

Part II  
Money and Public Finance  
Lecture 8  
Zero lower bound issues

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Monetary and Fiscal Policy Issues in General Equilibrium  
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# Motivation

- Since the intensification of the financial crisis in autumn 2008, major central banks are constrained by the zero lower bound on their policy rates
- Consequently, central banks have embarked on non-standard monetary policies, ie policies which go beyond the standard interest-rate channel and operate through the **expansion and composition of CB balance sheets**
- This lecture explains why it is a significant challenge to ensure the effectiveness of non-standard monetary policies
- This is most easily seen by understanding the logic of the Eggertsson-Woodford irrelevance result of open-market operations at the zero bound

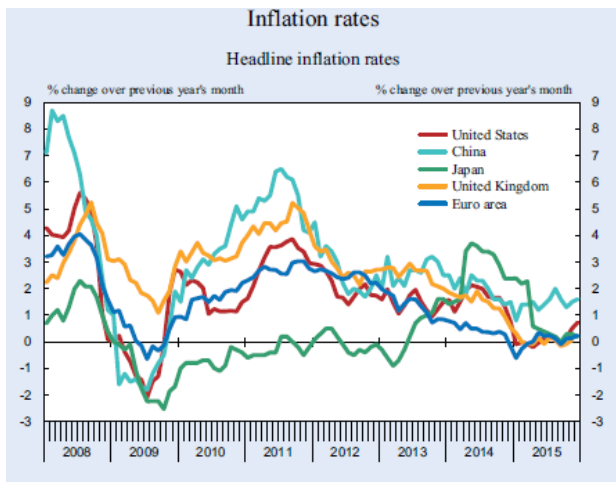
# Motivation

## *Key references:*

- Eggertsson, G. and Woodford, M., The zero bound on interest rates and optimal monetary policy, *Brookings papers on Economic Activity*, 1, 139 - 211, 2003.
- Eggertsson, G. and Woodford, M., Policy options in a liquidity trap, *American Economic Review*, 94/2, 76-79, 2004.
- Curdia, V. and Woodford, M., The Central-Bank Balance Sheet as an Instrument of Monetary Policy, *Journal of Monetary Economics*, 58, 1, 47-74, 2011.

# Motivation

## Inflation rates

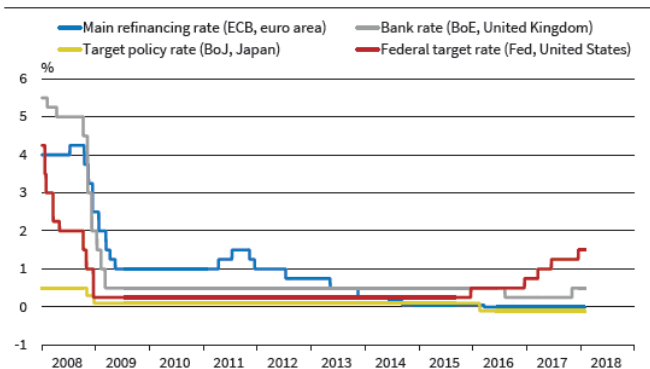


Source: EEAG Report on the European Economy, 2016

# Motivation

## Central bank policy rates

Central Bank Interest Rates



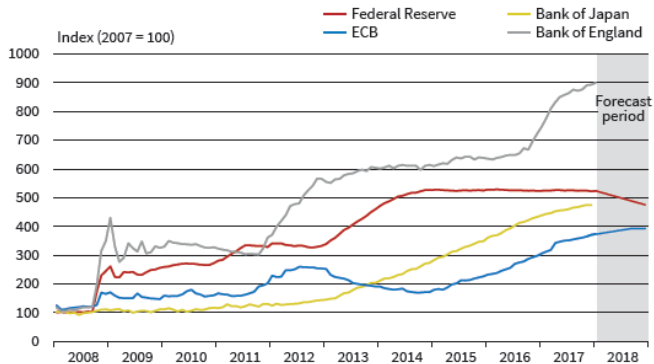
Source: European Central Bank; Federal Reserve Bank of St. Louis; Bank of England; Bank of Japan;  
last accessed on 27 January 2018.

