Designing QE in a fiscally sound monetary union

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[1] The views expressed in this paper do not necessarily reflect those of the ECB.

Overview of the presentation

1. Motivation
2. The model
3. Results
4. Conclusion

Main points of the paper

- Consider a tractable model of a **monetary union (with potentially asymmetric member countries**) in which the single short-term rate reaches the lower bound constraint
- How to **design QE?** (Portfolio composition? Risk Sharing?)
- **Goal:** replicate the allocations and welfare levels that would have prevailed under an unconstrained Taylor-type interest rate rule
- **Results** depend on (in)completeness of MU:

  Clear-cut results if MU has a **sound fiscal structure**

  Complexities arise if fiscal framework is incomplete
  (*needs future work in a strategic setting*)

EA QE: starting points

Theory : **No obvious theoretical reference point**

(Standard) **Dimension 1: Single economy**
"The problem with QE is it works in practice but it doesn't work in theory" (Ben Bernanke)

(Extra) **Dimension 2: Monetary union**
"...Usually, the fiscal implications are dealt with easily within a one-country framework, between the central bank and the treasury. But in the euro area, there is no European treasury..." (Mario Draghi)

$\rightarrow$ **What is lacking?**
Monetary union models which reconcile **Eggertsson/Woodford** with
1) **Tobin** (portfolio balance channel) and
2) **Mundell** (non-strategic issues) and **Chari/Kehoe** (strategic issues)

EA QE: starting points

Reality (2014): **Monetary Policy**

Inflation at risk to be too low for too long, while MP close to the effective lower bound

What to do?

- **Standard QE recipe** (of stand alone economies)?

  CB to support aggregate demand by purchasing longer-term gov't debt (**portfolio rebalancing**) plus **forward guidance (signalling)**

EA QE: starting points

Reality (2014)**: Many fiscal policies**

Fiscal framework suffers from weak governance of national policies and **no appetite for a fiscal union**

- Very uneven distribution of fiscal space (and since 2010 loss of market access as a reality)
- Missing notion of aggregate fiscal stance (which matters at ZLB)
- Unclear notion of riskiness of national debt
- Absence of area-wide safe (parts of) gov't debt (SBBS; Eurobonds)
- Treaty logic ("no bail out"): government budget constraints to be kept separate

EA QE: starting points

Reality (2014)**: Many fiscal policies**

Spirit of no bail-out idea got modified in the course of IMF-type conditional support:

- Logic for programme countries follows **Farhi/Tirole (2016)**, i.e. if fiscal positions of member countries are very different, **ex post solidarity** is reasonable, but this is different from **unconditional ex-ante risk sharing**

EA QE: Challenges and design issues

- Motivation of EA QE is clear: area-wide inflation outlook
  $\rightarrow$ SAPI-criteria (sustained adjustment in the path of inflation)
- Yet, design of **QE in a (fiscally) incomplete MU is non-trivial**
  $\rightarrow$ it touches on **the critical intersection of MP and FP**
- How to find a compromise between **Stimulus** vs. **Incentives?**
  $\rightarrow$ *Brunnermeier et al (2016) "The euro and the battle of ideas"*

EA QE: Challenges and design issues

How to find a compromise between **Stimulus** vs. **Incentives**?

- **Stimulus-camp: QE needed to boost demand in order to avoid losses from missing the inflation objective**

  Avoidance of these losses is particularly important in a MU, since nominal anchoring is key

- **Incentives-camp: QE is critically seen since it invites for detrimental free-riding of governments**

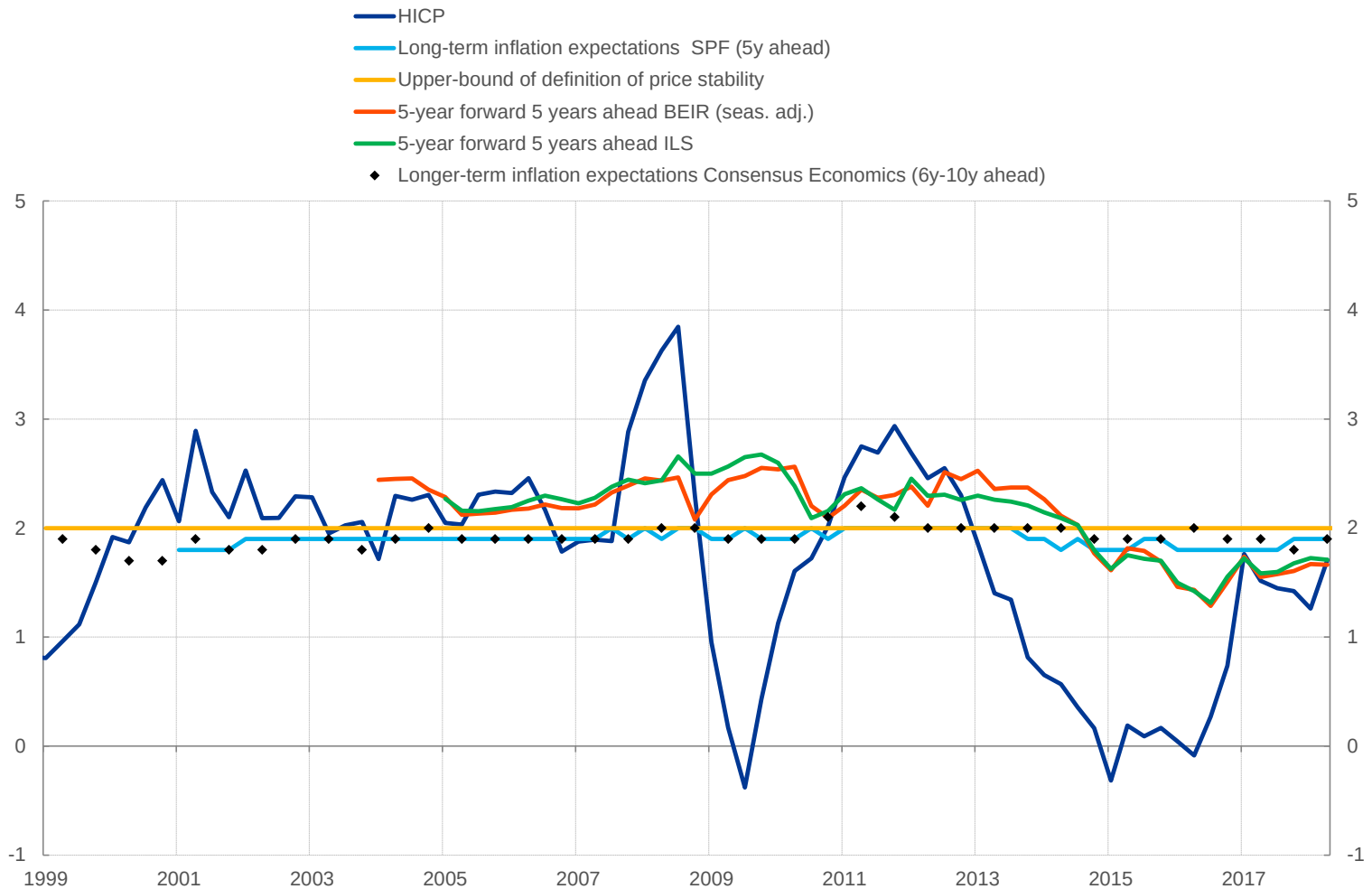
  Erosion of fiscal framework is particularly costly in a MU

