Designing QE in a fiscally sound monetary union

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\textsuperscript{1}The views expressed in this paper do not necessarily reflect those of the ECB.
Overview of the presentation

1. Motivation
2. The model
3. Results
4. Conclusion
Consider a tractable model of a **monetary union (with potentially asymmetric member countries)** in which the single short-term rate reaches the lower bound constraint

**Goal:** replicate the allocations and welfare levels that would have prevailed under an unconstrained Taylor-type interest rate rule

**Results** depend on (in)completeness of MU:

- **Clear-cut results** if MU has a **sound fiscal structure**
- **Complexities arise if fiscal framework is incomplete** *(needs future work in a strategic setting)*
**Theory**: No obvious theoretical reference point

(Standard) **Dimension 1: Single economy**

"The problem with QE is it works in practice but it doesn’t work in theory" (Ben Bernanke)

(Extra) **Dimension 2: Monetary union**

"... Usually, the fiscal implications are dealt with easily within a one-country framework, between the central bank and the treasury. But in the euro area, there is no European treasury..." (Mario Draghi)

→ **What is lacking?**

Monetary union models which reconcile Eggertsson/Woodford with

1) **Tobin** (portfolio balance channel) and

2) **Mundell** (non-strategic issues) and **Chari/Kehoe** (strategic issues)
Motivation

Reality (2014): Monetary Policy

Inflation at risk to be too low for too long, while MP close to the effective lower bound

What to do?

- **Standard QE recipe** (of stand alone economies)?
  
  CB to support aggregate demand by purchasing longer-term gov’t debt (**portfolio rebalancing**) plus **forward guidance** (**signalling**)
**Reality** (2014): Many fiscal policies

**Fiscal framework suffers from weak governance of national policies and no appetite for a fiscal union**

- Very uneven distribution of fiscal space (and since 2010 loss of market access as a reality)
- Missing notion of aggregate fiscal stance (which matters at ZLB)
- Unclear notion of riskiness of national debt
- Absence of area-wide safe (parts of) govt’ debt (SBBS; Eurobonds)
- Treaty logic (“no bail out”): government budget constraints to be kept separate
**Reality (2014):** Many fiscal policies

Spirit of no bail-out idea got modified in the course of IMF-type conditional support:

- Logic for programme countries follows Farhi/Tirole (2016), i.e. if fiscal positions of member countries are very different, **ex post solidarity** is reasonable, but this is different from unconditional **ex-ante risk sharing**
Motivation of EA QE is clear: area-wide inflation outlook → SAPI-criteria (sustained adjustment in the path of inflation)

Yet, design of QE in a (fiscally) incomplete MU is non-trivial → it touches on the critical intersection of MP and FP

How to find a compromise between Stimulus vs. Incentives?
→ Brunnermeier et al (2016) "The euro and the battle of ideas"
How to find a compromise between **Stimulus** vs. **Incentives**?

- **Stimulus-camp**: QE needed to boost demand in order to avoid losses from missing the inflation objective
  
  Avoidance of these losses is particularly important in a MU, since nominal anchoring is key

- **Incentives-camp**: QE is critically seen since it invites for detrimental free-riding of governments
  
  Erosion of fiscal framework is particularly costly in a MU
  
  (see: Chari/Kehoe, 2008)
**EA QE: Challenges and design issues**

**Effective compromise is possible:**

HICP inflation and inflation expectations (% p.a.)

![Graph showing HICP inflation and inflation expectations](image-url)

Sources: ECB, Eurostat and Consensus Economics Forecast.
Quarterly values: 1999Q1 - 2018Q2
Effective compromise is possible:

→ Eurosystsem has exploited that QE in a MU is a multidimensional tool and has been mindful of incompleteness of EMU.

→ Key parameters of PSPP (in addition to standard ones, known e.g. from US) carefully calibrated at the boundary of MP and FP.