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# Motivation

## Central bank balance sheets

Balance Sheets Sizes of Major Central Banks



Source: Federal Reserve; Bank of Japan; European Central Bank; Bank of England; Swiss National Bank; last accessed on 27 January 2018; EEAG calculations and forecast.

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# Benchmark: Irrelevance of open-market operations

## Overview of the Eggertsson-Woodford model

### MIU model with:

- satiation level of real balances if  $i = 0$
- complete financial markets
- representative agent
- no capital
- nominal price rigidities à la Calvo (1983)

### Monetary policy:

- follows Taylor-rule as long as  $i > 0$
- at  $i = 0$ : monetary policy may switch to a monetary base-supply rule ('quantitative easing')

### Question:

→ Does quantitative easing represent an additional tool of monetary policy at the zero bound ( $i = 0$ )?

# Benchmark: Irrelevance of open-market operations

## Model ingredients

Problem of the representative household:

$$\max E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ u(C_T, M_T/P_T; \xi_T) - \int_0^1 v(H_T(j); \xi_T) dj \right]$$

subject to the **budget constraint**:

$$\begin{aligned} & E_t \sum_{T=t}^{\infty} Q_{t,T} \left[ P_T C_T + \frac{i_T}{1+i_T} M_T \right] \\ & \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T} \left[ \int_0^1 \Pi_T(i) di + \int_0^1 w_T(j) H_T(j) dj - T_T^h \right] \end{aligned}$$

**Dixit-Stiglitz aggregate of consumption** and **Dixit-Stiglitz price index**:

$$\begin{aligned} C_t &= \left[ \int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}} \\ P_t &= \left[ \int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \end{aligned} \quad (1)$$

$\xi_t$  : vector of preference shocks



# Benchmark: Irrelevance of open-market operations

## Model ingredients

**Demand side of equilibrium conditions** (anticipating:  $Y_t = C_t$ ):

**Consumption-Euler equation:**

$$u_c(Y_t, M_t/P_t; \xi_t) = \beta E_t \left[ u_c(Y_{t+1}, M_{t+1}/P_{t+1}; \xi_{t+1})(1 + i_t) \frac{P_t}{P_{t+1}} \right] \quad (2)$$

**Optimal choice of real balances:**

$$\frac{u_m(Y_t, M_t/P_t; \xi_t)}{u_c(Y_t, M_t/P_t; \xi_t)} = \frac{i_t}{1 + i_t}$$

*Assume:*

→ if  $i_t > 0$  this eqn has a unique solution of real balances  $L(Y_t, i_t; \xi_t)$

→ at  $i_t = 0$ : **existence of satiation level**, ie  $L(Y, 0; \xi) = \bar{m}(Y; \xi)$  is the minimum level of real balances at which  $u_m = 0$  so that  $L$  is continuous at  $i = 0$ , leading to the complementary slackness condition:

$$\frac{M_t}{P_t} \geq L(Y_t, i_t; \xi_t) \quad (3)$$

$$i_t \geq 0 \quad (4)$$

$$\left[ \frac{M_t}{P_t} - L(Y_t, i_t; \xi_t) \right] \cdot i_t = 0$$

# Benchmark: Irrelevance of open-market operations

## Model ingredients

**Demand side of equilibrium conditions** (anticipating:  $Y_t = C_t$ ):

**Boundary conditions for optimal expenditure plans:**

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} [u_c(Y_T, M_T/P_T; \xi_T) \cdot Y_T + u_m(Y_T, M_T/P_T; \xi_T) \cdot M_T/P_T] < \infty \quad (5)$$

$$\lim_{T \rightarrow \infty} \beta^{T-t} E_t [u_c(Y_T, M_T/P_T; \xi_T) \cdot D_T/P_T] = 0 \quad (6)$$

