Johannes-Gutenberg Universität Mainz

Master in International Economics and Public Policy 1st Semester

# Advanced Macroeconomics

2023/2024 winter term

Klaus Wälde (lecture) and Motoaki Takahashi (tutorial)
www.macro.economics.uni-mainz.de

# Contents

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# 1 Introduction

## 1.1 What is macroeconomics?

- Economic growth
- Business cycles
- Employment and unemployment
- and much more ...
  - Money, nominal rigidities and inflation
  - Central banks
  - Savings and wealth distributions
  - Fiscal policy

## 1.2 Which macroeconomic topics are covered here?

Contents

- Macroeconomic topics
  - economic growth (part I)
  - business cycles (part II)
  - unemployment (part III)
- Behavioural models
  - conflicting motivations (dual self model)
  - time inconsistency
  - stress (if time permits)

Methods

- "Advanced" in "Advanced Macroeconomics" means
  - understanding economic arguments based on economic models and
  - taking psychological reasoning for decision making into account (behavioural economics)

## 1.3 The structure of parts I to III

1. What do we know from data?
   (a) ...
   (b) ...
   (c) Questions for economic theory
2. What theory can explain
   (a) ...
   (b) ...
   (c) What have we learned?
3. How psychological views can contribute
   (a) ...
   (b) ...
   (c) What have we learned?

# Part I
# Economic growth

## 2 The convergence debate

### 2.1 Facts about income levels

- What do we know about income levels (as opposed to growth)?
- What do we know about differences in income levels?
- see e.g. World Development Review by World Bank (2020)

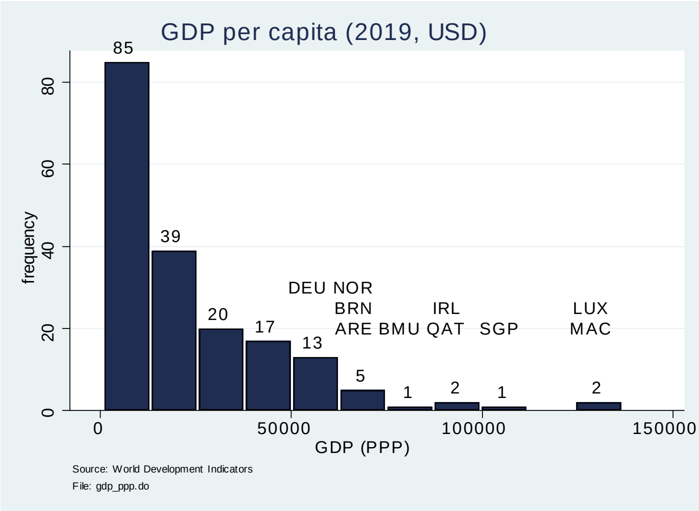



**Figure 1** *GDP per capita in 2019*
*Legend: LUX Luxembourg, MAC Macao China, SGP Singapore, IRL Ireland, QAT Qatar, BMU Bermuda, NOR Norway, BRN Brunei Darussalam, ARE United Arab Emirates, DEU Deutschland*

| | Gross National Income | | Gross National Income, PPP | |
|---|---|---|---|---|
| Country | billon $ | $ per capita | billon $ | $ per capita |
| Macao | 48.8 | 75610 | 75.1 | 117340 |
| Singapore | 329.7 | 58390 | 515.1 | 90320 |
| Bermuda | 7.6 | 117740 | 5.5 | 86450 |
| Switzerland | 736.5 | 87950 | 631.3 | 73620 |
| Germany | 3966.1 | 48600 | 4770.4 | 57410 |
| Turkey | 748.6 | 9690 | 2240.7 | 26860 |
| Mozambique | 15.0 | 490 | 39.8 | 1310 |
| Central African Republic | 2.4 | 520 | 5.0 | 1060 |
| Somalia | 4.9 | 320 | 13.9 | 900 |
| Burundi | 3.0 | 280 | 9.1 | 790 |

**Table 1** *By how much do countries differ in GDP per capita?*
*Factor of (almost) 100 between poorest and richest countries*
*Source: World Development Indicators, 2019*

## 2.2 Facts about income growth

- How do countries evolve over time?

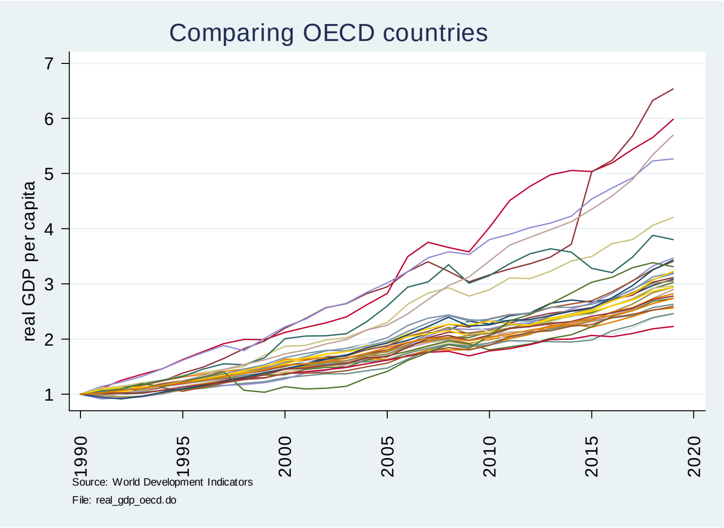



**Figure 2** *Development of real GDP per capita in OECD countries (Organisation for Economic Co-operation and Development – www.oecd.org)*

## 2.3 Is there convergence?

- Question: Is there convergence of income per capita over time?
- Are poor countries catching up?

