\hat{a}, t)$ and $p(\hat{a}, t)$ can be retransformed by Edgeworth's method of translation (Benhabib and Bisin, 2017, sect. 1.2, Wackerly, 2008, ch. 6.4, Wälde, 2012, theorem 7.3.2)
- Density $g(a, t)$ of wealth with trend is

$$g(a, t) = \frac{p(a(t)/\Gamma(t), t)}{\Gamma(t)}.$$

5. The empirical fit

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5.1. Data and quantitative phase diagram

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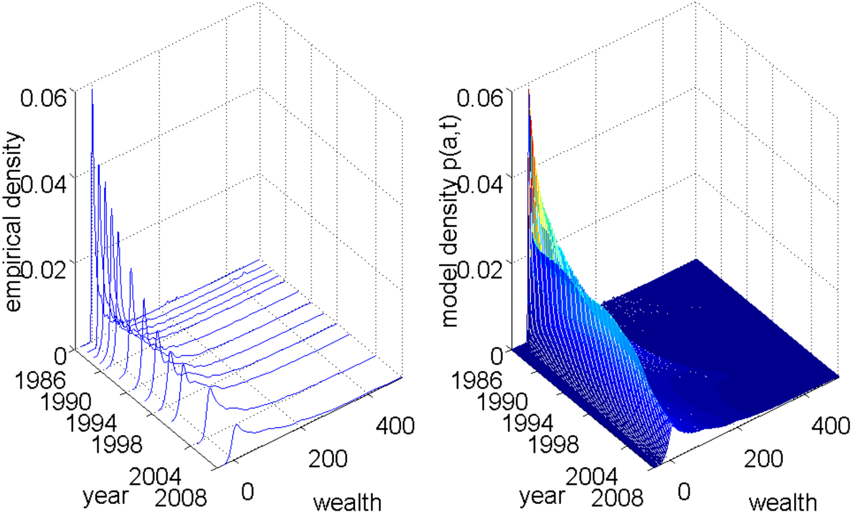
5.1. Data and quantitative phase diagram

- Some descriptive statistics
  - Wealth distributions from the NLSY79 for all waves that provide information on wealth
  - Fairly equal distribution of wealth when individuals are young in 1986
  - Steady increase in the spread as the cohort becomes older
  - (see next slide)

5. The empirical fit

5.1. Data and quantitative phase diagram



5. The empirical fit

5.1. Data and quantitative phase diagram

- Parameter values

μ	s	$\hat{w}$	g	$\hat{b}/\hat{w}$	ξ	ρ	σ	r^{low}	r^{int}
22%	1.19%	2281$	3.4%	30%	97%	1%	1	3.5%	4.5%

μ:	individual job finding rate	s:	separation rate
$\hat{w}$:	average wage in 1986	g:	growth rate of av. real wage
$\hat{b}/\hat{w}$:	benefit replacement rate		

ξ: share of $\hat{b}$ needed for minimum consumption, $\hat{c}^{min} = \xi \hat{b}$

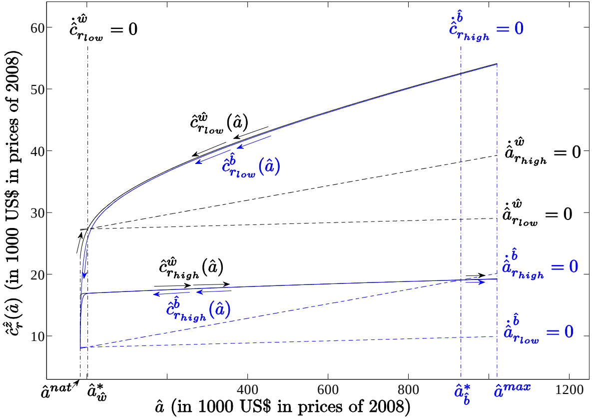
ρ:	time preference rate	σ:	risk aversion
r^{low}:	low interest rate	r^{int}:	intermediate interest rate

Time unit is one month, percentages are monthly (μ, s) or annual (ρ, g)

5. The empirical fit

5.1. Data and quantitative phase diagram

- Quantitative phase diagram



- Why interest rate uncertainty is so successful ...