→ these eqns have the structure of implementability constraints, ie we have substituted out for prices in the HH budget constraint, using the FOCs and anticipating that the equilibrium stochastic discount factor will be written as (see eq (8) below):

$$Q_{t,T} = \beta^{T-t} \frac{u_c(Y_T, M_T/P_T; \xi_T)}{u_c(Y_t, M_t/P_t; \xi_t)}$$

*notice:*  $D_t = B_t + M_t$  : nominal value of all government liabilities (base money plus government debt) held by the private sector

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Supply side of equilibrium conditions:

**Goods market equilibrium** (for each differentiated good  $i$  in some industry  $j$ ):

$$y_t(i) = A_t \cdot f[h_t(i)]$$

**Nominal profits of supplier of good  $i$ :**

$$\Pi \left[ p_t(i), p_t^j, P_t, Y_t, M_t/P_t; \tilde{\xi}_t \right] = p_t(i) \cdot Y_t \cdot (p_t(i)/P_t)^{-\theta} - w_t(i) \cdot h_t(i)$$

where

$$h_t(i) = f^{-1} [y_t(i)/A_t] = f^{-1} \left[ Y_t \cdot (p_t(i)/P_t)^{-\theta} / A_t \right]$$

and (using the FOC of the optimal labour supply):

$$w_t(i) = \frac{v_h \left( f^{-1} \left[ Y_t \cdot (p_t^j/P_t)^{-\theta} / A_t \right]; \xi_t \right)}{u_c(Y_t, M_t/P_t; \xi_t)} \cdot P_t$$

$\tilde{\xi}_t$ : full vector of disturbances, encompassing both technology shocks and preference shocks ( $\xi_t$ )

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Supply side of equilibrium conditions:

#### Calvo pricing:

$\alpha$  : fraction of industries with unchanged prices

$p_t^*$  : new price, set in all industries allowed to revise prices

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1 \left[ p_t^*, p_t^*, P_T, Y_T, M_T/P_T; \tilde{\xi}_T \right] = 0 \quad (7)$$

#### Stochastic discount factor:

$$Q_{t,T} = \beta^{T-t} \frac{u_c(Y_T, M_T/P_T; \xi_T)}{u_c(Y_t, M_t/P_t; \xi_t)} \quad (8)$$

#### Law of motion of aggregate price index:

$$P_t = \left[ (1 - \alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (9)$$

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Monetary policy:

#### Taylor rule:

$$i_t = \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) \quad (10)$$

→ Assume:  $\phi \geq 0$  for all values of its arguments

→ eq (10) implies a unique path of the monetary base, as long as  $\phi > 0$

→ if  $\phi = 0$ , eq (10) implies only a lower bound on  $M$ , but **effectiveness of ‘quantitative easing’** to be studied by switch to eq (11), ie

#### Monetary base-supply rule:

$$M_t = P_t \cdot L \left( Y_t, \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t); \tilde{\xi}_t \right) \cdot \psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t), \quad (11)$$

where the multiplicative factor  $\psi$  satisfies:

$$\begin{aligned} \psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) &= 1 && \text{if } \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) > 0 \\ \psi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) &\geq 1 && \text{otherwise} \end{aligned}$$

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Monetary policy:

Which assets to be bought by CB when it varies the monetary base ?

→ **non-restrictive assumption**: any of  $k$  different types of securities, distinguished by exogenously given state-contingent returns

**Vector of central bank portfolio shares** in state-contingent assets:

$$\omega_t^m = \omega^m(P_t/P_{t-1}, Y_t; \tilde{\xi}_t) \quad (12)$$

→ components of vector  $\omega_t^m$  sum to 1

→ nominal value of CB holdings of various securities (at the end of period  $t$ ):

$$M_t \cdot \omega_t^m$$

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Asset pricing:

#### State-dependent pay-off structure of securities:

A vector of asset holdings  $\mathbf{z}_{t-1}$  at the end of period  $t-1$  delivers in period  $t$ :