- (Strongly) limited risk sharing (singleness of MP vs. incentives for sound national FPs)
- Portfolio weights (purchases guided by capital key)
- Issuer and issue limits (123-related concerns, avoidance of strategic role in debt restructuring)
EA QE: Challenges and design issues

→ EA QE complements a broad range of other non-standard tools

- **TLTRO’s**: long-term provision of liquidity to banks
- **NIRP**
- (Chained) **Forward guidance**
- **ABSPP, CBPP, CSPP**
- **ELA**: provision of emergency liquidity, no risk sharing
- **OMT**: country-specific support, risk-shared, conditionality
Research agenda:

Role of key parameters to be assessed by model-based work which → recognises current trade-offs (recall: Stimulus vs. Incentives) → allows for feasible changes of EA architecture over time

5PR as a reference point for short vs. long-term outcomes:

"...Progress will have to follow a sequence of short- and longer-term steps, but it is vital to establish and agree the full sequence today. The measures in the short-term will only increase confidence now if they are the start of a larger process, a bridge towards a complete and genuine EMU.” (5PR)

Example: EA safe assets would affect trade-offs
Our approach

→ Analytics of such agenda are tricky
→ Proceed stepwise, use backward induction

**Step 1 (Current paper: ”Designing QE in a fiscally sound monetary union”)**

- Assume, counterfactually, MU has a complete fiscal framework
- → How to design EA QE in an extended 2-country monetary union model à la Benigno (2004) with
  i) portfolio balance channel (s.t. QE works!) and
  ii) (occasionally) binding lower bound constraint but maintain iii) standard and stable fiscal feedback rules

**Step 2 (work in progress: strategic issues)**

- Relax iii) and reconsider design of EA QE in an incomplete fiscal set-up
- Idea: consider variation à la Chari/Kehoe (2008) and allow for Nash vs optimal outcomes, i.e. expansionary effects of EA QE to be weighted against adverse incentive effects under non-cooperative FP’s
Our approach

Step 3 (work in progress: non-strategic issues)

- Use country-specific QE in normal times even when interest rates are not constrained
- Idea: create sufficient country-specific instruments in a monetary union, opposing the shortage of instruments as described by Mundell
- Questions: how to optimally design QE in a monetary union above the lower bound? Is the same welfare level as in a single economy for all member states possible?
How to design QE?

Particularly relevant benchmark in a monetary union:

- Outcomes that would have been realised if there had been no lower bound constraint on the common short-term interest rate
Key features

- Analytical starting point: 3-equation New Keynesian model delivers ineffectiveness result of QE at the ZLB
- We embed this model as a parametric special case in a 2-country monetary union model with banks, extending Benigno (2004)
- HH accumulate wealth via deposits (with banks) and real balances, and consume differentiated goods from both countries (\(N, S\)) with home bias
- Banks, acting like mutual funds, invest in short- and long-term government bonds of both countries
- Passive fiscal policy: short- and long-term bonds follow well-behaved feedback rules
Organigram
Real effects of QE

- **Issue:** irrelevance proposition of Wallace (1981) and Eggertsson and Woodford (2003)
  → QE is ineffective at the lower bound constraint

- Tobin and Brainard (1963) observe imperfect substitutability: positive relationship between relative portfolio shares and asset returns

We model the portfolio balancing channel via:

1. imperfect substitutability between bonds of different maturities due to portfolio adjustment costs (Harrison, 2012; Andrés et al., 2004), e.g.:
   - preferences (“preferred habitat” à la Vayanos und Vila, 2009)
   - regulation requirements
   - transaction costs

2. further imperfect substitutability between domestic and foreign long-term bond holdings due to home bias
Deposit rate

- Deposits are claims against the bank’s portfolio of short- and long-term bonds issued in both countries subject to portfolio adjustment costs and home bias in long-term holdings.