5. The empirical fit

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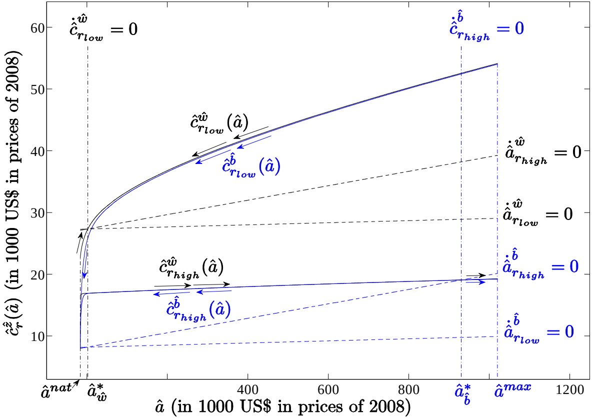
- Quantitative phase diagram



- Why interest rate uncertainty is so successful ...

$$c(t) = \frac{\rho - (1-\sigma) r}{\sigma} \left\{ a(t) + \int_t^\infty e^{-r[\tau - t]} w(\tau)\, d\tau \right\}$$

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

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5.2. Targeting wealth distributions and measuring the fit

- From probabilities to population shares via law of large numbers
  - Probability of an individual to be of a financial type i is p_i
  - Share of individuals of financial type i in population is p_i (as well)

5. The empirical fit

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- Starting point: fit the model distribution to the wealth distribution in 2008
  - (i) Two initial subdensities $p^{\hat{z}}\left(\hat{a}, 0\right)$ for wealth in 1986, one for $\hat{z} = \hat{w}$ and one for $\hat{z} = \hat{b}$
  - Solve Fokker-Planck equations for each of the n financial types
  - We obtain $2n$ wealth densities (n types times 2 initial interest rates) for 22 years later in 2008
  - For each financial type, add trend and obtain wealth density $g_i\left(a, t\right)$

5. The empirical fit

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  - (ii) For a given exogenous number n of financial types, we determine population shares/ probabilities p_i
  - We do so by maximizing our measure of fit

$$F(t) = 1 - \frac{\int_{-\infty}^{\infty} \left| g^{\text{model}}(a, t) - g^{\text{data}}(a, t) \right| da}{2}$$

[Digression] Our measure of fit

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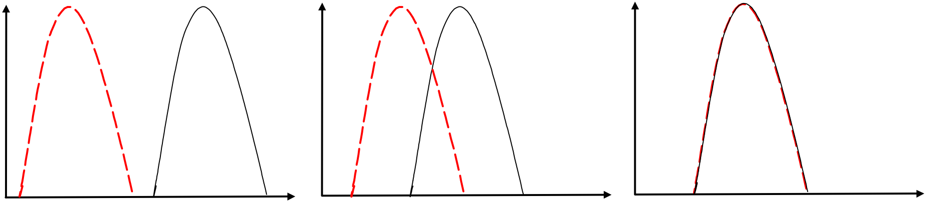

Figure: *Zero fit with $F(t) = 0$ in left figure, some fit (say $F(t) = 1/4$) in the middle and perfect fit in right panel*

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

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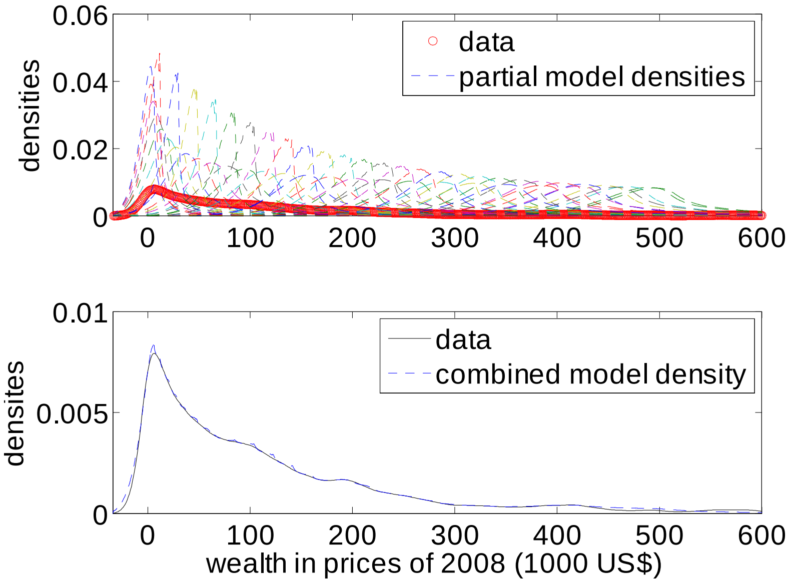
where model density is

$$g^{\text{model}}(a,t) = \Sigma_{i=1}^{2n} p_i g_i(a,t)$$

  - (iii) The number n of financial types is chosen
  - The optimal number turns out to be $n = 26$ with two initial conditions each
  - This gives $2n = 52$ densities of wealth for 2008

