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Bachelor of Science in Wirtschaftswissenschaften

Wealth distributions - Problem Set 3

Summer 2022

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Excercise 1 - Determine the evolution of k_t

This excercise is aimed at the paper by Bossmann et al. (2007). Moreover, we inted to determine the dynamics of the capital stock k_t. Based on the assumptions, we know that

$$k_{t+1} = \frac{K_{t+1}/n}{L_{t+1}/n} = \frac{\Sigma_{i=1}^{n} s_{it}/n}{\Sigma_{i=1}^{n} l_{it+1}/n}. \tag{1}$$

If we make use of

$$\begin{aligned} n g_t &= \tau \Sigma_{i=1}^{n} b_{it} \\ s_{it} &= (1-\alpha)\left(w_t l_{it} + b_{it} + g_t\right) \\ b_{it+1} &= \frac{(1-\beta) s_{it} \left[1 + r_{t+1}\right]}{1+\tau} \end{aligned}$$

and assume a sufficiently large economy ($n \to \infty$, $\Sigma_{i=1}^{n} l_{it}/n = E\left(l_{it}\right) = 1$) and substituting further for w_t and r_t from

$$r_t = \frac{\partial f(k_t)}{\partial k_t} - \delta = \gamma A k_t^{\gamma-1} - \delta, \tag{2}$$

$$w_t = f(k_t) - k_t \frac{\partial f(k_t)}{\partial k_t} = (1-\gamma) A k_t^{\gamma}, \tag{3}$$

we obtain after rearranging

$$k_{t+1} = c_1 k_t^{\gamma} + c_2 k_t, \tag{4}$$

where

$$\begin{aligned} c_1 &\equiv (1-\gamma\beta)(1-\alpha) A \\ c_2 &\equiv (1-\alpha)(1-\beta)(1-\delta) \end{aligned}$$

Your job is now to prove (4) is correct!

Excercise 2 - Prove $\frac{c_3+c_5}{1-c_4} = k$

Again this excercise deals with the paper by Bossmann et al. (2007). The idea is that students should not only look at the slides but also actively read the paper to answer this question. This excercise aims at proving

$$\frac{c_3 + c_5}{1 - c_4} = \overline{k}.$$

For full disclosure keep in mind

$$\begin{aligned} c_3 &\equiv c_3 \equiv (1-\alpha)\overline{w} \\ c_4 &\equiv \frac{(1-\alpha)(1-\beta)(1+\overline{r})}{1+\tau} \\ c_5 &\equiv \frac{\tau(1-\alpha)(1-\beta)(1+\overline{r})}{1+\tau}\overline{k} \\ \Delta &\equiv \Delta \equiv (1-\alpha)(1-\beta)(1+\overline{r}). \end{aligned}$$

and $\bar{k}$ being the long-run value of k

a) Please show at first that $(1-\Delta)\,\bar{k} = c_3$. In order to do so try to prove $(1-\Delta) = \frac{c_3}{k}$.

1. For the LHS, write down the expression for $1-\Delta$ and insert $\overline{r}$ from equation (16)[1] and rearrange
2. For the RHS, insert $\overline{k}$ from equation (15) and substituting $\overline{w}$ from equation (17) into $(1-\alpha)\,\bar{w}$ and rewrite

b) Use the definitions of c_4 and c_5 from above, rewrite them with the definition of Δ and apply them to the initial equation $\frac{c_3+c_5}{1-c_4}$. Then use the result from before (i.e. $(1-\Delta)\,\bar{k} = c_3$) and rearrange to obtain the final result.

[1] Equation (15),(16) and (17) can be found in the paper.