→ Rates of return on deposits are weighted averages of short-term and long-term rates and thus heterogeneous across the union:

\[ \hat{R}_{D,t}^{N} = \frac{1}{1 + \delta} \hat{R}_{S,t} + \frac{\delta}{1 + \delta} \left[ \omega N \hat{R}_{L,t+1}^{N} + (1 - \omega N) \hat{R}_{L,t+1}^{S} \right] \]

- Compared with New Keynesian benchmark, non-negativity of deposit rates replaces ZLB constraint on short-term interest rates.
Central bank

Stylised balance sheet of the central bank in our monetary union:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds</td>
<td>Money in circulation</td>
</tr>
<tr>
<td>$\alpha B_{SC}^N$</td>
<td>$\alpha M_N^N$</td>
</tr>
<tr>
<td>$(1 - \alpha) B_{SC}^S$</td>
<td>$(1 - \alpha) M_S^S$</td>
</tr>
<tr>
<td>Long-term bonds</td>
<td></td>
</tr>
<tr>
<td>$\alpha Q_N^N$</td>
<td></td>
</tr>
<tr>
<td>$(1 - \alpha) Q_S^S$</td>
<td></td>
</tr>
</tbody>
</table>

- **Conventional MP**: short-term Taylor-type interest rate rule (reacting to union-wide inflation rate and output gap)
- Short-term bonds are perfect substitutes to ensure same short-term rate across countries
- **Unconventional MP**: (potentially) country-specific purchases of long-term bonds (“QE”)
- Monetary union allows (via TARGET-balances): $B_{SC}^N + Q^N \neq M^N$
  $\rightarrow$ Additional funding channel for $c^N \neq y^N$
Risk sharing

- Current assumption:
  Regular CB income on short-term bond holdings: **shared**
  QE-related CB income on long-term bond holdings: **not shared**

- Deeper analysis of risk sharing requires strategic setting
Symmetric monetary union

- $N = S$

- Model consists of

\[
\hat{c}_t^N = \hat{c}_{t+1}^N - \sigma \left[ \hat{R}_{D,t}^N - \hat{\pi}_{c,t+1}^N - \hat{r}_{n,t}^N \right]
\]

(1)

\[
\hat{\pi}_{c,t}^N = \beta \hat{\pi}_{c,t+1}^N + \frac{\varepsilon - 1}{\chi} (\psi + \frac{1}{\sigma}) \hat{c}_t^N
\]

(2)

\[
\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1 - \rho_R) \left[ \phi_\pi \hat{\pi}_{c,t}^N + \phi_y \hat{c}_t^N \right] + \varepsilon_{R,t}
\]

(3)

and

\[
\hat{R}_{D,t}^N = \hat{R}_{S,t} + \tilde{\nu}_1 \left[ \hat{b}_{LP,t}^N - \hat{b}_{SP,t}^N \right]
\]

(4)

and further equations

**Special case:** In the absence of portfolio adjustment costs ($\tilde{\nu}_1 = 0$), model is isomorphic to New Keynesian 3-equation model:

→ Eggertsson/Woodford: QE is ineffective, while forward guidance is not
Symmetric monetary union

General case ($\tilde{\nu}_1 > 0$):

- **Unconstrained interest rate rule outcomes** can be replicated via QE-augmented policy rule
- **Caveat:** Initial shock is not too large (such that unconstrained deposit rates remain non-negative: $R_{D,t}^{N*} \geq 1$)
- **QE** remains effective until **yield curve becomes flat** (leading in the limit to zero deposit rates)

**Intuition for Replicability:**

- deposit rates drive dynamics in consumption Euler equation
- use appropriately scaled QE purchases to replicate **unconstrained deposit rates** and, hence, **unconstrained outcomes of all welfare relevant variables**
  → see: **Proposition 1**
Proposition 1: Consider the equilibrium allocation $A^{N*} = \{\hat{c}^{N*}_t, \hat{h}^{N*}_t, \hat{m}^{N*}_t\}_{t=0}^{\infty}$ of welfare relevant variables in a symmetric monetary union that results from an unconstrained interest rate rule consistent with $R_{D,t}^{N*} \geq 1$, leading to a welfare level $W^{N*}$. If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a QE-augmented policy rule which respects the lower bound and replicates $A^{N*}$ and, thus, $W^{N*}$.

