Midterm Exam

Advanced Macroeconomic Theory 2 Part 1

9 June 2017

Please note

- The exam consists of 2 questions. Please answer ***both*** of them.
- Solution sheets are handed out before the exam. Only answers on the solution sheets will be scored.
- This is a closed-book exam. No extra material is allowed. You are not allowed to talk to or to contact another person.

Good Luck!

1 Growth and knowledge spillovers (40 Minutes)

Consider a growth model with knowledge spillovers, where consumption is given by the following Dixit-Stiglitz preferences

$$c(t) \equiv \left[\int_{i=0}^{n(t)} c(i,t)^{\theta} di \right]^{\frac{1}{\theta}} \tag{1.1}$$

where $\theta \in (0,1)$ is a parameter that determines the elasticity of substitution between varieties, indexed by $i \in [0, n(t)]$, and the household allocates consumption expenditure between each variety subject to

$$\int_{i=0}^{n(t)} p(i,t)c(i,t)di = e(t), \tag{1.2}$$

where $e(t)$ is the household's consumption expenditure over the basket over $n(t)$ varieties. In the production sector, a firm produces variety i and maximises profits

$$\pi(i,t) = p(i,t)x(i,t) - w(t)l(i,t) \tag{1.3}$$

where $p(i,t) > 0$ is the selling price of $x(i,t)$, $w(t) > 0$ is the labour cost per worker, and $l(i,t)$ is labour employed to produce variety i. Firm i uses the following production technology,

$$x(i,t) = l(i,t). \tag{1.4}$$

The R&D sector develops new varieties according to,

$$\dot{n}(t) = \varphi L_R(t) K(n(t)) \tag{1.5}$$

where $\varphi > 0$ is the productivity of the R&D process, $L_R(t)$ is the number of workers engaged in the research sector, and $K(n(t)) = n(t)^2$ is a function capturing knowledge spillovers. Also note the following key conditions in equilibrium:

Goods market-clearing:	$x(i,t) = c(i,t)$,
Labour market-clearing:	$L = L_R(t) + \int_{i=0}^{n(t)} l(i,t)di$,
Free-entry into R&D:	$v(t) = \frac{w(t)}{\varphi K(n(t))}$.

1. Using (1.1) and (1.2), compute the optimal demand for a variety i, as a function of its price $p(i,t)$, the household's expenditure $e(t)$, and the price index $P \equiv \int_{i=0}^{n(t)} p(i,t)^{1-\varepsilon} di$, where $\varepsilon \equiv \frac{1}{1-\theta}$.

2. What is the optimal price for a variety i, resulting from maximising profits with respect to $x(i,t)$? Use (1.3) and (1.4), your answer to (1) above, and the goods market-clearing condition.

3. Rewrite $\dot{n}(t)$ using your answers to (1) and (2), as well as equations (1.4) and (1.5), the labour market-clearing condition and the free-entry condition.

4. (a) Now assuming that $e(t) = 1$, draw the phase diagram for $v(t)$ and $n(t)$, using your answer to (3) and the equation below,

$$\dot{v}(t) = v(t)\rho - \frac{1-\theta}{n(t)}. \tag{1.6}$$

   Give an economic interpretation to the model's dynamics with knowledge spillovers.

   (b) Under what conditions is the $\dot{v}(t) = 0$ line above the $\dot{n}(t) = 0$ line?

2 Search and matching (20 Minutes)

Imagine an economy with a fixed labour force N. Firms either fill positions or keep them vacant. Workers are either employed or unemployed,

$$N = N^u(t) + L(t), \tag{2.1}$$

where $N^u(t)$ and $L(t)$ are the stock of unemployed and employed workers at t, and N is the (fixed) total number of workers. The unemployment rate and the vacancy rate are defined as

$$u(t) = \frac{N^u(t)}{N}, \quad v(t) = \frac{N^v(t)}{N}, \tag{2.2}$$

the job finding probability at t is defined as

$$p\left(\theta(t)\right) \equiv \frac{m\left(u(t), v(t)\right)}{u(t)} = m\left(1, \theta(t)\right), \quad \text{with} \;\; p'(\theta(t)) > 0 \tag{2.3}$$

where $\theta(t) = \frac{v(t)}{u(t)}$ stands for labour market tightness. The matching function $m(.)$ is assumed to have a Cobb-Douglas representation, with parameter $\eta \in (0, 1)$ on vacancies $v(t)$. The rate with which a vacancy is filled is given by

$$q\left(\theta(t)\right) \equiv \frac{m\left(u(t), v(t)\right)}{v(t)} = m\left(\frac{1}{\theta(t)}, 1\right) \equiv \frac{p(\theta(t))}{\theta(t)} \quad \text{with} \;\; q'(\theta(t)) < 0. \tag{2.4}$$

The change in unemployment over time is described by a differential equation

$$\dot{N}^u(t) = \lambda L(t) - m\left(N^u(t), N^v(t)\right) \tag{2.5}$$

where $\lambda > 0$ is the worker's probability to lose a job.

1. Derive the change in the unemployment rate

   $$\dot{u}(t) = \lambda[1 - u(t)] - \theta(t)q(\theta(t))u(t)$$

   starting from equation (2.5). What is the economic interpretation of this equation?

2. Draw the phase diagram for $\theta(t)$ and $u(t)$, using the other dynamic equation below together with your answer to (1) above,

   $$\frac{\dot{\theta}(t)}{\theta(t)} = \frac{1}{1-\eta}\left[\rho + \lambda + \beta\theta(t)^{\eta} - \frac{1-\beta}{k}\left(y - b\right)\theta(t)^{\eta-1}\right]. \tag{2.6}$$

   What happens to the phase diagram if a positive technology shock increases y?
   (Hint: use implicit differentiation to determine $\partial\theta(t)/\partial y$ when $\dot{\theta}(t) = 0$)
   (Note: equation (2.6) admits a single solution $\theta^* > 0$, you do not need to compute it)

3. What are the channels through which a shock to productivity propagates in the model?