$\mathbf{a}'_t \mathbf{z}_{t-1}$  units of money and

$\mathbf{b}'_t \mathbf{z}_{t-1}$  units of consumption goods and

a vector  $\mathbf{F}_t \mathbf{z}_{t-1}$  of securities tradeable in period- $t$  asset markets

**Gross nominal return on  $j$ th asset** between  $t-1$  and  $t$ :

$$R_t(j) = \frac{a_t(j) + P_t b_t(j) + \mathbf{q}'_t \mathbf{F}_t(\cdot, j)}{q_{t-1}(j)}, \quad (13)$$

where  $\mathbf{q}_t$  is the vector of nominal asset prices in period- $t$  trading

**Absence of arbitrage opportunities** implies:

$$\mathbf{q}'_t = \sum_{T \geq t+1}^{\infty} E_t Q_{t,T} [\mathbf{a}'_T + P_T \mathbf{b}'_T] \prod_{s=t+1}^{T-1} \mathbf{F}_s \quad (14)$$

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Asset pricing:

**Example 1:** *One-period riskless nominal bond*

$b_t(j)$  and  $F_t(\cdot, j)$  are zero in all states, while  $a_t(j)$  is the same in all states

**Example 2:** *One period real (or indexed) bond*

$a_t(j)$  and  $F_t(\cdot, j)$  are zero in all states, while  $b_t(j)$  is the same in all states



# Benchmark: Irrelevance of open-market operations

## Model ingredients

Monetary policy (continued):

**Nominal transfer of CB to Treasury** in period  $t$ :

$$T_t^{cb} = \mathbf{R}'_t \boldsymbol{\omega}_{t-1}^m M_{t-1} - M_{t-1}, \quad (15)$$

assuming that no interest is paid on base money and with the vector of returns  $\mathbf{R}_t$  as defined in eq (13)

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Fiscal policy:

→ Recall:  $D_t = B_t + M_t$  denotes nominal value of all government liabilities (base money  $M_t$  plus government debt  $B_t$ ) held by the private sector

→ Assume: gov't debt can be issued through different types  $j$  of securities with portfolio share  $\omega_j^f$ , implying the gov't budget constraint

$$D_t = \mathbf{R}'_t \boldsymbol{\omega}_{t-1}^f B_{t-1}^S - T_t^{cb} - T_t^h,$$

where  $T_t^h$  denotes the primary surplus and  $B_t^S$  denotes the sum of all government debt held by the private sector and the central bank

→ securities issued by gov't (with weights  $\boldsymbol{\omega}^f$ ) form a subset of those the CB may purchase (with weights  $\boldsymbol{\omega}^m$ )

# Benchmark: Irrelevance of open-market operations

## Model ingredients

### Fiscal policy:

**Fiscal policy rule 1)** determines the **evolution of**  $D_t$ :

$$\frac{D_t}{P_t} = d \left( \frac{D_{t-1}}{P_{t-1}}, \frac{P_t}{P_{t-1}}, Y_t, \tilde{\xi}_t \right) \quad (16)$$

**Fiscal policy rule 2)** specifies the **composition of**  $B_t$ :

$$\omega_t^f = \omega^f \left( \frac{P_t}{P_{t-1}}, Y_t, \tilde{\xi}_t \right) \quad (17)$$

# Benchmark: Irrelevance of open-market operations

## Equilibrium

### Rational expectations equilibrium:

is defined as a set of stochastic processes  $\{p_t^*, P_t, Y_t, i_t, \mathbf{q}_t, M_t, \omega_t^m, D_t, \omega_t^f\}$ ,

satisfying the demand and supply side conditions, the asset-pricing equations and the monetary and fiscal policy specifications listed above and

with each variable specified as a function of the history of exogenous disturbances  $(\tilde{\xi})$

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result:

Instead of the complete set

$$\{p_t^*, P_t, Y_t, i_t, \mathbf{q}_t, M_t, \omega_t^m, D_t, \omega_t^f\}$$

consider the subset

$$\{p_t^*, P_t, Y_t, i_t, \mathbf{q}_t, D_t\},$$

leading to:

### **Proposition (Eggertsson and Woodford, 2003, p. 157):**

The set of paths for the variables  $\{p_t^*, P_t, Y_t, i_t, \mathbf{q}_t, D_t\}$  that are consistent with the existence of a rational expectations equilibrium is independent of the specification of the functions  $\psi$  (eq 11),  $\omega^m$  (eq 12), and  $\omega^f$  (eq 17).