Corollary 1: Features of the QE-augmented policy rule:
1. If $R_{S,t}^{*} \geq 1$, set $R_{S,t} = R_{S,t}^{*}$ and if $R_{S,t}^{*} < 1$, set $R_{S,t} = 1$
2. For $t < t_1$, set $q_t^{N} = 0$, while for $t \geq t_1$ set $q_t^{N} \geq 0$
Symmetric monetary union

Experiment 1: **MU with symmetric shocks and symmetric structures**

![Graphs showing economic variables](image)

- \( y^N \)
- \( \text{cpi}^N \)
- \( R_S \)
- \( R_D^N \)
- \( R_L^N \)
- \( q^N \)

Legend:
- **no ZLB**
- **ZLB no QE**
- **ZLB with QE**

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Motivation

Model

Results

Conclusion
Symmetric monetary union

Comment 1: QE augmented policy rule preserves standard assignments of active MP and passive FP even if short-term rate reaches lower bound

Comment 2: For large shocks (s.t. $R_{D,t}^{N*} < 1$), QE becomes ineffective, but forward guidance remains effective (see appendix)
Asymmetric monetary union

- $N \neq S$ in terms of a) shocks or b) structures

- Additional features: Current account imbalances (funded by CB via TARGET-balances or privately by integrated financial markets; see appendix)

- QE: CB has two instruments ($q_t^N$, $q_t^S$) for asymmetric monetary union:
  → Proposition 1 can be extended to Proposition 2:
**Proposition 2:** Consider the equilibrium allocation of welfare relevant variables, consisting of the pair \( A^{N*} = \{ \hat{c}_t^{N*}, \hat{h}_t^{N*}, \hat{m}_t^{N*} \} \) \( \infty \) and \( A^{S*} = \{ \hat{c}_t^{S*}, \hat{h}_t^{S*}, \hat{m}_t^{S*} \} \) \( t=0 \), that results from an unconstrained interest rate rule consistent with \( R^{N*}_{D,t} \geq 1 \) and \( R^{S*}_{D,t} \geq 1 \), leading to welfare levels \( W^{N*} \) and \( W^{S*} \). If the lower bound constraint on short-term interest rates makes it not feasible to implement this allocation with a conventional policy rule, then there exists a QE-augmented policy rule which respects the lower bound and replicates \( A^{N*} \) and \( A^{S*} \) and, thus, \( W^{N*} \) and \( W^{S*} \).

**Corollary 2:** Features of the QE-augmented policy rule:
1. If \( R^{*}_{S,t} \geq 1 \), set \( R_{S,t} = R^{*}_{S,t} \) and if \( R^{*}_{S,t} < 1 \), set \( R_{S,t} = 1 \)
2. For \( t < t_1 \) set \( q^{N}_t = q^{S}_t = 0 \), while for \( t \geq t_1 \) set \( q^{N}_t \geq 0 \) and \( q^{S}_t \geq 0 \)
Asymmetric monetary union

Experiment 2: **MU with asymmetric shocks, but symmetric structures** (*here: homogeneous transmission channel*)

Shock realises only in $N$:

$\rightarrow$ purchases with **symmetric** portfolios (*“capital key”*): $q^S = q^N$
Experiment 3: **MU with symmetric shocks, but asymmetric structures** (*here: heterogeneous transmission channel*)

Larger home bias in LT bonds in S ($\omega_S > \omega_N$):

$\rightarrow$ purchases with **asymmetric** portfolios (≠ “capital key”): $q^S > q^N$
Asymmetric monetary union

How to read Experiment 2 vs. 3?

