Capital Income Risk and the Dynamics of the Wealth Distribution

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1. Introduction

- In recent years, there has been a rising concern about an increase in inequality, i.e. a concern about changes in wealth distributions over time
- Indirect evidence from the success of Piketty's (2014) "Capital in the Twenty-First Century"

$\Longrightarrow$ Understanding the determinants of the wealth distribution and its evolution over time is of enormous academic and public interest

1. Introduction

Our questions

- How can we explain the *evolution* of wealth distributions?
- Should the density of wealth or wealth shares be targeted?

Our framework

- We work with a partial equilibrium model
- We allow for
    - idiosyncratic labour income risk,
    - *idiosyncratic capital income risk and*
    - *ex-ante heterogeneity in financial ability*

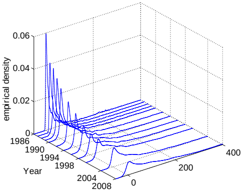


Figure 1 *The evolution of the wealth distribution of the NLSY79 cohort*

1. Introduction

Existing literature related to our questions

- Determinants of a *cross-sectional* distribution of wealth (with a fat right tail)
  - idiosyncratic labour income risk with “superstar state”: Castañeda et al. (2003)
  - idiosyncratic capital income risk: Angeletos (2007), Benhabib, Bisin and Zhu (2011, 2015), Benhabib and Bisin (2018)
- The dynamics of distributions
  - Bayer and Wälde (2010a) study the dynamics of wealth distributions in a Bewley-Huggett-Aiyagari model (without capital income risk)
  - Gabaix et al. (2016) study the dynamics of *income* inequality
  - Kaymak and Poschke (2016) present how top 1%, 5% and 10% wealth shares evolve over time (without capital income risk)
  - Benhabib, Bisin and Luo (2017, sect. 6) study how wealth shares evolve over time (with capital income risk)

1. Introduction

Preview of findings

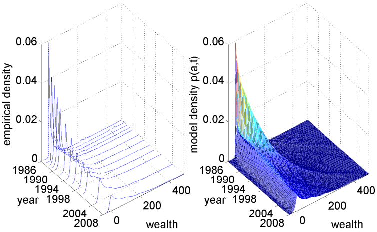


Figure 2 *The evolution of the wealth density in the data and in the model when targeting the 2008 wealth density*

- When targeting the wealth density, the fit is 96.1% and the fit for (non-targeted) Lorenz curve is 92%
- When targeting the Lorenz curve, the fit is 99.5% and the fit for the (non-targeted) wealth density is 74.5%

1. Introduction

Structure of the talk

2. The model
3. Fokker-Planck equations
4. Matching the evolution of wealth densities
5. Robustness checks
6. Concluding remarks

2. The model

- We consider an individual, who
  - faces indiosyncratic labour income risk

$$dz(t) = gz(t)\,dt + [w(\Gamma(t)) - z(t)]\,dq_\mu(t) + [b(\Gamma(t)) - z(t)]\,dq_s(t) \tag{1}$$

  where $\Gamma(t) \equiv \Gamma_0 e^{gt}$,
  - faces idiosyncratic interest rate risk

$$dr(t) = \left[r_{\text{high}} - r(t)\right] dq_{\text{low}}(t) + \left[r_{\text{low}} - r(t)\right] dq_{\text{high}}(t), \tag{2}$$

  where q_μ, q_s, q_{low}, and q_{high} are four Poisson processes with positive arrival rates μ, s, λ^{low}, and λ^{high}, respectively,
  - is financially constrained according to

$$da(t) = \{r(t)\,a(t) + z(t) - c(t)\}\,dt. \tag{3}$$

  - has preferences represented by a CRRA utility function

$$u(c(t)) = \begin{cases} \frac{c(t)^{1-\sigma}-1}{1-\sigma}, & \sigma > 0, \sigma \neq 1, \\ \ln(c(t)), & \sigma = 1. \end{cases} \tag{4}$$