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result:

#### Implication of the Eggertsson-Woodford proposition:

At the zero bound ( $i = 0$ ) the RE equilibrium is independent of:

i) the extent to which quantitative easing is used  
(ie the **length of the CB balance sheet**)

and

ii) the nature of assets that the CB buys through open-market operations  
(ie the **portfolio composition of the CB balance sheet**)

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result:

#### Sketch of the proof, using 2 main insights:

##### **Insight 1** (concerning $\psi$ ):

Each of the equilibrium conditions can be written in a way that no longer makes reference to the money supply

##### **Insight 2** (concerning $\omega^m$ and $\omega^f$ ):

As long as FP determines the path of *total privately held government liabilities*  $D_t/P_t$  through a rule of type eq (16), the portfolio shares  $\omega^m$  and  $\omega^f$  do not matter

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result:

**Background for insight 1** (concerning  $\psi$ ):

$M_t$ , as specified by the monetary base-supply rule (eq 11), affects the equilibrium conditions (2), (5), (6), (7), (8), and (14) through  $M_t/P_t$  in two distinct ways:

$$(i) \quad u_c(Y_t, M_t/P_t; \xi_t) \quad (ii) \quad u_m(Y_t, M_t/P_t; \xi_t) \cdot M_t/P_t$$

**ad term i):** because of satiation, for  $m \geq L(Y, 0; \xi) = \bar{m}(Y; \xi)$  holds

$$u(Y, m; \xi) = u(Y, \bar{m}(Y; \xi); \xi),$$

implying  $u_c(Y, m; \xi) = u_c(Y, \bar{m}(Y; \xi); \xi)$ .

Hence, the term  $u_c(Y_t, M_t/P_t; \xi_t)$  can be replaced against

$$\lambda \left( Y_t, P_t/P_{t-1}, \tilde{\xi}_t \right) \equiv u_c \left( Y_t, L \left( Y_t, \phi(P_t/P_{t-1}, Y_t; \tilde{\xi}_t); \xi_t \right); \xi_t \right),$$

using that eq (3) holds as strict equality at all levels of real balances at which  $u_c$  depends on them. Hence, we can write  $u_c$  independently of  $\psi$ .



# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result:

**Background for insight 1** (concerning  $\psi$ ):

**ad term ii):** Similarly, the term  $u_m(Y_t, M_t/P_t; \xi_t) \cdot M_t/P_t$  can be replaced against

$$\begin{aligned} & \mu \left( Y_t, P_t/P_{t-1}, \tilde{\xi}_t \right) \\ \equiv & u_m \left( Y_t, L \left( Y_t, \phi \left( P_t/P_{t-1}, Y_t; \tilde{\xi}_t \right); \xi_t \right); \xi_t \right) \cdot L \left( Y_t, \phi \left( P_t/P_{t-1}, Y_t; \tilde{\xi}_t \right); \xi_t \right), \end{aligned}$$

since  $M_t/P_t$  must equal  $L \left( Y_t, \phi \left( P_t/P_{t-1}, Y_t; \tilde{\xi}_t \right); \xi_t \right)$  below the satiation level, whereas  $u_m = 0$  otherwise.

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result: Comments and intuition - Remark I

*Does the IRR-result not ignore portfolio balance effects?*

**No**

→ CB can issue base money to buy assets with different risk characteristics

→ Yet, asset pricing theory, *in a GE model with a representative agent*, precludes portfolio effects

→ Why? the overall risk in the economy remains unchanged and agents cannot 'escape' the requirement that private and public sector budget constraints must be consistently linked

→ Crucial for this reasoning to go through: base money offers at the margin no liquidity premium (ie is without non-pecuniary returns). This assumption will be satisfied at the zero bound.