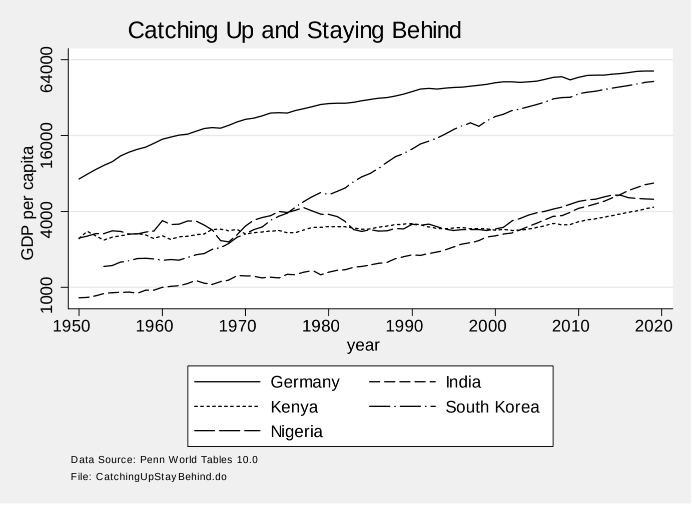



**Figure 3** *Catching up and staying behind of some countries (log (!) of GDP per capita over time)*

- Systematic approach (Baumol, 1986)