- Lower bound applies symmetrically if structures are symmetric → QE according to capital key
- Asymmetric structures create asymmetric private demand patterns for long-term bonds which do not fully realise due to the lower bound → Asymmetric QE needs to make up for the asymmetric patterns

Recall: no scope for opportunistic behaviour by assumption! → capital key becomes a natural margin for QE design under current circumstances

→ Paper is consistent with the ECB offering a range of distinct facilities, e.g.: 
  QE: unconditional area-wide stimulus, guided by capital key, to lift inflation
  OMT: conditional support for structural reforms, country-specific
New Keynesian 3-equation model extended to a 2-country monetary union model with banks

Effectiveness of QE at the lower bound via portfolio adjustment costs?
   Idea: non-negativity of deposit rates replaces the non-negativity of short term policy rate

Sound fiscal governance structure:
   QE portfolio of CB can be adjusted to replicate unconstrained outcomes resulting from a standard Taylor-like interest rate rule

Key modelling challenge: incorporate strategic trade-offs arising from current fiscal incompleteness of EMU

1st best: make MP more effective via balanced reforms of EA architecture
Thank you for your attention!
BACKGROUND: Forward guidance

Experiment 4: **Approximating** unconstrained outcomes with QE and FG
BACKGROUND: Households (1)

The representative household in $N$ obtains utility from overall consumption ($c^N$) and real money balances ($\frac{M^N}{P^N_c}$), and disutility from hours worked ($h^N$). The country-specific CPI is given by $P^N_c$.

The lifetime utility function is:

$$
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \phi^N_t \left[ \left( \frac{c^N_t - \zeta c^N_{t-1}}{1 - \sigma^{-1}} \right)^{1-\sigma^{-1}} - \left( \frac{h^N_t}{1 + \psi} \right)^{1+\psi} + \frac{\chi^{-1}_m}{1 - \sigma^{-1}_m} \left( \frac{M^N_t}{P^N_{c,t}} \right)^{1-\sigma^{-1}_m} \right]
$$

s.t. $D^N_t + M^N_t + P^N_{c,t}c^N_t = R^N_{D,t-1}D^N_{t-1} + M^N_{t-1} + W^N_th^N_t + \Gamma^N_t$

- Variables denoted in per-capita terms (sizes of $N$ and $S$ are $\alpha$ and $1-\alpha$).
- Nominal variables are deflated with the country-specific consumer price.
- Only $N$ equations are shown. Those for $S$ look symmetrical (with the exception that the terms of trade $T_t$ take the opposite sign).
The optimality conditions in log-linear terms are:

\[(1 - \zeta \beta) \hat{MUC}_t^N = -\frac{1}{\sigma(1 - \zeta)} \left[ \hat{c}_t^N - \zeta \hat{c}_{t-1}^N \right] + \frac{\zeta \beta}{\sigma(1 - \zeta)} \left[ \hat{c}_{t+1}^N - \zeta \hat{c}_t^N \right] + \zeta \beta \hat{r}_{n,t+1}^N \]

\[\hat{MUC}_t^N = \hat{MUC}_{t+1}^N + \left[ \hat{R}_{D,t}^N - \hat{r}_{c,t+1}^N - \hat{r}_{n,t}^N \right] \]

\[\psi \hat{h}_t^N = \hat{w}_t^N + \hat{MUC}_t^N \]

\[\hat{m}_t^N = -\sigma_m \hat{MUC}_t^N - \frac{\sigma_m \beta}{1 - \beta} \hat{R}_{D,t}^N \]

where the natural rate of interest is defined as \(\hat{r}_{n,t}^N \equiv -(\hat{\phi}_{t+1}^N - \hat{\phi}_t^N)\) and follows an exogenous AR(1) process:

\[\hat{r}_{n,t}^N = \rho r \hat{r}_{n,t-1}^N + \varepsilon_{n,t}^N \]

