Midterm Exam
Advanced Macroeconomic Theory 2 Part 1

9 June 2017

Please note

• The exam consists of 2 questions. Please answer both of them.

• Solution sheets are handed out before the exam. Only answers on the solution sheets will be scored.

• This is a closed-book exam. No extra material is allowed. You are not allowed to talk to or to contact another person.

Good Luck!
1 Growth and knowledge spillovers (40 Minutes)

Consider a growth model with knowledge spillovers, where consumption is given by the following Dixit-Stiglitz preferences

\[ c(t) \equiv \left[ \int_{i=0}^{n(t)} c(i, t)^\theta di \right]^{\frac{1}{\theta}} \]  

(1.1)

where \( \theta \in (0, 1) \) is a parameter that determines the elasticity of substitution between varieties, indexed by \( i \in [0, n(t)] \), and the household allocates consumption expenditure between each variety subject to

\[ \int_{i=0}^{n(t)} p(i, t)c(i, t)di = e(t), \]  

(1.2)

where \( e(t) \) is the household’s consumption expenditure over the basket over \( n(t) \) varieties.

In the production sector, a firm produces variety \( i \) and maximises profits

\[ \pi(i, t) = p(i, t)x(i, t) - w(t)l(i, t) \]  

(1.3)

where \( p(i, t) > 0 \) is the selling price of \( x(i, t) \), \( w(t) > 0 \) is the labour cost per worker, and \( l(i, t) \) is labour employed to produce variety \( i \). Firm \( i \) uses the following production technology,

\[ x(i, t) = l(i, t). \]  

(1.4)

The R&D sector develops new varieties according to,

\[ \dot{n}(t) = \varphi L_R(t) K(n(t)) \]  

(1.5)

where \( \varphi > 0 \) is the productivity of the R&D process, \( L_R(t) \) is the number of workers engaged in the research sector, and \( K(n(t)) = n(t)^2 \) is a function capturing knowledge spillovers. Also note the following key conditions in equilibrium:

- Goods market-clearing: \( x(i, t) = c(i, t) \),
- Labour market-clearing: \( L = L_R(t) + \int_{i=0}^{n(t)} l(i, t)di, \)
- Free-entry into R&D: \( v(t) = \frac{1}{\varphi K(n(t))}. \)

1. Using (1.1) and (1.2), compute the optimal demand for a variety \( i \), as a function of its price \( p(i, t) \), the household’s expenditure \( e(t) \), and the price index \( P \equiv \int_{i=0}^{n(t)} p(i, t)^{1-\varepsilon}di, \) where \( \varepsilon \equiv \frac{1}{1-\theta}. \)

2. What is the optimal price for a variety \( i \), resulting from maximising profits with respect to \( x(i, t) \)? Use (1.3) and (1.4), your answer to (1) above, and the goods market-clearing condition.

3. Rewrite \( \dot{n}(t) \) using your answers to (1) and (2), as well as equations (1.4) and (1.5), the labour market-clearing condition and the free-entry condition.

4. (a) Now assuming that \( e(t) = 1 \), draw the phase diagram for \( v(t) \) and \( n(t) \), using your answer to (3) and the equation below,

\[ \dot{v}(t) = v(t)\rho - \frac{1-\theta}{n(t)}. \]  

(1.6)

Give an economic interpretation to the model’s dynamics with knowledge spillovers.

(b) Under what conditions is the \( \dot{v}(t) = 0 \) line above the \( \dot{n}(t) = 0 \) line?
2 Search and matching (20 Minutes)

Imagine an economy with a fixed labour force \( N \). Firms either fill positions or keep them vacant. Workers are either employed or unemployed,

\[
N = N^u(t) + L(t),
\]

where \( N^u(t) \) and \( L(t) \) are the stock of unemployed and employed workers at \( t \), and \( N \) is the (fixed) total number of workers. The unemployment rate and the vacancy rate are defined as

\[
u(t) = \frac{N^u(t)}{N}, \quad v(t) = \frac{N^v(t)}{N},
\]

the job finding probability at \( t \) is defined as

\[
p(\theta(t)) \equiv m\left(u(t),v(t)\right) = m\left(1,\theta(t)\right), \quad \text{with} \quad p'(\theta(t)) > 0
\]

where \( \theta(t) = \frac{v(t)}{u(t)} \) stands for labour market tightness. The matching function \( m(\cdot) \) is assumed to have a Cobb-Douglas representation, with parameter \( \eta \in (0,1) \) on vacancies \( v(t) \). The rate with which a vacancy is filled is given by

\[
q(\theta(t)) \equiv m\left(u(t),v(t)\right) = m\left(1,\theta(t)\right) = \frac{p(\theta(t))}{\theta(t)} \quad \text{with} \quad q'(\theta(t)) < 0.
\]

The change in unemployment over time is described by a differential equation

\[
\dot{N}^u(t) = \lambda L(t) - m\left(N^u(t),N^v(t)\right)
\]

where \( \lambda > 0 \) is the worker’s probability to lose a job.

1. Derive the change in the unemployment rate

\[
\dot{u}(t) = \lambda[1 - u(t)] - \theta(t)q(\theta(t))u(t)
\]

starting from equation (2.5). What is the economic interpretation of this equation?

2. Draw the phase diagram for \( \theta(t) \) and \( u(t) \), using the other dynamic equation below together with your answer to (1) above,

\[
\frac{\dot{\theta}(t)}{\theta(t)} = \frac{1}{1 - \eta} \left[ \rho + \lambda + \beta \theta(t)^\eta - \frac{1 - \beta}{k} (y - b) \theta(t)^{\eta - 1} \right].
\]

What happens to the phase diagram if a positive technology shock increases \( y \)?

(Hint: use implicit differentiation to determine \( \partial \theta(t)/\partial y \) when \( \theta(t) = 0 \))

(Note: equation (2.6) admits a single solution \( \theta^* > 0 \), you do not need to compute it)

3. What are the channels through which a shock to productivity propagates in the model?