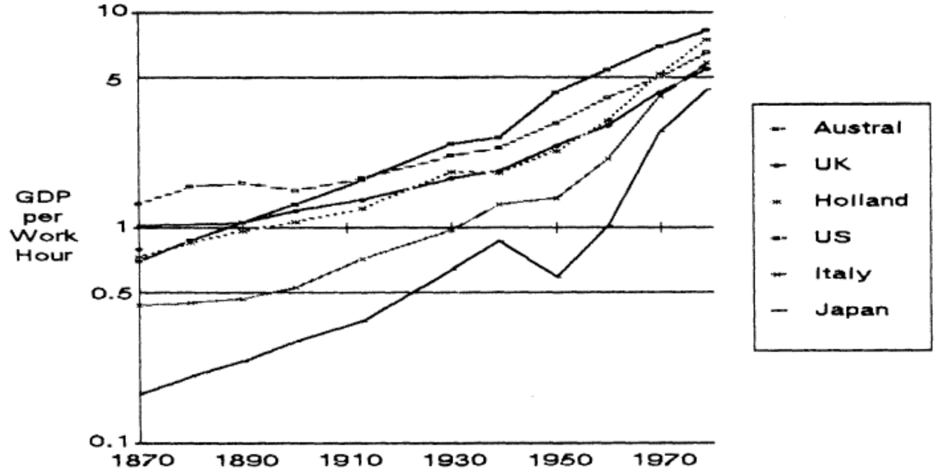



**Figure 4** *Convergence among selected OECD countries (Baumol, 1986, fig. 1)*

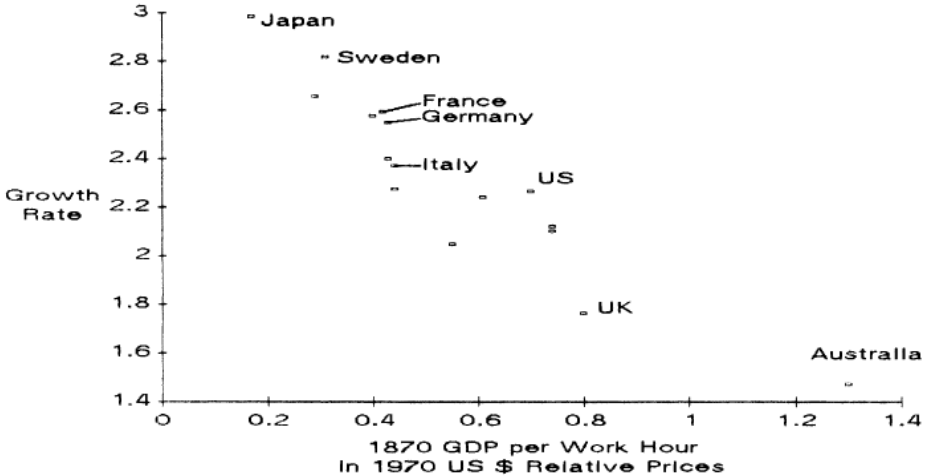



**Figure 5** *Productivity growth rate, 1870 - 1979 vs. 1870 level (Baumol, 1986, fig. 2)*

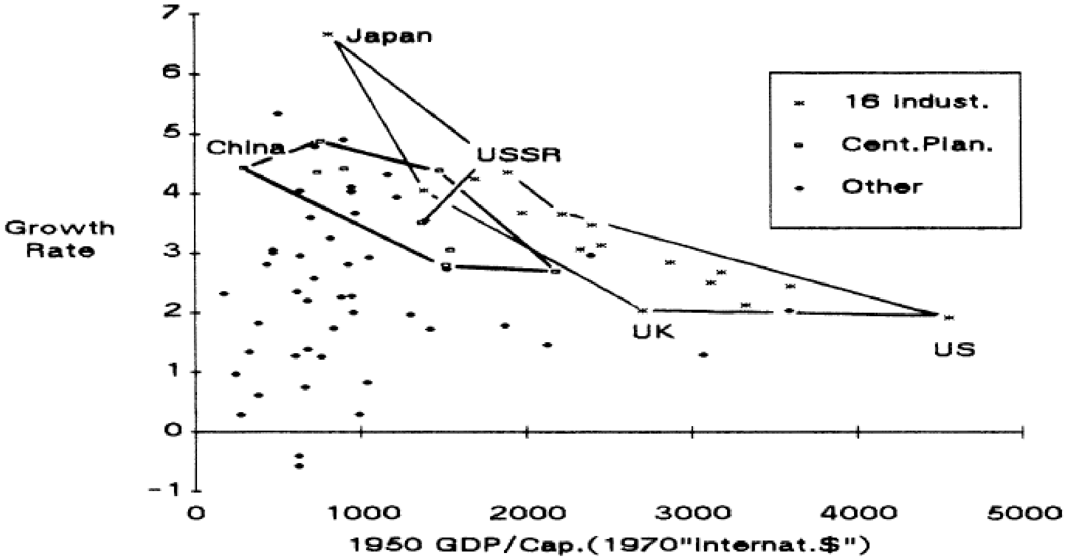



**Figure 6** *Growth rate, 1950 - 1980, Gross domestic product per capita vs. 1950 level, 72 countries (Baumol, 1986, fig. 3)*

- Summary of Baumol findings
  - There is convergence of GDP per capita among industrialized countries
  - Convergence not so clear for centrally planned economies
  - No convergence for less developed countries
- Big subsequent discussion on convergence or not
  - View in 2006 (see introduction in Sala-i-Martin, 2006): no convergence (see fig. Ia below)
  - Once population weights are used, this result disappears (see fig. Ib below)
  - Evidence for convergence reappears for the entire sample of countries