(on this reasoning in the spirit of Modigliani-Miller, see also: *Wallace (1981)*)

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result: Comments and intuition - Remark II

*Does the IRR-result not ignore fiscal effects of open-market operations?*

**No**

- CB can substitute base money for interest-bearing government debt
- Yet, at the zero bound this will not change the path of taxes consistent with intertemporal solvency
  
- Moreover, the evolution of total privately held public sector liabilities  $D_t$  remains unaffected because of the crucial assumption summarized in eq (16)

# Benchmark: Irrelevance of open-market operations

## Irrelevance result

### Irrelevance result: Comments and intuition - Remark III

*Does the IRR-result not contradict findings in the spirit of Auerbach and Obstfeld (2003) where permanent changes to the monetary base are effective at the zero bound?*

**No**

→ The IRR-result stresses that expansionary policies are ineffective **unless they change expectations about the conduct of MP when the zero bound is no longer binding**

→ Under a standard Taylor-rule, for example, the increase in the monetary base would not be permanent, ie it would be reversed as soon as the zero bound is no longer binding

→ *Auerbach and Obstfeld (2003)* in line with the insight that the zero bound constraint, as long as it is binding, can only be mitigated through a **credible commitment regarding an expansionary monetary policy in the future**

# Forward guidance

How to restore effectiveness of monetary policy in the Eggertsson-Woodford economy? **Forward guidance**

→ At the zero bound constraint, effectiveness of monetary policy to be restored through a **credible commitment regarding future monetary policy**, ie monetary policy has to be more expansionary when **the zero bound is no longer binding**

# Forward guidance

**Example:** Zero bound constraint binds for 15 quarters; **Inflation targeting rule** ( $\pi^* = 0$ ) vs. **Optimal policy** (with commitment to stay expansionary after the zero bound ceases to bind)

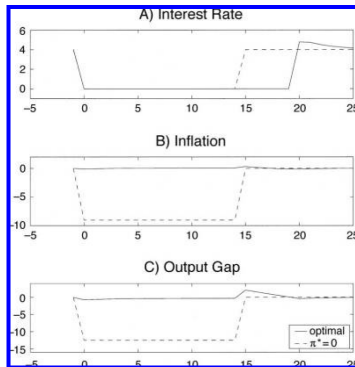


FIGURE 1. RESPONSES TO AN EXOGENOUS DISTURBANCE THAT LOWERS THE NATURAL RATE OF INTEREST FROM 4 PERCENT TO -2 PERCENT

# Forward guidance

## Implications:

→ **Pricel-level targeting rule** is a good proxy for optimal rule, while inflation targeting rule is not (since the latter treats target misses as bygones are bygones)

→ CB purchases of assets with longer maturities can be indirectly effective via **signalling effects** (to the extent that the reversal of such purchases during the exit may be perceived as costly)

→ in general: **forwardlooking communication** is crucial

# Policy options beyond the benchmark economy

**What is needed to depart from Eggertsson-Woodford benchmark economy such that the size and composition of CB balance sheets become relevant?**

**Curdia and Woodford (2011)** offer a minimum set of assumptions:

- Needed is some non-trivial heterogeneity among private agents in terms of financial imperfections such that intermediaries matter for allocation of resources
- Specific assumption: private sector consists of borrowers and savers; financial contracting feasible only via specialized intermediaries
- If private intermediation sufficiently disrupted, CB can step in during emergencies as intermediary and 'credit policy' (ie targeted CB interventions in impaired market segments) becomes an effective tool, alongside (standard) interest rate policy and reserve-supply policy
- Credit policy to be more effective than quantitative easing in its narrow sense (ie expansions of base money used for purchases of sovereign debt)
- Exit considerations from credit policy to be separated from decisions on policy rates