  (see: Chari/Kehoe, 2008)

EA QE: Challenges and design issues

Effective compromise is possible:

HICP inflation and inflation expectations (% p.a.)



Sources: ECB, Eurostat and Consensus Economics Forecast.
Quarterly values: 1999Q1 - 2018Q2

EA QE: Challenges and design issues

Effective compromise is possible:

→ Eurosystem has exploited that **QE in a MU is a multidimensional tool** and has been **mindful of incompleteness of EMU**

→ **Key parameters** of PSPP (in addition to *standard* ones, known e.g. from US) carefully calibrated at the boundary of MP and FP

- **(Strongly) limited risk sharing** (singleness of MP vs. incentives for sound national FPs)
- **Portfolio weights** (purchases guided by **capital key**)
- **Issuer and issue limits** (123-related concerns, avoidance of strategic role in debt restructuring)

EA QE: Challenges and design issues

→ **EA QE complements a broad range of other non-standard tools**

- **TLTRO's**: long-term provision of liquidity to banks
- **NIRP**
- (Chained) **Forward guidance**
- **ABSPP, CBPP, CSPP**
- **ELA**: provision of emergency liquidity, no risk sharing
- **OMT**: country-specific support, risk-shared, conditionality

EA QE: design issues

Research agenda:

Role of **key parameters** to be assessed by model-based work which
→ **recognises current trade-offs** (*recall:* **Stimulus vs. Incentives**)
→ allows for **feasible changes of EA architecture over time**

5PR as a reference point for short vs. long-term outcomes:

"...Progress will have to follow a sequence of short- and longer-term steps, but it is vital to establish and agree the full sequence today. The measures in the short-term will only increase confidence now if they are the start of a larger process, a bridge towards a complete and genuine EMU." (5PR)

Example: EA safe assets would affect trade-offs

Our approach

→ Analytics of such agenda are tricky
→ Proceed **stepwise**, use **backward induction**

Step 1 (*Current paper:* "Designing QE in a fiscally sound monetary union")

- Assume, counterfactually, MU has a **complete** fiscal framework
- → How to design EA QE in an extended 2-country monetary union model à la **Benigno (2004)** with

  i) **portfolio balance channel** (s.t. QE works!) and
  ii) **(occasionally) binding lower bound constraint**
  but maintain iii) **standard and stable fiscal feedback rules**

Step 2 (*work in progress*: strategic issues)

- Relax iii) and reconsider design of EA QE in an **incomplete fiscal set-up**
- Idea: consider variation à la **Chari/Kehoe (2008)** and allow for Nash vs optimal outcomes, i.e. **expansionary effects of EA QE to be weighted against adverse incentive effects under non-cooperative FP's**

Our approach

Step 3 (*work in progress*: non-strategic issues)

- Use **country-specific QE in normal times** even when interest rates are not constrained
- Idea: create sufficient country-specific instruments in a monetary union, **opposing the shortage of instruments** as described by **Mundell**
- Questions: how to **optimally** design QE in a monetary union above the lower bound? Is the **same welfare level as in a single economy** for all member states possible?

Model benchmark

How to design QE?

Particularly relevant benchmark in a **monetary union**:

- Outcomes that would have been realised if there had been no lower bound constraint on the **common** short-term interest rate