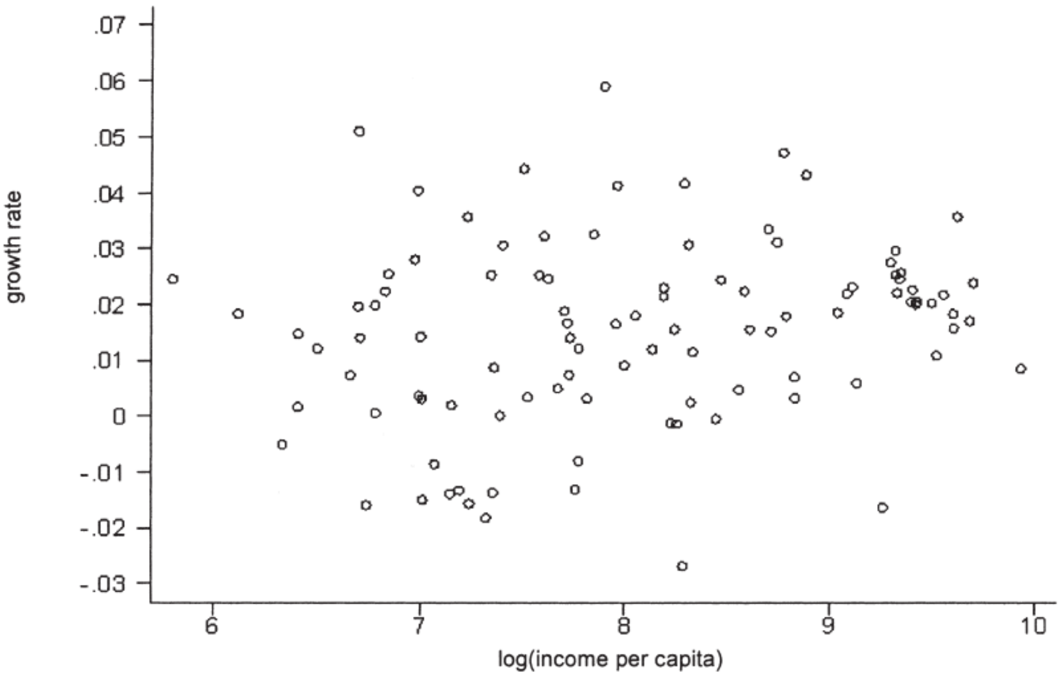



**Figure 7** *Growth vs. initial income (unweighted) (Sala-i-Martin, 2006, fig. Ia)*

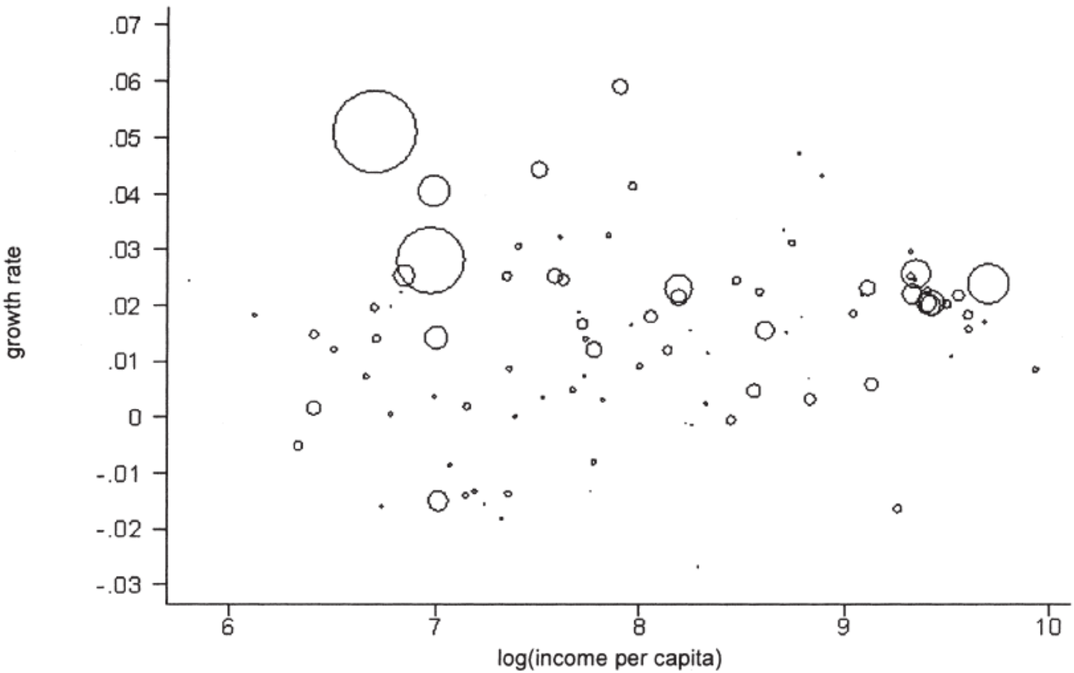



**Figure 8** *Growth vs. initial income (Population-Weighted) (Sala-i-Martin, 2006, fig. Ib)*

- The approach of Sala-i-Martin
  - Main contribution consisted in constructing a world distribution of income (WDI)
  - Using this WDI, he finds convergence of GDP per capita
- Findings on poverty
  - Figure IV shows that absolute poverty (income per capita below 1$ per day) fell from 1970 to 2000
  - What about poverty rates per region? Figure VII shows that it fell for all regions but for Africa
  - What about a more comprehensive measure of the distribution of income? Figure VIII looks at Gini coefficient
  - Inequality of the WDI fell over time but only by very little
  - There is convergence, but very slow

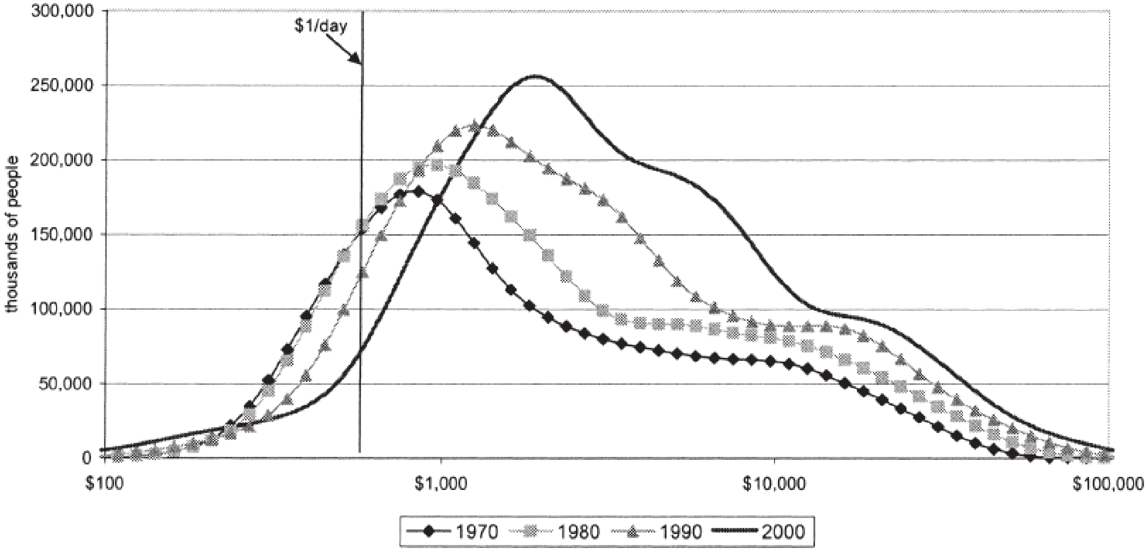



**Figure 9** *The WDI in various years (Sala-i-Martin, 2006, fig. IV)*

- Horizontal axis shows (real) annual income per capita
- Vertical axis counts number of individuals
- Dots, squares, triangles and line show histogram (not density)