2. The model

- We assume
  - labour income grows at a rate of g (in order to match wealth levels)
  - changes in the interest rate are not anticipated
  - each individual draws the (arrival) rates of the jumping processes from a distribution before becoming economically active $\Rightarrow$ the ex-ante heterogeneity in financial ability (including type dependence and scale dependence as Gabaix et al. (2016))
  - there is minimum consumption level

$$c^{\min}(t) = \xi b(t), \ \xi \in (0,1) \tag{5}$$

  - there is a natural borrowing limit

$$a^{\text{nat}}(t) = -\frac{(1-\xi)\, b(t)}{r_{\text{high}} - g}. \tag{6}$$

2. The model

- Optimal behaviour is described by two Keynes-Ramsey rules, which
  - describe how optimal consumption evolves over time when employed and when unemployed

$$
\begin{aligned}
dc_r^w(a) &= \frac{c_r^w(a)}{\sigma}\left\{r-\rho+s\left[\left(\frac{c_r^w(a)}{c_r^b(a)}\right)^{\sigma}-1\right]\right\}dt \\
&\quad+\left[c_r^b(a)-c_r^w(a)\right]dq_s. \qquad (7a)
\end{aligned}
$$

$$
\begin{aligned}
dc_r^b(a) &= \frac{c_r^b(a)}{\sigma}\left\{r-\rho-\mu\left[1-\left(\frac{c_r^b(a)}{c_r^w(a)}\right)^{\sigma}\right]\right\}dt \\
&\quad+\left[c_r^w(a)-c_r^b(a)\right]dq_\mu. \qquad (7b)
\end{aligned}
$$

2. The model

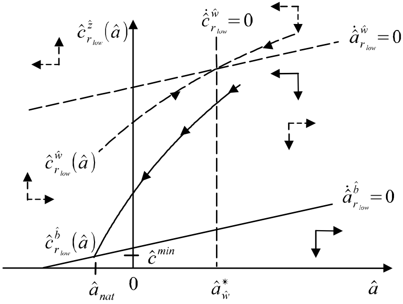
- When we detrend the KRRs, we obtain a two-dimensional ODE system for optimal consumption

$$\frac{d\hat{c}_r^{\hat{w}}(\hat{a})}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g + \frac{s}{\sigma}\left[\left(\frac{\hat{c}_r^{\hat{w}}(\hat{a})}{\hat{c}_r^{\hat{b}}(\hat{a})}\right)^{\sigma} - 1\right]}{(r-g)\,\hat{a} + \hat{w} - \hat{c}_r^{\hat{w}}(\hat{a})}\hat{c}_r^{\hat{w}}(\hat{a}), \tag{8a}$$

$$\frac{d\hat{c}_r^{\hat{b}}(\hat{a})}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g - \frac{\mu}{\sigma}\left[1 - \left(\frac{\hat{c}_r^{\hat{b}}(\hat{a})}{\hat{c}_r^{\hat{w}}(\hat{a})}\right)^{\sigma}\right]}{(r-g)\,\hat{a} + \hat{b} - \hat{c}_r^{\hat{b}}(\hat{a})}\hat{c}_r^{\hat{b}}(\hat{a}). \tag{8b}$$

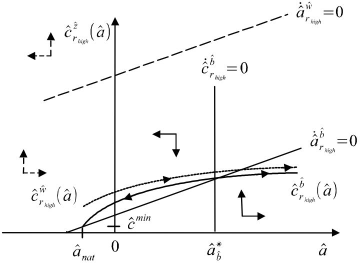
2. The model

- Phase diagrams



Low interest rate regime

$r < \rho + \sigma g$



High interest rate regime

$r > \rho + \sigma g$

3. Fokker-Planck equations

- We are interested in the joint density of wealth and time $g(a, t)$
- We derive a system of two partial differential equations (because of two-state employment status) describing how wealth densities change over time (when employed and when unemployed)
  - If time was fixed, we would obtain the cross-sectional distribution of wealth
  - If the level of wealth was fixed, then the equations would tell us how the share of population owning such a fixed level of wealth or less evolves over time
- These are called *Fokker-Planck equations* (or Kolmogorov forward equations)
- We solve the FPEs by the methods of characteristic (Nagel 2013)