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit



5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

- Measuring the fit for years other than 2008

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- Measuring the fit for years other than 2008
  - When p_i maximize fit in 2008, what about fit $F(t)$ for other years?

t	1986	1987	1988	1989	1990	1992	1994
$F(t)$	100	72.2	61.6	58.7	58.2	63.4	68.8

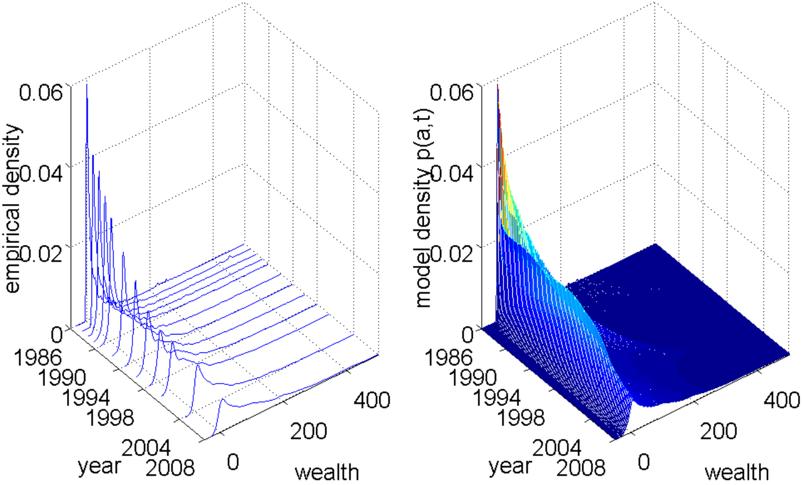
t	1996	1998	2000	2004	2008
$F(t)$	73.8	79.1	81.5	85.8	96.2

  - This is quantitative version of the above figure

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

- ... the quantitative version of above figure



5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

- Measuring the fit for all years
  - When we choose p_i to maximize fit in 2008, what about fit $F(t)$ for all years?

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  - This is quantitative version of above figure
  - Fit is perfect by construction for 1986
  - Between 1986 and 2008, fit first falls and then rises
  - In 2008, the fit is close to perfect again

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

- Targeting all years individually

5. The empirical fit

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- Targeting all years individually

t	1987	1988	1989	1990	1992	1994
Fin. types (n)	18	37	52	23	37	37
$F(t)$	86.1	93.0	92.5	91.7	92.3	94.3

t	1996	1998	2000	2004	2008
Fin. types (n)	37	32	32	29	26
$F(t)$	94.3	93.2	93.9	96.3	96.2

  - Fit tends to increase over the years
  - The worst fit we obtain is 86% for 1987
  - Best fit now reaches 96.3% for 2004
  - (The rise in the fit over time should be expected as the system, ceteris paribus, has more time to adjust to any given empirical wealth distribution)

5. The empirical fit

5.2. Targeting wealth distributions and measuring the fit

- Targeting the overall fit ...

- Targeting the overall fit ...
  - ... by maximizing the average of $F(t)$ over all 11 waves from 1987 to 2008
  - The average $F(t)$ lies at 88.9%
  - Better average fit as compared to the average over the fits (74.9%) when we target 2008
  - Individual fits range from 81.6% to 92.2%

5. The empirical fit

5.3. Robustness checks

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- Lorenz curves and Gini coefficients

5. The empirical fit

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- Lorenz curves and Gini coefficients
  - When we target the density in 2008
    - the wealth shares in the model in 2008 differ on average 3.9% from data wealth shares
    - For all waves, the average difference is at 8.3%
  - When we target the average over all years
    - the difference for 2008 increases to 5.7%
    - For all years the average difference is 2.6% only

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- The effect of the high interest rate
  - What is the effect of a broader range of the idiosyncratic interest rate?
  - We targeted 2008 under a high interest rate of 8% (instead of 4.5%)
  - This implies that a_b^* moves to the left (66,300 US$ instead of 930,132 US$)
  - Fit increases slightly to $F(2008) = 97.3\%$
  - As with a rate of 4.5%, the unemployed accumulate wealth beyond a_b^*. As this range is now much larger, the right tail becomes fatter.
  - Overall, however, our general findings are confirmed

5. The empirical fit

5.3. Robustness checks

- The effect of risk aversion

5. The empirical fit

5.3. Robustness checks

- The effect of risk aversion
  - The drop in consumption in the high interest rate regime is due to the drop in the present value of labour income (discussed earlier)
  - Nevertheless look into the effect of risk aversion
  - For σ equal to 0.8, the fit $F\,(2008)$ drops to 89.4%
  - For σ equal to 1.2, the fit remains basically unchanged at 96.3% (as compared to the 96.2%)

5. The empirical fit

5.3. Robustness checks

- Is financial ability time-invariant?