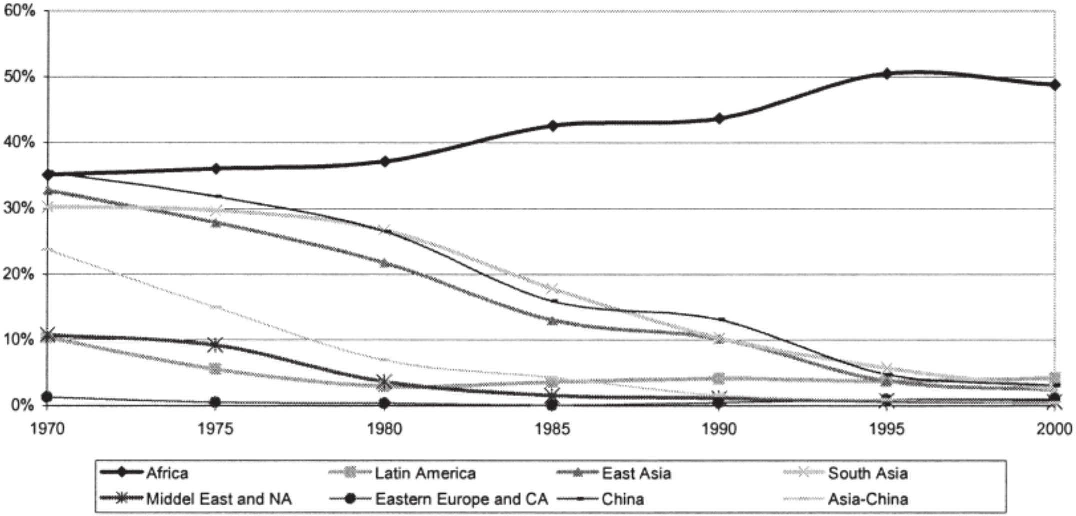



**Figure 10** *Regional poverty rates ($1.5 a day line) (Sala-i-Martin, 2006, fig. VII)*

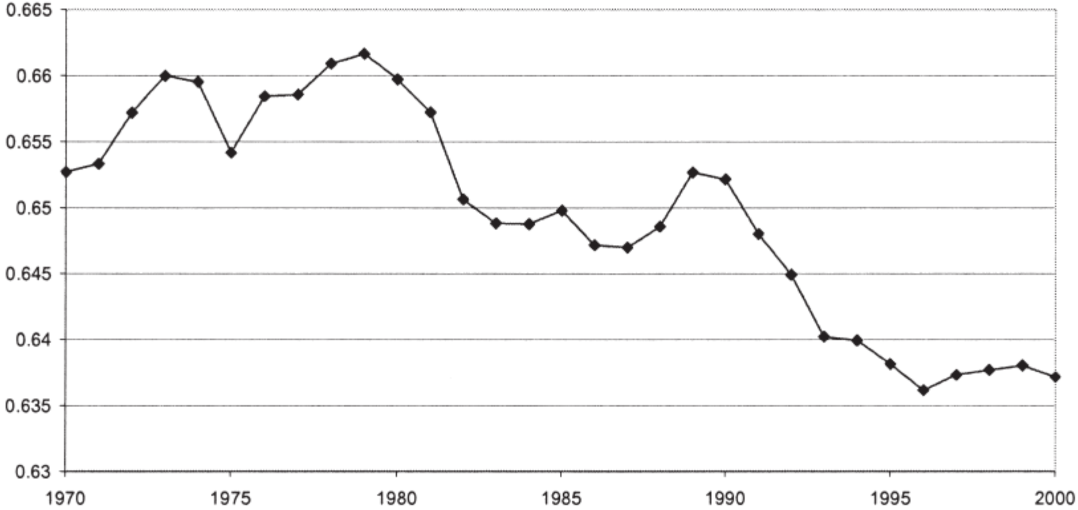



**Figure 11** *World Income Inequality: Gini (Sala-i-Martin, 2006, fig. VIII)*

- What do these findings mean?
  - Very strong (if not irresponsible) conclusion: Policy does not need to do anything (more) with respect to convergence or development cooperation
  - Countries converge, inequality goes down, we just need to wait
  - This could by called cynical given the huge ratios in income per capita
  - My reading: comforting that world moves “in right direction” but more effort would be welcome to speed up the process

## 2.4 Questions for economic theory

- Why are some countries richer than others?
- Why do countries grow?
- Why do some countries grow faster than others?
- Can countries grow faster only temporarily or also permanently?