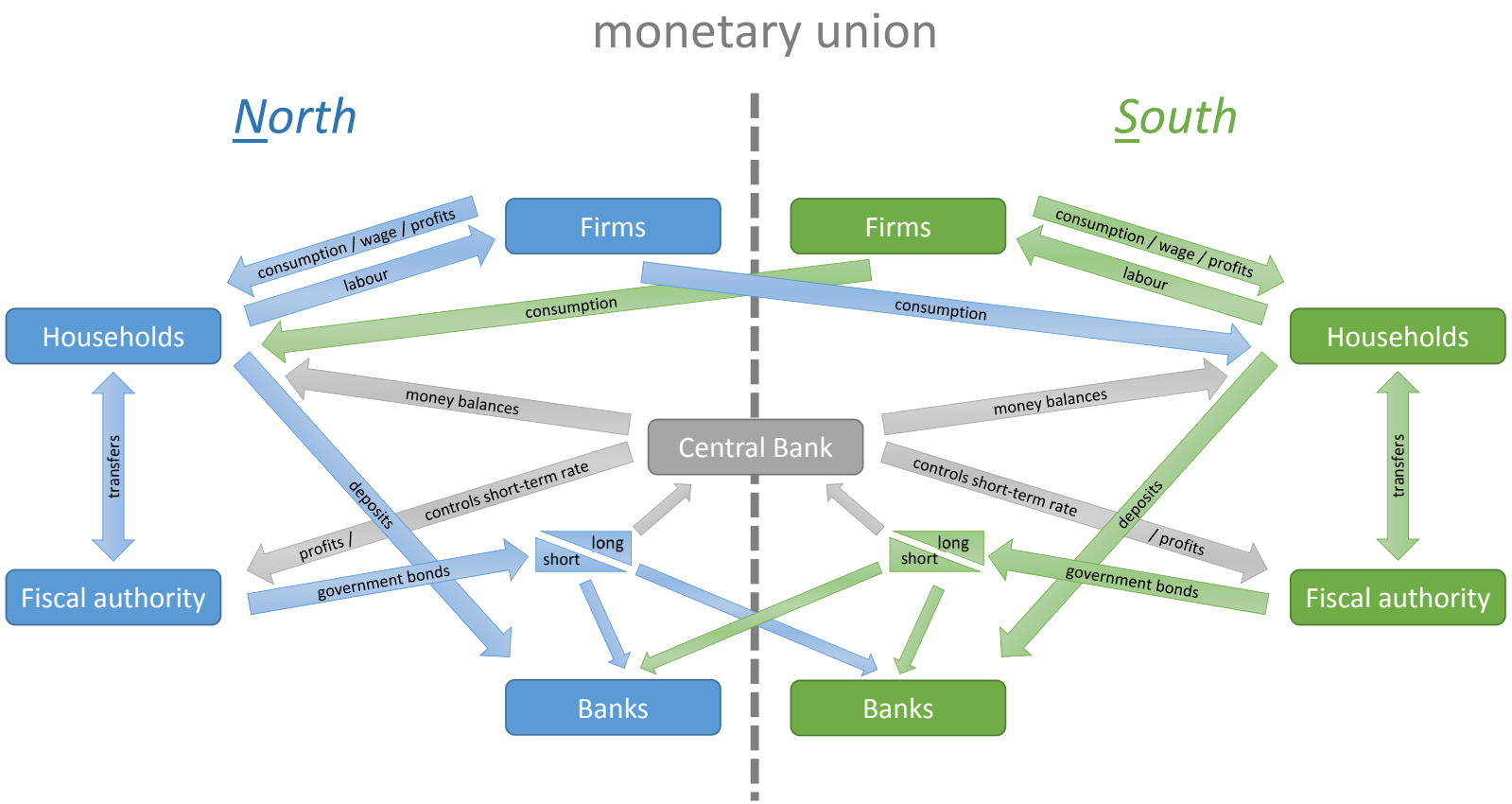
Key features

- Analytical starting point: 3-equation New Keynesian model delivers ineffectiveness result of QE at the ZLB
- We embed this model as a **parametric special case in a 2-country monetary union model with banks**, extending Benigno (2004)

- HH accumulate wealth via **deposits** (with banks) and **real balances**, and consume differentiated goods from both countries (N, S) with home bias
- Banks, acting like mutual funds, invest in **short- and long-term** government bonds of both countries

- Passive fiscal policy: short- and long-term bonds follow **well-behaved** feedback rules

Organigram


monetary union
North
South
Firms
Firms
consumption / wage / profits
labour
consumption
consumption
consumption / wage / profits
labour
Households
Households
money balances
money balances
Central Bank
transfers
transfers
controls short-term rate
controls short-term rate
deposits
deposits
profits /
/ profits
long
short
long
short
government bonds
government bonds
Fiscal authority
Fiscal authority
Banks
Banks

Real effects of QE

- Issue: **irrelevance proposition** of Wallace (1981) and Eggertsson and Woodford (2003)
  $\rightarrow$ QE is ineffective at the lower bound constraint
- Tobin and Brainard (1963) observe **imperfect substitutability**: positive relationship between relative portfolio shares and asset returns

We model the **portfolio balancing channel** via:

1. **imperfect substitutability** between bonds of **different maturities** due to **portfolio adjustment costs** (Harrison, 2012; Andrés et al., 2004), e.g.:
   - preferences ("preferred habitat" à la Vayanos und Vila, 2009)
   - regulation requirements
   - transaction costs
2. further imperfect substitutability between **domestic** and **foreign** long-term bond holdings due to **home bias**

Deposit rate

- Deposits are claims against the bank's portfolio of short- and long-term bonds issued in both countries subject to portfolio adjustment costs and home bias in long-term holdings.

  $\rightarrow$ Rates of return on deposits are **weighted averages** of short-term and long-term rates and thus **heterogeneous** across the union:

$$\hat{R}^{N}_{D,t} = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta}{1+\delta}\left[\omega_N \hat{R}^{N}_{L,t+1} + (1-\omega_N)\hat{R}^{S}_{L,t+1}\right]$$

- Compared with New Keynesian benchmark, **non-negativity of deposit rates replaces ZLB constraint on short-term interest rates**.

Central bank

Stylised balance sheet of the central bank in our monetary union:

Assets		Liabilities	
Short-term bonds	αB_{SC}^N	Money in circulation	αM^N
	$(1-\alpha) B_{SC}^S$		$(1-\alpha) M^S$
Long-term bonds	αQ^N		
	$(1-\alpha) Q^S$		

- **Conventional MP**: short-term Taylor-type interest rate rule (reacting to union-wide inflation rate and output gap)
- Short-term bonds are perfect substitutes to ensure same short-term rate across countries
- **Unconventional MP**: (potentially) country-specific purchases of long-term bonds (**"QE"**)
- Monetary union allows (via TARGET-balances): $B_{SC}^N + Q^N \neq M^N$
  $\rightarrow$ Additional funding channel for $c^N \neq y^N$

Risk sharing

- Current assumption:
  Regular CB income on short-term bond holdings: **shared**
  QE-related CB income on long-term bond holdings: **not shared**

- Deeper analysis of risk sharing requires strategic setting

Symmetric monetary union

- $N = S$
- Model consists of

$$\hat{c}_t^N = \hat{c}_{t+1}^N - \sigma \left[\hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N \right] \tag{1}$$

$$\hat{\pi}_{c,t}^N = \beta \hat{\pi}_{c,t+1}^N + \frac{\varepsilon - 1}{\chi} (\psi + \frac{1}{\sigma}) \hat{c}_t^N \tag{2}$$

$$\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1 - \rho_R) \left[\phi_\pi \hat{\pi}_{c,t}^N + \phi_y \hat{c}_t^N \right] + \varepsilon_{R,t} \tag{3}$$

and

$$\hat{R}_{Dt}^N = \hat{R}_{St} + \widetilde{\nu}_1 \left[\hat{b}_{LPt}^N - \hat{b}_{SPt}^N \right] \tag{4}$$

and further equations

Special case: In the absence of portfolio adjustment costs ($\widetilde{\nu}_1 = 0$), model is isomorphic to New Keynesian 3-equation model:

$\rightarrow$ Eggertsson/Woodford: QE is ineffective, while forward guidance is not