- \(\sigma > 0\) elasticity of intertemporal substitution
- \(\psi > 0\) wage elasticity of labor supply
- \(\sigma_m > 0\) interest elasticity of money demand
- \(\zeta \in [0, 1]\) habit formation in consumption
The consumption bundle $c^N$ is assumed to be given by a CES function that consists of domestic $c^N_D$ and foreign goods $c^N_F$:

$$c^N \equiv \left[ \lambda^N \eta \left( c^N_D \right)^{\eta-1} + (1 - \lambda^N) \eta \left( c^N_F \right)^{\eta-1} \right]^{\frac{1}{\eta-1}}$$

- $\lambda^N \in [0, 1]$ share of domestic goods in the consumption basket consumed by the household (a natural index of openness)
- $\eta > 0$ elasticity of substitution between *Domestic* and *Foreign* goods.

Aggregate demand in $N$ (log-linearised already):

$$\hat{y}^N_t = \lambda^N \hat{c}^N_t + (1 - \lambda^N) \hat{c}^S_t + \eta (1 - \lambda^N)(\lambda^N + \lambda^S) \hat{T}_t$$

- Consumer prices are: $\hat{\pi}^N_{c,t} = \lambda^N \hat{\pi}^N_{p,t} + (1 - \lambda^N) \hat{\pi}^S_{p,t}$
BACKGROUND: Firms

In each country, a continuum of monopolistically competitive firms sell their differentiated goods in the domestic and foreign market. Only labour enters the production function (in log-linear terms):

\[ \hat{y}_t^N = \hat{h}_t^N \]

The NK Phillips curve features nominal price rigidity à la Rotemberg:

\[ \hat{\pi}_{p,t}^N = \beta \hat{\pi}_{p,t+1}^N + \frac{\varepsilon - 1}{\chi} \left[ \hat{\pi}_{t}^N + (1 - \lambda_H) \hat{T}_t \right] \]

with law of motion for the terms of trade \( T_t \equiv \frac{P_{S,p,t}}{P_{N,p,t}} \)

\[ \hat{T}_t = \hat{T}_{t-1} + \hat{\pi}_{p,t}^S - \hat{\pi}_{p,t}^N \]
BACKGROUND: Banks

In each country, banks accept deposits and invest in short- and long-term bonds of both countries, facing portfolio adjustment costs and home bias for long-term bonds. Short-term bonds are perfect substitutes.

- The profit maximisation is given by:

\[
\max \mathbb{E}_t \left[ R_{S,t} B_{SP,t}^N + R_{L,t+1} B_{LD,t}^N + R_{L,t+1} B_{LF,t}^N - R_{D,t} D_t^N \right. \\
\left. \quad - \frac{\nu_1}{2} \left( \delta \frac{B_{SP,t}^N}{B_{LP,t}^N} - 1 \right)^2 P_P^N,t - \frac{\nu_2}{2} \left( \frac{\omega_N}{1 - \omega_N} \frac{B_{LF,t}^N}{B_{LD,t}^N} - 1 \right)^2 P_P^N,t \right]
\]

s.t. \( D_t^N = B_{SP,t}^N + B_{LP,t}^N \)

\( B_{SP,t}^N = B_{SD,t}^N + B_{SF,t}^N \)

\( B_{LP,t}^N = B_{LD,t}^N + B_{LF,t}^N \)

The optimality conditions yield (in log-linear terms):

- **Deposit rate:** weighted average of short- and long-term rates

\[
\hat{R}_{D,t}^N = \frac{1}{1 + \delta} \hat{R}_{S,t} + \frac{\delta}{1 + \delta} \left[ \omega_N \hat{R}_{L,t+1}^N + (1 - \omega_N) \hat{R}_{L,t+1}^S \right]
\]

- **Maturity** and **regional spreads:** similarly proportional to portfolio shares
Fiscal policy requires to finance debt payments (interest+principal) and lump-sum transfers to domestic households using debt and seigniorage.