# 3 Economic growth theory

## 3.1 Neoclassical growth theory

### 3.1.1 Some background

- Exogenous saving rate
  - Solow (1956)
- Optimal saving
  - Cass (1965) and Koopmans (1965)
  - Ramsey (1928)
- Textbook
  - Aghion and Howitt (1998), “Endogenous growth theory”, ch. 1.1 and 1.2

### 3.1.2 The Solow growth model

- Production technology

  - We employ a simple Cobb-Douglas structure

$$Y(t) = K(t)^{\alpha} [A(t) L(t)]^{1-\alpha}$$

    where

  - $K(t)$ is the capital stock ("number of machines") at a
  - point in time $t$
  - $A(t)$ is labour productivity
  - $L(t)$ is the number of workers (headcount or hours worked) and
  - $\alpha$ is the output elasticity of capital where $0 < \alpha < 1$

- Capital accumulation
  - Describing ....
  - .... preferences of households by a constant saving rate $s$ and
  - ... depreciation by a constant depreciation rate $\delta$, net investment is given by

$$\dot{K}(t) \equiv \frac{dK(t)}{dt} = sK(t)^{\alpha} [A(t) L(t)]^{1-\alpha} - \delta K(t)$$

  - Population $L(t)$ (which is identical to the number of workers) grows at a constant rate $n$ and labour productivity $A(t)$ grows at a constant rate $g$,

$$L(t) = L_0 e^{nt}, \quad A(t) = A_0 e^{gt} \tag{3.1}$$

    where
  - $L_0$ is the initial population size (i.e. when at the point in time 0 when we start analysing our economy) and
  - $A_0$ is initial labour productivity

- The dynamics of the economy
  - Analysing the dynamics of this economy is difficult as many variables are growing
  - There is no such thing as a steady state (being defined as constant endogenous variables)
  - We therefore work with an auxiliary variable $\tilde{k}(t)$ such that there is hope that this variable will become constant under certain circumstances
  - This auxiliary variable is defined as

$$\tilde{k}(t) \equiv \frac{K(t)}{A(t)L(t)} \tag{3.2}$$

  - The growth rate of $\tilde{k}$ is (see tutorial 5.1)

$$\frac{d\tilde{k}(t)/dt}{\tilde{k}(t)} = s\tilde{k}(t)^{-(1-\alpha)} - \delta - g - n \tag{3.3}$$

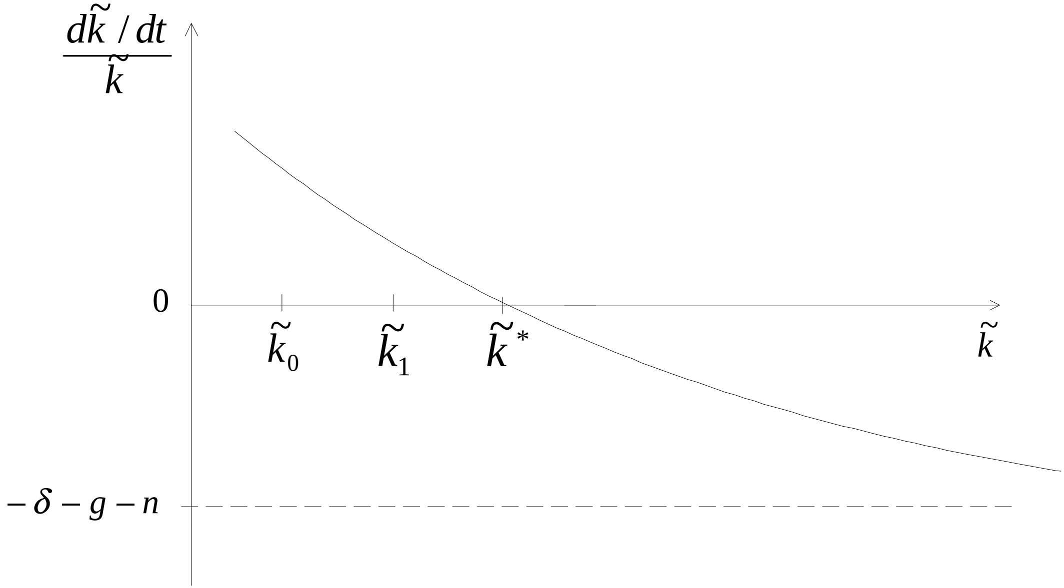



**Figure 12** *Transitional dynamics in the Solow growth model*

- Growth rates and catching up
  - The figure shows that the growth rate $\frac{d\tilde{k}(t)/dt}{\tilde{k}(t)}$ at $\tilde{k}_0$ is higher than at $\tilde{k}_1$
  - This is the famous prediction that poor countries catch up relative to rich countries:
    * Consider a (economically) poor country with capital per effective worker of $\tilde{k}_0$ and a rich country at $\tilde{k}_1$
    * The marginal productivity of capital at $\tilde{k}_0$ is higher than at $\tilde{k}_1$
    * Investment at $\tilde{k}_0$ leads to faster increase of capital stock and faster growth of GDP
    * Distances (relative and absolute) in GDP per capita reduce over time between poor and rich country
    * Yet, distance is never zero (apart from the steady state which is reached at infinity)