Symmetric monetary union

General case ($\widetilde{\nu}_1 > 0$):

- **Unconstrained interest rate rule outcomes** can be **replicated via QE-augmented policy rule**
- **Caveat:** Initial shock is not too large (such that unconstrained deposit rates remain non-negative: $R^{N*}_{D,t} \geq 1$)
- **QE** remains **effective** until **yield curve becomes flat** (leading in the limit to zero deposit rates)

Intuition for Replicability:

- deposit rates drive dynamics in consumption Euler equation
- use appropriately scaled QE purchases to **replicate unconstrained deposit rates** and, hence, **unconstrained outcomes of all welfare relevant variables**

  $\rightarrow$ *see:* **Proposition 1**

Symmetric monetary union

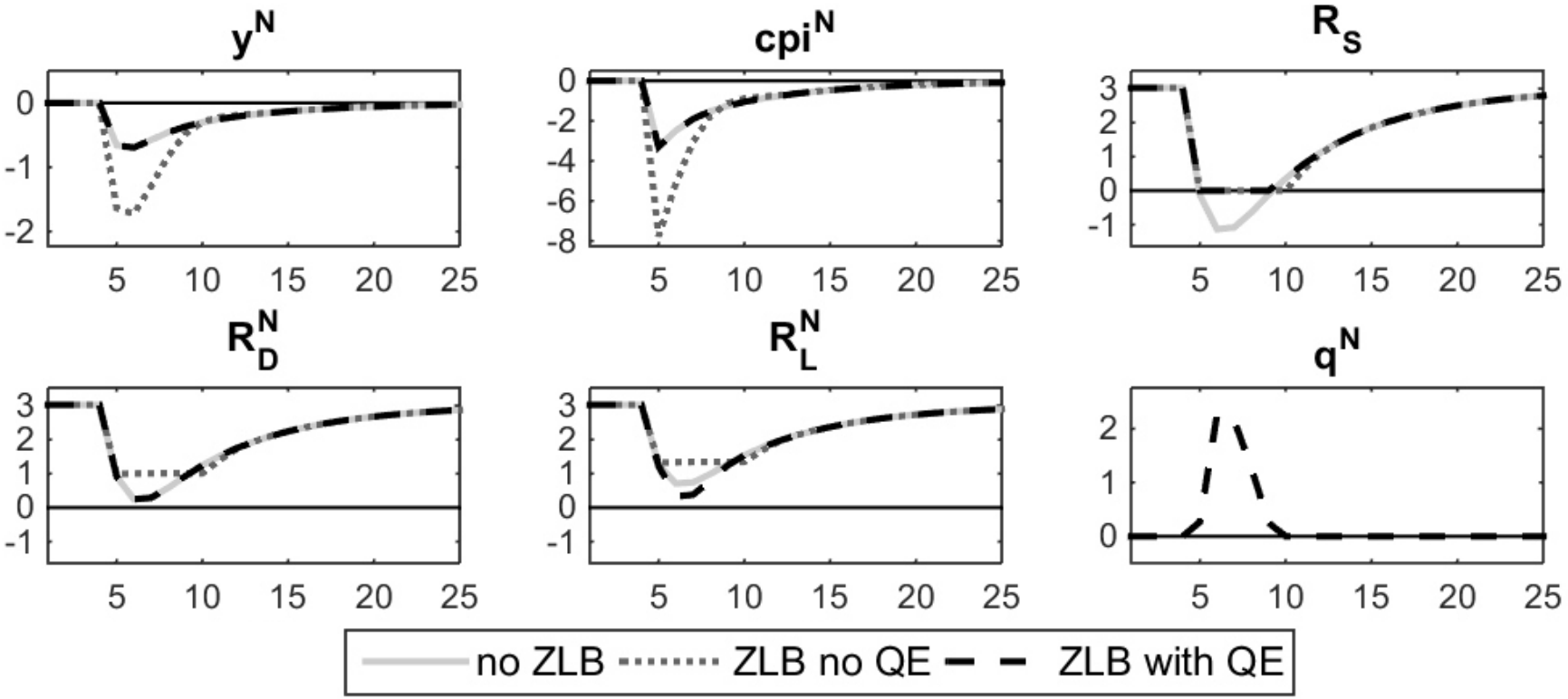
Proposition I: *Consider the equilibrium allocation* $A^{N*} = \left\{\hat{c}_t^{N*}, \hat{h}_t^{N*}, \hat{m}_t^{N*}\right\}_{t=0}^{\infty}$ *of welfare relevant variables in a symmetric monetary union that results from an unconstrained interest rate rule consistent with* $R_{D,t}^{N*} \geq 1$*, leading to a welfare level* W^{N*}*. If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a QE-augmented policy rule which respects the lower bound and replicates* A^{N*} *and, thus,* W^{N*}*.*

Corollary 1: Features of the QE-augmented policy rule:

1. If $R_{S,t}^{*} \geqslant 1$, set $R_{S,t} = R_{S,t}^{*}$ and if $R_{S,t}^{*} < 1$, set $R_{S,t} = 1$
2. For $t < t_1$, set $q_t^N = 0$, while for $t \geqslant t_1$ *set* $q_t^N \geqslant 0$

Symmetric monetary union

Experiment 1: **MU with symmetric shocks and symmetric structures**



Symmetric monetary union

Comment 1: QE augmented policy rule preserves standard assignments of active MP and passive FP even if short-term rate reaches lower bound

Comment 2: For large shocks (s.t. $R_{D,t}^{N*} < 1$), QE becomes ineffective, but forward guidance remains effective (see appendix)

Asymmetric monetary union

- $N \neq S$ in terms of a) **shocks** or b) **structures**
- Additional features: Current account imbalances (funded by CB via **TARGET-balances** or privately by **integrated financial markets**; see appendix)
- QE: CB has two instruments (q_t^N, q_t^S) for asymmetric monetary union:
  $\rightarrow$ **Proposition 1** can be extended to **Proposition 2:**

Asymmetric monetary union

Proposition 2: *Consider the equilibrium allocation of welfare relevant variables, consisting of the pair* $A^{N*} = \left\{\hat{c}_t^{N*}, \hat{h}_t^{N*}, \hat{m}_t^{N*}\right\}_{t=0}^{\infty}$ *and* $A^{S*} = \left\{\hat{c}_t^{S*}, \hat{h}_t^{S*}, \hat{m}_t^{S*}\right\}_{t=0}^{\infty}$, *that results from an unconstrained interest rate rule consistent with* $R_{D,t}^{N*} \geq 1$ *and* $R_{D,t}^{S*} \geq 1$, *leading to welfare levels* W^{N*} *and* W^{S*}. *If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a QE-augmented policy rule which respects the lower bound and replicates* A^{N*} *and* A^{S*} *and, thus,* W^{N*} *and* W^{S*}.

