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Bachelor of Science in Wirtschaftswissenschaften

Wealth distributions - Problem Set 4

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1. Exercise 1-Optimal Consumption

Consider the following objective function,

$$U = \int_t^\infty e^{-\rho(\tau - t)} u\left(c\left(\tau\right)\right) d\tau,$$

and dynamic budget constraint

$$\dot{a}(\tau) = r(\tau)a(\tau) + w(\tau) - c(\tau).$$

Provide an interpretation to these equations.

1. (a) Compute the Keynes-Ramsey rule for the individual's savings problem, that is, find the optimal consumption rule for the problem above by maximising lifetime utility with respect to its budget constraint

   (b) What is the Keynes-Ramsey rule if the instantaneous utility function takes a CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

2. Exercise 2 - Deriving the Pareto distribution

Assuming you have a process for age

$$dx = bdt - xdq_\delta \tag{1}$$

and a process for wealth

$$da_t\left(x\right) = \left(r - \tau - \alpha\right) a_t\left(x\right) dt - \left[a_t\left(x\right) - \bar{a}\right] dq_\delta. \tag{2}$$

As age is *the only* driver of wealth over time, please derive the stationary distribution for wealth.

Exercise 3 - Solving a differential equation

3. (a) Show the general way of solving a differential equation

   (b) Consider the budget constraint for wealth

$$da_t = (r - \tau - \alpha)\, a_t dt - [a_t - \bar{a}] dq_\delta. \tag{3}$$

   Show that

$$a_t = a_0 e^{(r-\tau-\alpha)x_t}$$

   fulfills the budget constraint and, hence, is a valid solution considering living individuals.

   (c) Consider the following differential equation

$$dx(t) = [a(t) - x(t)]dt + c_1(t)x(t)dz_1(t) + g_2(t)dz_2(t)$$

   where $z1$ and $z2$ are two correlated Brownian motions.

   i. What is the solution of this differential equation?

   ii. Use Ito's Lemma to show that your solution is a solution.