- Growth rates in the long-run equilibrium
  - (Note the difference between equilibrium and *long-run* equilibrium)
  - The steady state value of capital per effective labor $\tilde{k}$ is given by a constant (see tutorial 5.1),
$$\tilde{k}^* = \frac{K(t)}{A(t)L(t)} = \left(\frac{s}{\delta + g + n}\right)^{1/(1-\alpha)} \tag{3.4}$$
  - Computing the derivative with respect to time (see also tutorial 5.1) yields the long run growth rate of capital (note that the right-hand side of (3.4) is constant)
$$\frac{\dot{K}(t)}{K(t)} = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} = g + n$$
where the second equality used (3.1)

- Growth rate of GDP is given by

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \frac{\dot{K}(t)}{K(t)} + (1-\alpha)\left[\frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)}\right]$$
$$= \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} = g + n$$

i.e. driven only by growth rate of TFP and population (capital growth per se does not play a role)

- Growth rate of GDP per capita $y(t) \equiv Y(t)/L(t)$

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{L}(t)}{L(t)} = g + n - n = g$$

- When do inhabitants of a country become richer?
  - A country becomes richer in terms of GDP per capita only by an increase in total factor productivity $A(t)$!
  - Compare frequencies of growth rates of GDP and of GDP per capita to see the huge importance of population growth rate $n$ in tutorial 5.2

### 3.1.3 The Cass-Koopmans-Ramsey model

- Question
  - How could we explain the exogenous saving rate $s$, i.e.
  - what are its determinants?
- Why should we want to explain $s$?
  - There are large empirical differences across individuals, countries and over time
  - "One size fits NOT all"
  - Theoretical curiosity: A central variable should not be left unexplained

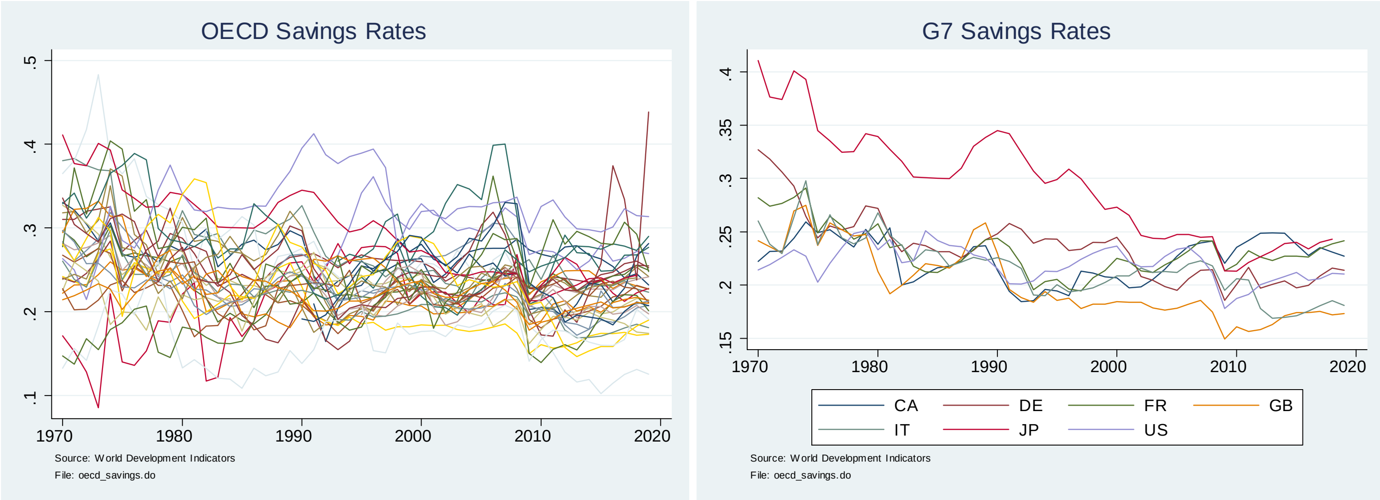



**Figure 13** *Savings rates around the world (left) and in G7 countries (right)*

- What the pictures tell us
  - Saving rates vary a lot over time
  - They also seem to differ systematically across countries (imagine a country-specific intercept in the right figure)
  - Where do these differences come from?
- Approach
  - Belief of (most) economists: Consumption and saving (and therefore saving rate) are optimally chosen
  - So – let us construct a maximization problem which explains saving rate endogenously
- Technical background
  - Wälde (2012), ch. 5.6.3

- Preferences
  - We study a central planner problem
  - This allows us to focus on optimally chosen $s$ in the simplest way (as opposed to decentralised economy)
  - Objective function is social welfare function $U(t)$

$$U(t) = \int_t^\infty e^{-\rho[\tau-t]} u(C(\tau))\, d\tau \tag{3.5}$$

  where
  - $u(C(\tau))$ is the instantaneous utility function,
  - $\rho > 0$ is the time preference rate (measuring impatience of individual) and
  - $e^{-\rho[\tau-t]}$ is the discount factor (or function)
  - Planning/ optimal behaviour starts in $t$ (like 'today') and goes up to infinity $\infty$