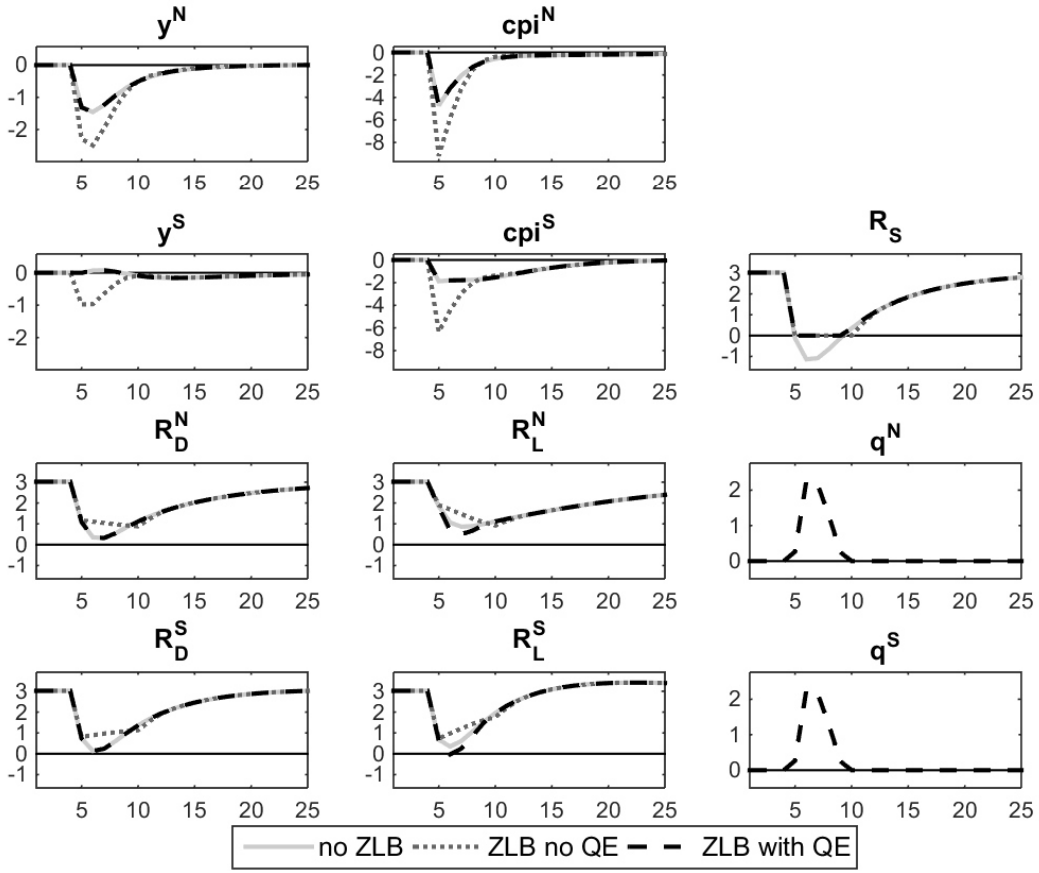
Corollary 2: Features of the QE-augmented policy rule:

1. If $R_{S,t}^* \geqslant 1$, set $R_{S,t} = R_{S,t}^*$ and if $R_{S,t}^* < 1$, set $R_{S,t} = 1$
2. For $t < t_1$ *set* $q_t^N = q_t^S = 0$, while for $t \geqslant t_1$ *set* $q_t^N \geqslant 0$ and $q_t^S \geqslant 0$

Asymmetric monetary union

Experiment 2: **MU with asymmetric shocks, but symmetric structures** (*here:* **homogeneous transmission channel**)

Shock realises only in N:

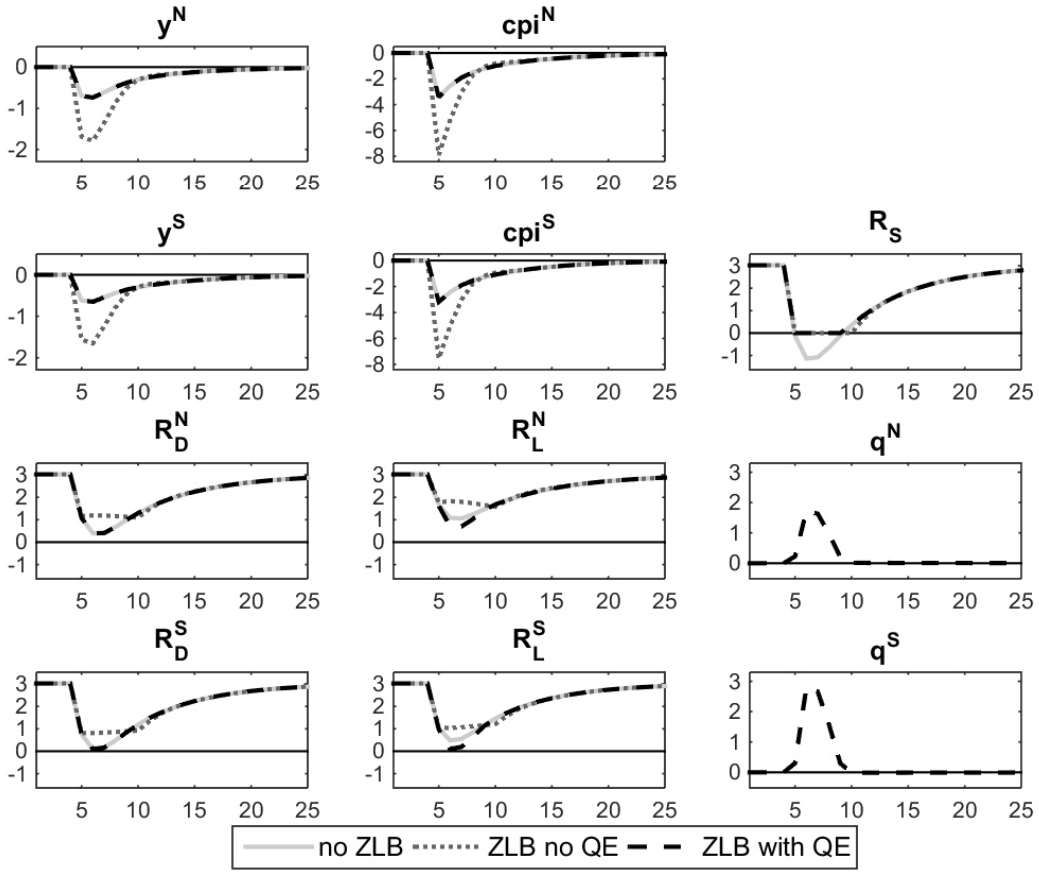


→ purchases with **symmetric** portfolios (= "**capital key**"): $q^S = q^N$

Asymmetric monetary union

Experiment 3: **MU with symmetric shocks, but asymmetric structures** (*here:* **heterogeneous transmission channel**)

Larger home bias in LT bonds in S ($\omega_S > \omega_N$):



→ purchases with **asymmetric** portfolios ($\neq$ "**capital key**"): $q^S > q^N$

Asymmetric monetary union

How to read Experiment 2 vs. 3?

- Lower bound applies symmetrically if structures are symmetric
  → QE according to capital key
- Asymmetric structures create asymmetric private demand patterns for long-term bonds which do not fully realise due to the lower bound
  → Asymmetric QE needs to make up for the asymmetric patterns

Recall: **no scope for opportunistic behaviour by assumption!**
→ **capital key** becomes a **natural margin for QE design under current circumstances**

→ Paper is consistent with the ECB offering a range of distinct facilities, e.g.:
QE: unconditional area-wide stimulus, guided by capital key, to lift inflation
OMT: conditional support for structural reforms, country-specific

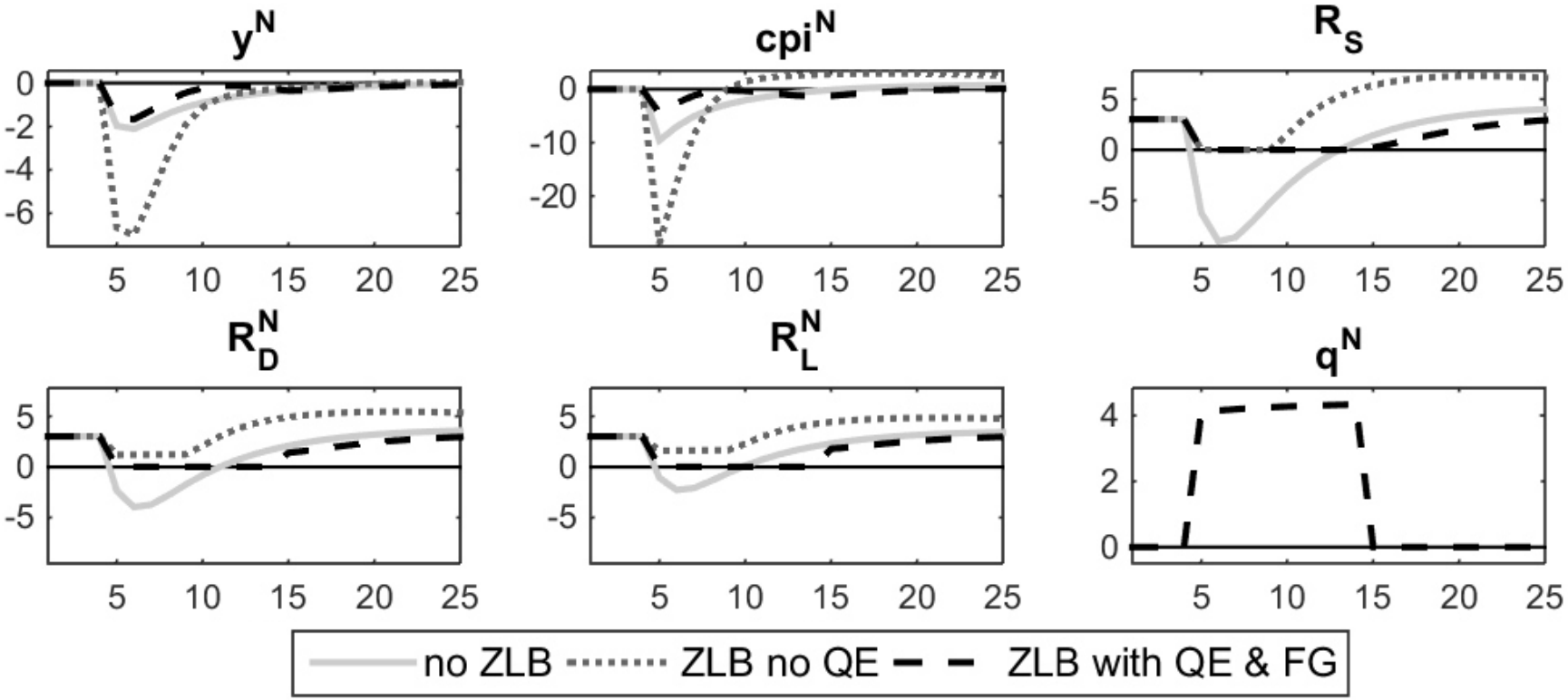
Conclusion

- New Keynesian 3-equation model extended to a 2-country monetary union model with banks
- Effectiveness of QE at the lower bound via portfolio adjustment costs?
  Idea: non-negativity of deposit rates replaces the non-negativity of short term policy rate
- Sound fiscal governance structure:
  QE portfolio of CB can be adjusted to replicate unconstrained outcomes resulting from a standard Taylor-like interest rate rule
- Key modelling challenge: incorporate strategic trade-offs arising from current fiscal incompleteness of EMU
- 1^{st} best: make MP more effective via balanced reforms of EA architecture

Thank you for your attention!

BACKGROUND: Forward guidance

Experiment 4: **Approximating** unconstrained outcomes with QE and FG



BACKGROUND: Households (1)

The representative household in N obtains utility from overall consumption (c^N) and real money balances ($\frac{M^N}{P_c^N}$), and disutility from hours worked (h^N). The country-specific CPI is given by P_c^N.