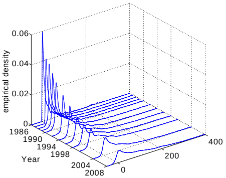
3. Fokker-Planck equations

$$\frac{\partial}{\partial t}p^{\hat{w}}\left(\hat{a},t\right)+\left[\left(r-g\right)\hat{a}+\hat{w}-\hat{c}_r^{\hat{w}}\left(\hat{a}\right)\right]\frac{\partial}{\partial\hat{a}}p^{\hat{w}}\left(\hat{a},t\right)$$
$$=\left[\frac{d\hat{c}_r^{\hat{w}}\left(\hat{a}\right)}{d\hat{a}}-\left(r-g\right)-s\right]p^{\hat{w}}\left(\hat{a},t\right)+\mu p^{\hat{b}}\left(\hat{a},t\right), \tag{9a}$$
$$\frac{\partial}{\partial t}p^{\hat{b}}\left(\hat{a},t\right)+\left[\left(r-g\right)\hat{a}+\hat{b}-\hat{c}_r^{\hat{b}}\left(\hat{a}\right)\right]\frac{\partial}{\partial\hat{a}}p^{\hat{b}}\left(\hat{a},t\right)$$
$$=sp^{\hat{w}}\left(\hat{a},t\right)+\left[\frac{d\hat{c}_r^{\hat{b}}\left(\hat{a}\right)}{d\hat{a}}-\left(r-g\right)-\mu\right]p^{\hat{b}}\left(\hat{a},t\right), \tag{9b}$$

- Remarks
  - $p(\hat{a},t)=p\left(\hat{a},t|\hat{z}=\hat{w}\right)p\left(\hat{z}=\hat{w}\right)+p\left(\hat{a},t|\hat{z}=\hat{b}\right)p\left(\hat{z}=\hat{b}\right)=p^{\hat{w}}\left(\hat{a},t\right)+p^{\hat{b}}\left(\hat{a},t\right)$
  - $\hat{c}_r^i\left(\hat{a}\right)$, $i\in\{\hat{b},\hat{w}\}$, is the optimal consumption resulted from the ODE system (8)
  - The density of wealth with trend is $g\left(a,t\right)=\frac{p(a(t)/\Gamma(t),t)}{\Gamma(t)}$

4. Matching the evolution of wealth densities

- Matching the evolution of wealth densities means
  - starting from an *initial density* and
  - matching a targeted density at a future point in time or
  - matching multiple densities simultaneously
- We use the data on wealth extracted from NLSY79 that contains 12 cross sections of wealth



- Parameters

μ	s	$\hat{w}$	g	$\hat{b}/\hat{w}$	ξ	ρ	σ	r_{low}	r_{high}
21.99%	1.19%	2280.8$	3.4%	30%	97%	1%	1	3.5%	4.5%

4. Matching the evolution of wealth densities

- We target the 2008 empirical density of wealth. How?
  - Use the 1986 empirical density as an initial condition
  - Solve the FPEs (9) for each financial type to obtain two densities $g_j(a, 2008)$ and $g_{j+1}(a, 2008)$ (each financial type might start with low or high interest rate)
  - Attach a probability p_j to each density $g_j(a, 2008)$
  - Choose pobabilities p_j to minimize the area between the model density and the empirical density, i.e. to maximize the following measure of fit

$$F\left(2008\right) = 1 - \frac{\int_{-\infty}^{\infty} \left| g^{\text{model}}\left(a, 2008\right) - g^{\text{data}}\left(a, 2008\right) \right| da}{2}. \qquad (10)$$

where

$$g^{\text{model}}\left(a, 2008\right) = \Sigma_{j=1}^{2n} p_j g_j\left(a, 2008\right),$$

and the integral is the L^1-norm or the total variation norm.