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5.3. Robustness checks

- Is financial ability time-invariant?
  - Financial ability i is captured by a pair of arrival rates λ_i^{low} and λ_i^{high}
    - Drawn at beginning of economic life
    - Arrival rates describe how quickly on average an individual moves from low to high returns (and back)
    - Over a period of 22 years with big changes on financial markets (dot-com bubble in the late 1990s or the internet access to almost all asset types), hard to argue that financial ability i is invariant

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  - Does financial ability change over time?
    - Our starting point is the fit $F(2008)$ for 2008 with 26 financial types of 96.2%
    - (Employing weights p_i) individuals spent 41.9% of their time (9.2 out of 22 years) in the high interest rate regime
    - With the same number of financial types and fitting 1998, individuals only spent 31.0% of their time in the high interest rate regime
  - Financial ability seems to have increased over time
  - Possible reasons are learning about or better access to financial markets

5. The empirical fit

5.3. The distribution of idiosyncratic interest rates

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5.3. The distribution of idiosyncratic interest rates

- Our method yields a relatively good fit of wealth distributions and their dynamics
- Does our idiosyncratic interest rate distribution have properties that are broadly consistent with empirical idiosyncratic interest rate distributions?

5. The empirical fit

5.3. The distribution of idiosyncratic interest rates

Asset	Mean	St.dev.		Source
T-bills	−0.38%	4.35%	US	Flavin Yamashita (2002)
Bonds	0.60%	8.40%		PSID: 1968 to 1992
Stocks	8.24%	24.15%		S&P 500: 1926 to 1992
Mortgage	0.00%	3.36%		
House	6.59%	14.24%		
Wealth [1]	7.92%	27.14%	US	Cao and Luo (2017)
Wealth [2]	5.94%	11.27%		PSID: 1984, 1989 and 1994
Private equity	13.1%	6.90%	US	Moskowitz and
Public equity	14.0%	17.00%		Vissing-Jorgensen (2002)
Risky assets	3.84%	25.47%	NO	Fagereng et al. (2016)
Safe asset	2.91%	3.15%		Admin. tax data:
Total assets	3.16%	5.30%		1993 to 2013

(1) with capital gains
(2) without capital gains

5. The empirical fit

5.3. The distribution of idiosyncratic interest rates

- Empirical interest rate distributions
  - Empirical means below 14% (average 5.5%)
  - Standard deviations between 3% and above 27% (average 12.6%)

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- In our model, we can compute time paths of interest rate distributions
  - For target year 2008, return is 4.3% with standard deviation 0.40%
  - Standard deviations for robustness checks of the same order of magnitude
  - The highest standard deviation (for high interest rate at 8%) is 1.41%

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- Is this low standard deviation an artefact?
  - We have a special structure (only two values, r^{low} and r^{high})?
  - With a continuous uniform distribution, $\sigma^{\text{uniform}} = \left(r^{\text{high}} - r^{\text{low}} \right) / \sqrt{12} = 0.29\%$

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- Hence, findings not driven by specific interest rate distribution
- Empirically plausible specifications for idiosyncratic interest rate "overexplain" wealth inequality

6. Conclusion

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- Simple model structure with extended parameter space ...
  - Take a model of precautionary savings
  - Allow for uncertainty in the interest rate
  - Allow for interest rates below the wage growth rate and above the stationary level
  - Allow for ex-ante heterogeneity in financial ability (transition rates for interest rate)

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  - Allow for uncertainty in the interest rate
  - Allow for interest rates below the wage growth rate and above the stationary level
  - Allow for ex-ante heterogeneity in financial ability (transition rates for interest rate)
- ... does the job
  - Model explains people becoming poorer and poorer over time and people accumulating wealth
  - Setup matches evolution of wealth pretty well (between 88.9% and 96.3%)
  - The implied interest rate in the model has a much lower standard deviation ($\approx$ 1%) than real world standard devitions ($\approx$ 12%)
  - Interest rate uncertainty seems to "overexplain" the real world wealth distribution

6. Conclusion

- Future research

Thank you!