The lifetime utility function is :

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \phi_t^N \left[\frac{\left(c_t^N - \varsigma c_{t-1}^N\right)^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{\left(h_t^N\right)^{1+\psi}}{1+\psi} + \frac{\chi_m^{-1}}{1-\sigma_m^{-1}} \left(\frac{M_t^N}{P_{c,t}^N} \right)^{1-\sigma_m^{-1}} \right]$$

$$s.t. \quad D_t^N + M_t^N + P_{c,t}^N c_t^N = R_{D,t-1}^N D_{t-1}^N + M_{t-1}^N + W_t^N h_t^N + \Gamma_t^N$$

- Variables denoted in per-capita terms (sizes of N and S are α and $1-\alpha$).
- Nominal variables are deflated with the country-specific consumer price.
- Only N equations are shown. Those for S look symmetrical (with the exception that the terms of trade T_t take the opposite sign).

BACKGROUND: Households (2)

The optimality conditions in log-linear terms are:

$$(1-\varsigma\beta)M\hat{U}C_t^N = -\frac{1}{\sigma(1-\varsigma)}\left[\hat{c}_t^N - \varsigma\hat{c}_{t-1}^N\right] + \frac{\varsigma\beta}{\sigma(1-\varsigma)}\left[\hat{c}_{t+1}^N - \varsigma\hat{c}_t^N\right] + \varsigma\beta\hat{r}_{n,t+1}^N$$

$$M\hat{U}C_t^N = M\hat{U}C_{t+1}^N + \left[\hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N\right]$$

$$\psi\hat{h}_t^N = \hat{w}_t^N + M\hat{U}C_t^N$$

$$\hat{m}_t^N = -\sigma_m M\hat{U}C_t^N - \frac{\sigma_m\beta}{1-\beta}\hat{R}_{D,t}^N$$

where the natural rate of interest is defined as $\hat{r}_{n,t}^N \equiv -(\hat{\phi}_{t+1}^N - \hat{\phi}_t^N)$ and follows an exogenous AR(1) process:

$$\hat{r}_{n,t}^N = \rho_r\hat{r}_{n,t-1}^N + \varepsilon_{n,t}^N$$

- $\sigma > 0$ elasticity of intertemporal substitution
- $\psi > 0$ wage elasticity of labor supply
- $\sigma_m > 0$ interest elasticity of money demand
- $\varsigma \in [0,1]$ habit formation in consumption

BACKGROUND: Households (3)

The consumption bundle c^N is assumed to be given by a CES function that consists of domestic c_D^N and foreign goods c_F^N:

$$c^N \equiv \left[\lambda_N^{\frac{1}{\eta}} (c_D^N)^{\frac{\eta-1}{\eta}} + (1-\lambda_N)^{\frac{1}{\eta}} (c_F^N)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

- $\lambda_N \in [0,1]$ share of domestic goods in the consumption basket consumed by the household (a natural index of openness)
- $\eta > 0$ elasticity of substitution between *Domestic* and *Foreign* goods.

Aggregate demand in N (log-linearised already):

$$\hat{y}_t^N = \lambda_N \hat{c}_t^N + (1-\lambda_N)\hat{c}_t^S + \eta(1-\lambda_N)(\lambda_N+\lambda_S)\hat{T}_t$$

- Consumer prices are: $\hat{\pi}_{c,t}^N = \lambda_N \hat{\pi}_{p,t}^N + (1-\lambda_N)\hat{\pi}_{p,t}^S$

BACKGROUND: Firms

In each country, a continuum of monopolistically competitive firms sell their differentiated goods in the domestic and foreign market. Only labour enters the production function (in log-linear terms):

$$\hat{y}_t^N = \hat{h}_t^N$$

The NK Phillips curve features nominal price rigidity à la Rotemberg:

$$\hat{\pi}_{p,t}^N = \beta \hat{\pi}_{p,t+1}^N + \frac{\varepsilon - 1}{\chi} \left[\hat{w}_t^N + (1 - \lambda_H) \hat{T}_t \right]$$

with law of motion for the terms of trade $\left(T_t \equiv \frac{P_{p,t}^S}{P_{p,t}^N} \right)$

$$\hat{T}_t = \hat{T}_{t-1} + \hat{\pi}_{p,t}^S - \hat{\pi}_{p,t}^N$$

BACKGROUND: Banks

In each country, banks accept deposits and invest in short- and long-term bonds of both countries, facing portfolio adjustment costs and home bias for long-term bonds. Short-term bonds are perfect substitutes.

- The profit maximisation is given by:

$$\max \mathbb{E}_t[R_{S,t}B^N_{SP,t} + R^N_{L,t+1}B^N_{LD,t} + R^S_{L,t+1}B^N_{LF,t} - R^N_{D,t}D^N_t - \frac{\nu_1}{2}\left(\delta\frac{B^N_{SP,t}}{B^N_{LP,t}} - 1\right)^2 P^N_{P,t} - \frac{\nu_2}{2}\left(\frac{\omega_N}{1-\omega_N}\frac{B^N_{LF,t}}{B^N_{LD,t}} - 1\right)^2 P^N_{P,t}]$$

$$\text{s.t. } D^N_t = B^N_{SP,t} + B^N_{LP,t}$$

$$B^N_{SP,t} = B^N_{SD,t} + B^N_{SF,t}$$

$$B^N_{LP,t} = B^N_{LD,t} + B^N_{LF,t}$$

The optimality conditions yield (in log-linear terms):

- **Deposit rate:** weighted average of short- and long-term rates

$$\hat{R}^N_{D,t} = \frac{1}{1+\delta}\hat{R}_{S,t} + \frac{\delta}{1+\delta}\left[\omega_N\hat{R}^N_{L,t+1} + (1-\omega_N)\hat{R}^S_{L,t+1}\right]$$

- **Maturity** and **regional spreads:** similarly proportional to portfolio shares

BACKGROUND: Fiscal policy

Fiscal policy requires to finance debt payments (interest+principal) and lump-sum transfers to domestic households using debt and seigniorage.