  - Iterate these steps for $n = 2, ..., 131$

4. Matching the evolution of wealth densities

- The optimal number of financial types is 30 (60 interest rate paths)
- Wealth density in 2008 is (almost) perfectly matched by model

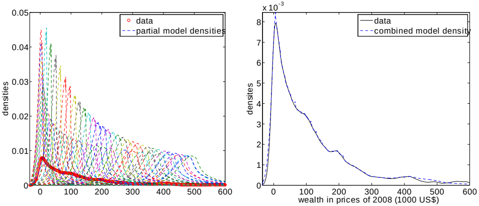


Figure 3 *The wealth distribution in 2008 in the data and in the model*

- The (implied) mean return is 4.3% and the standard deviation is 0.42%
- While the mean return is empirically plausible, the standard deviation seems to be too low

$\Rightarrow$ this baseline calibration seems to "overexplain" the dynamics of the wealth distribution

4. Matching the evolution of wealth densities

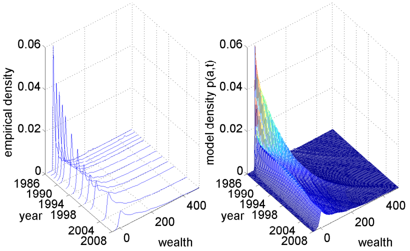


Figure 4 *The evolution of the wealth density in the data and in the model when targeting the 2008 wealth density*

- Quantitative measure of fit

1986	1987	1988	1989	1990	1992	1994	1996	1998	2000	2004	2008
100	73.5	63.0	60.6	61.4	66.3	72.1	77.2	81.9	84.4	87.4	96.1

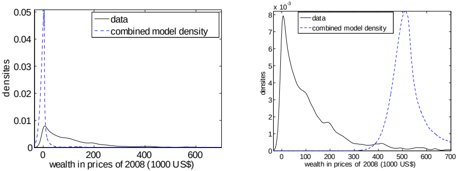
5. Robustness checks

- The goodness of fit is attributed to both labour income risk and capital income risk
- We therefore inquire into the role of pure labour income risk, pure capital income risk and the financial types

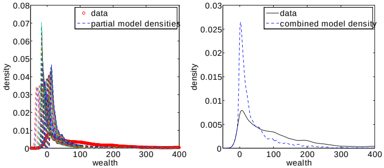
We find

- The fit,
  (i) when there is only labour income risk, is below 30%
  (ii) when there is only capital income risk, is below 66%
  (iii) when there is only capital income risk and when we allow for "awesome" or "superstar" realizations of the interest rate, increases to 96.7%
  (iv) when there is only one financial type (average), is below 60%

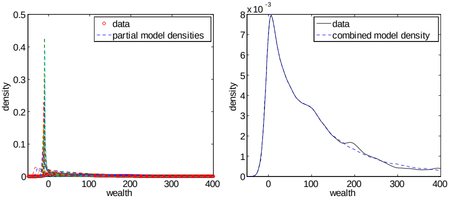
5. Robustness checks



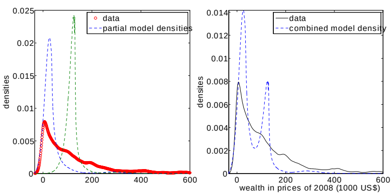
(i)



(ii)



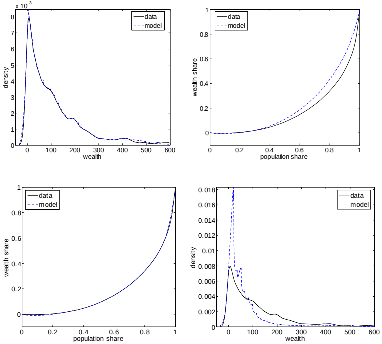
(iii)



(iv)

5. Robustness checks

- When targeting the 2008 density (96.1%), the non-targeted Lorenz curve in the data is matched (92%) by model quite well (top figures)
- But when targeting the Lorenz curve (99.5%), the density fit (74.5%) is not very convincing (bottom figures)
- This shows a strong trade-off between fitting densities and Lorenz curves



6. Concluding remarks

Our questions

- How can we explain the *evolution* of the wealth distribution?
- Should the density of wealth or wealth shares be targeted?

6. Concluding remarks

Our answers

- For an empirically plausible distribution of interest rate, both
    - labour income risk and
    - capital income risk

are essential for explaining the dynamics of the wealth distribution

- With interest rate distributions including *"awesome"* or *"superstar states"*, capital income risk alone can generate extremely good fits for the dynamics of the wealth distribution
- Targeting density seems to produce a better overall fit if one would like to obtain a good fit of both the density and the wealth shares

Thank you!