- Long-term bonds are modelled as consols $B_{\text{consols}}^N$ with value $V^N$ with no maturity and one nominal unit as return each period.
- Nominal outstanding long-term debt: $B_{LGt}^N = V_t^N B_{\text{consols},t}^N$.
- The return is given by: $R_{L,t}^N = \frac{1+V_t^N}{V_{t-1}^N}$.

The government budget constraint is:

$$B_{SG,t}^N + B_{LG,t}^N + S_t^N = R_{S,t-1}^N B_{SG,t-1}^N + R_{L,t}^N B_{LG,t-1}^N + P_{c,t}^N \tau_t^N$$

The fiscal rules keep the real debt structure constant and determine lump-sum transfers as a stable feedback with $\theta > 0$ (log-linearised):

$$\hat{b}_{LGt}^N = \hat{b}_{SGt}^N$$

$$\frac{\delta}{\hat{b}_{LP}^N} \hat{\tau}_t^N = -\theta \left[ \hat{R}_{S,t-1}^N - \hat{\tau}_{c,t}^N + \hat{b}_{SG,t-1}^N \right]$$

Short-term debt is the clearing residual in the government budget constraint.
BACKGROUND: Monetary policy

The central bank controls the short-term interest rate $R_S$ via a Taylor-like rule which responds to the union-wide aggregates

$$\hat{R}_{S,t} = \rho_R \hat{R}_{S,t-1} + (1 - \rho_R)(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \varepsilon_{R,t}$$

with $\alpha$ being the size of North and $1 - \alpha$ the size of South:

$$\hat{\pi}_{c,t} = \alpha \hat{\pi}_{c,t}^N + (1 - \alpha) \hat{\pi}_{c,t}^S$$

$$\hat{y}_t = \alpha \hat{y}_t^N + (1 - \alpha) \hat{y}_t^S$$

Standard monetary policy is symmetric, yet unconventional bond purchases can potentially be asymmetric with some functional form:

$$\tilde{q}_t^N = f^N(\cdot) + \varepsilon_{q,t}^N$$

Seigniorage and income/losses from bond purchases can be distributed according to country size or back to the country of origin.
BACKGROUND: Seigniorage and market clearing

Central bank balance sheet with $M_t = \alpha M^N_t + (1 - \alpha) M^S_t$:

$$M_t = \alpha \left( B^N_{SC,t} + Q^N_t \right) + (1 - \alpha) \left( B^S_{SC,t} + Q^S_t \right)$$

Aggregate seigniorage in $N$ is then determined by:

$$\alpha S^N_t = (1 - (1 - \alpha)\mu_1)(R^N_{S,t-1} - 1)\alpha B^N_{SC,t-1} + \alpha \mu_1 (R^N_{S,t-1} - 1)(1 - \alpha) B^S_{SC,t-1} + (1 - (1 - \alpha)\mu_2)(R^N_{L,t-1} - 1)\alpha Q^N_{t-1} + \alpha \mu_2 (R^S_{L,t-1} - 1)(1 - \alpha) Q^S_{t-1}$$

- $\mu_1 \in [0, 1]$ degree of income/loss sharing from regular seigniorage
- $\mu_2 \in [0, 1]$ degree of income/loss sharing from QE bond purchases

Market clearing on the bond markets implies in each country:

- Short-term bonds: $B^N_{SG,t} = B^N_{SD,t} + \frac{1-\alpha}{\alpha} B^S_{SF,t} + B^N_{SC,t}$
- Long-term bonds: $B^N_{LG,t} = B^N_{LD,t} + \frac{1-\alpha}{\alpha} B^S_{LF,t} + Q^N_t$
**BACKGROUND: Current account**