- Long-term bonds are modelled as consols $B^N_{consols}$ with value V^N with no maturity and one nominal unit as return each period.
- Nominal outstanding long-term debt: $B^N_{LGt} = V^N_t B^N_{consols,t}$
- The return is given by: $R^N_{L,t} = \frac{1+V^N_t}{V^N_{t-1}}$

The government budget constraint is:

$$B^N_{SG,t} + B^N_{LG,t} + S^N_t = R_{S,t-1} B^N_{SG,t-1} + R^N_{L,t} B^N_{LG,t-1} + P^N_{c,t} \tau^N_t$$

The fiscal rules keep the real debt structure constant and determine lump-sum transfers as a stable feedback with $\theta > 0$ (log-linearised):

$$\hat{b}^N_{LGt} = \hat{b}^N_{SGt}$$

$$\frac{\delta}{\bar{b}^N_{LP}} \hat{\tau}^N_t = -\theta \left[\hat{R}_{S,t-1} - \hat{\pi}^N_{c,t} + \hat{b}^N_{SG,t-1} \right]$$

Short-term debt is the clearing residual in the government budget constraint.

BACKGROUND: Monetary policy

The central bank controls the short-term interest rate R_S via a Taylor-like rule which responds to the union-wide aggregates

$$\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1-\rho_R)(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \varepsilon_{R,t}$$

with α being the size of *North* and $1-\alpha$ the size of *South*:

$$\hat{\pi}_{c,t} = \alpha \hat{\pi}^N_{c,t} + (1-\alpha)\hat{\pi}^S_{c,t}$$
$$\hat{y}_t = \alpha \hat{y}^N_t + (1-\alpha)\hat{y}^S_t$$

Standard monetary policy is symmetric, yet unconventional bond purchases can potentially be asymmetric with some functional form:

$$\tilde{q}^N_t = f^N(.) + \varepsilon^N_{q,t}$$

- Seigniorage and income/losses from bond purchases can be distributed according to country size or back to the country of origin.

BACKGROUND: Seigniorage and market clearing

Central bank balance sheet with $M_t = \alpha M_t^N + (1-\alpha) M_t^S$:

$$M_t = \alpha \left(B_{SC,t}^N + Q_t^N \right) + (1-\alpha) \left(B_{SC,t}^S + Q_t^S \right)$$

Aggregate seigniorage in N is then determined by:

$$\begin{aligned} \alpha S_t^N = \quad & (1-(1-\alpha)\mu_1)\,(R_{S,t-1}-1)\alpha B_{SC,t-1}^N + \alpha\mu_1 (R_{S,t-1}-1)(1-\alpha) B_{SC,t-1}^S \\ & + (1-(1-\alpha)\mu_2)\,(R_{L,t}^N-1)\alpha Q_{t-1}^N + \alpha\mu_2 (R_{L,t}^S-1)(1-\alpha) Q_{t-1}^S \end{aligned}$$

- $\mu_1 \in [0,1]$ degree of income/loss sharing from regular seigniorage
- $\mu_2 \in [0,1]$ degree of income/loss sharing from QE bond purchases

Market clearing on the bond markets implies in each country:

- Short-term bonds: $B_{SG,t}^N = B_{SD,t}^N + \frac{1-\alpha}{\alpha} B_{SF,t}^S + B_{SC,t}^N$
- Long-term bonds: $B_{LG,t}^N = B_{LD,t}^N + \frac{1-\alpha}{\alpha} B_{LF,t}^S + Q_t^N$

BACKGROUND: Current account

Current account $P^N_{p,t}\Omega^N_t = P^N_{c,t}c^N_t - P^N_{p,t}[y^N_t - \Xi^N_t]$ funded via five channels:

$$
\begin{aligned}
P^N_{p,t}\Omega^N_t = & \frac{1-\alpha}{\alpha}\left[M^S_t - M^S_{t-1} - (B^S_{SC,t} - B^S_{SC,t-1}) - (Q^S_t - Q^S_{t-1})\right] \\
& + \mu_1(1-\alpha)(R_{S,t-1}-1)\left[B^S_{SC,t-1} - B^N_{SC,t-1}\right] \\
& + \mu_2(1-\alpha)\left[(R^S_{L,t}-1)Q^S_{t-1} - (R^N_{L,t}-1)Q^N_{t-1}\right] \\
& + \frac{1-\alpha}{\alpha}\left[B^S_{SF,t} - R_{S,t-1}B^S_{SF,t-1}\right] - \left[B^N_{SF,t} - R_{S,t-1}B^N_{SF,t-1}\right] \\
& + \frac{1-\alpha}{\alpha}\left[B^S_{LF,t} - R^N_{L,t}B^S_{LF,t-1}\right] - \left[B^N_{LF,t} - R^S_{L,t}B^N_{LF,t-1}\right]
\end{aligned}
$$

1. new money holdings in S exceed new money creation in S
2. If CB income shared across union:
   a) more regular seigniorage generated in S than in N
   b) more QE income generated in S than in N
3. If financial markets integrated:
   a) Banks in S buy more new short-term debt issued in N than vice versa
   b) Banks in S buy more new long-term debt issued in N than vice versa

BACKGROUND: Calibration

Parameter	Value	Description
α	0.5	Relative country size of *North*
λ_N	0.8	Home bias of consumption in *North*
ω_N	0.7	Home bias of bonds in *North*
η	1.0	Substitutability of domestic and foreign goods
β	0.9925	Household discount factor
σ	6.0	Elasticity of inter-temporal substitution
ς	0.7	Habit formation parameter in consumption
ψ	2.0	Frisch elasticity of labour supply
σ_m	1.0	Interest elasticity of money demand
ε	5.0	Elasticity of substitution across goods
χ	28.65	Price adjustment cost parameter
ν_1	0.0038	Short-long portfolio balance cost parameter
ν_2	0.0127	Domestic-foreign portfolio balance cost parameter
θ	0.5	Adjustment parameter in the fiscal transfer rule
μ_1	1.0	Degree of income sharing from seigniorage
μ_2	0.0	Degree of income sharing from bond purchases
ϕ_π	1.5	Inflation coefficient in the interest rate rule
ϕ_y	0.5	Output coefficient in the interest rate rule
ρ_R	0.5	Smoothing parameter in the interest rate rule
ρ_n	0.85	Smoothing parameter for the natural rate
$\bar{T}$	1.0	Steady state of the terms of trade
$\bar{m}_b$	0.2	Steady state ratio of money to short-term bonds
$\bar{b}^N_{LP}$	0.6	Steady state ratio of long-term bonds to output
δ	3.0	Steady state ratio of long- to short-term bonds