**Current account** $P_{p,t}^N \Omega_t^N = P_{c,t}^N c_t^N - P_{p,t}^N [y_t^N - \Xi_t^N]$ funded via five channels:

$$P_{p,t}^N \Omega_t^N = \frac{1 - \alpha}{\alpha} \left[ M_t^S - M_{t-1}^S - (B_{SC}^S,t - B_{SC}^S,t-1) - (Q_t^S - Q_{t-1}^S) \right]$$

$$+ \mu_1 (1 - \alpha) (R_{S,t-1} - 1) \left[ B_{SC}^S,t-1 - B_{SC}^S,t-1 \right]$$

$$+ \mu_2 (1 - \alpha) \left[ (R_{L,t} - 1) Q_{t-1}^S - (R_{L,t} - 1) Q_{t-1}^N \right]$$

$$+ \frac{1 - \alpha}{\alpha} \left[ B_{SF}^S,t - R_{S,t-1} B_{SF}^S,t-1 \right] - \left[ B_{SF}^N,t - R_{S,t-1} B_{SF}^N,t-1 \right]$$

$$+ \frac{1 - \alpha}{\alpha} \left[ B_{LF}^S,t - R_{L,t} B_{LF}^S,t-1 \right] - \left[ B_{LF}^N,t - R_{L,t} B_{LF}^N,t-1 \right]$$

1. new money holdings in $S$ exceed new money creation in $S$
2. If CB income shared across union:
   a) more regular seigniorage generated in $S$ than in $N$
   b) more QE income generated in $S$ than in $N$
3. If financial markets integrated:
   a) Banks in $S$ buy more new short-term debt issued in $N$ than vice versa
   b) Banks in $S$ buy more new long-term debt issued in $N$ than vice versa
## BACKGROUND: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>Relative country size of <em>North</em></td>
</tr>
<tr>
<td>$\lambda_N$</td>
<td>0.8</td>
<td>Home bias of consumption in <em>North</em></td>
</tr>
<tr>
<td>$\omega_N$</td>
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<td>Home bias of bonds in <em>North</em></td>
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<tr>
<td>$\eta$</td>
<td>1.0</td>
<td>Substitutability of domestic and foreign goods</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9925</td>
<td>Household discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>6.0</td>
<td>Elasticity of inter-temporal substitution</td>
</tr>
<tr>
<td>$\zeta$</td>
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<td>Habit formation parameter in consumption</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.0</td>
<td>Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\sigma_m$</td>
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<td>Interest elasticity of money demand</td>
</tr>
<tr>
<td>$\varepsilon$</td>
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<td>Elasticity of substitution across goods</td>
</tr>
<tr>
<td>$\chi$</td>
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<td>Price adjustment cost parameter</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>0.0038</td>
<td>Short-long portfolio balance cost parameter</td>
</tr>
<tr>
<td>$\nu_2$</td>
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<td>Domestic-foreign portfolio balance cost parameter</td>
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<tr>
<td>$\theta$</td>
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<td>Adjustment parameter in the fiscal transfer rule</td>
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<tr>
<td>$\mu_1$</td>
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<td>Degree of income sharing from seigniorage</td>
</tr>
<tr>
<td>$\mu_2$</td>
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<td>Degree of income sharing from bond purchases</td>
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<tr>
<td>$\phi_\pi$</td>
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<td>Inflation coefficient in the interest rate rule</td>
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<tr>
<td>$\phi_y$</td>
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<td>Output coefficient in the interest rate rule</td>
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<tr>
<td>$\rho_R$</td>
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<td>Smoothing parameter in the interest rate rule</td>
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<tr>
<td>$\rho_n$</td>
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<td>Smoothing parameter for the natural rate</td>
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<tr>
<td>$\bar{T}$</td>
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<td>Steady state of the terms of trade</td>
</tr>
<tr>
<td>$\bar{m}_b$</td>
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<td>Steady state ratio of money to short-term bonds</td>
</tr>
<tr>
<td>$\bar{b}_N^{LP}$</td>
<td>0.6</td>
<td>Steady state ratio of long-term bonds to output</td>
</tr>
<tr>
<td>$\delta$</td>
<td>3.0</td>
<td>Steady state ratio of long- to short-term bonds</td>
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</tbody>
</table>