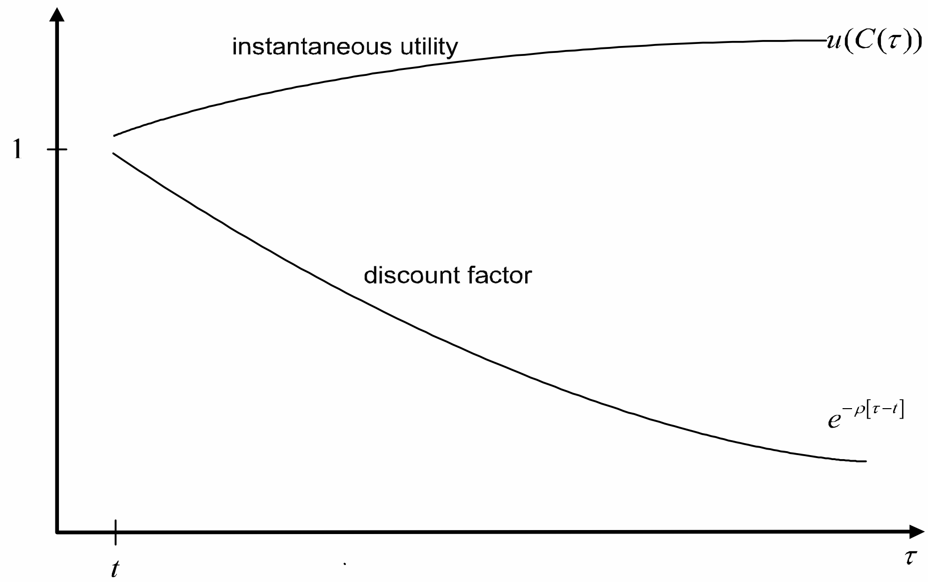



**Figure 14** *The discount factor* $e^{-\rho[\tau-t]}$ *and (and example) of instantaneous utility as a function of time* $\tau$

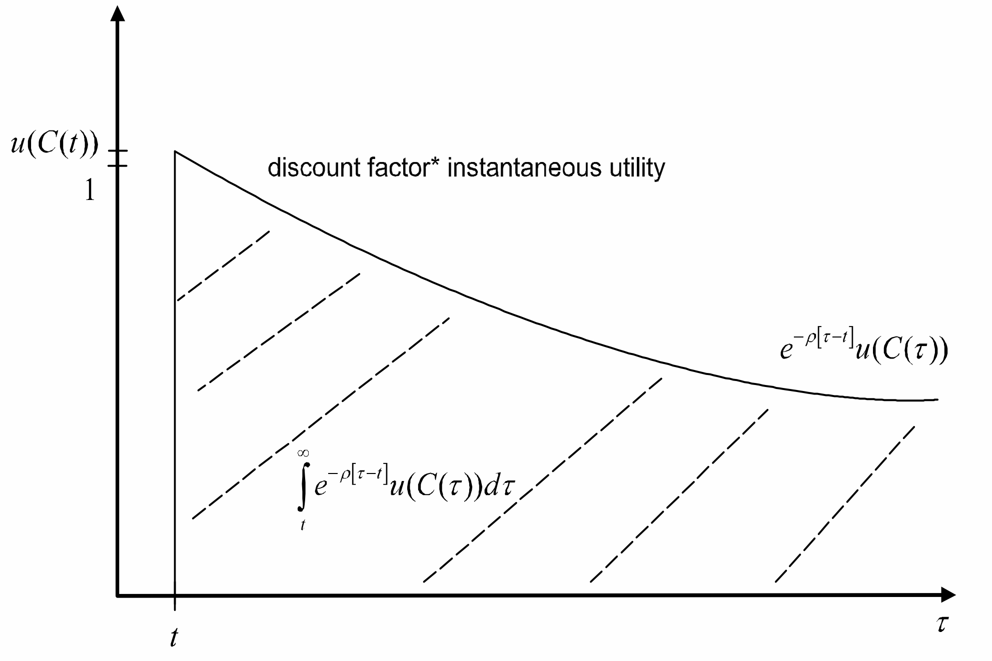



**Figure 15** *The objective function is shown by the shaded area (and is maximized by the choice of the consumption path* $C(\tau)$*)*

- Instantaneous utility
  - Functional form ...
$$u\left(C(\tau)\right)=\frac{\left(C(\tau)\right)^{1-\sigma}-1}{1-\sigma},\ \ \sigma>0 \tag{3.6}$$
  - ... implies constant intertemporal elasticity of substitution of $1/\sigma$
  - (Note that the logarithmic utility function is a special case of this functional form,
$$\lim_{\sigma\to 1} u\left(C(\tau)\right)=\ln C(\tau),$$
    apply L'Hôpital's rule to find this)

- The resource constraint
  - Maximization problem becomes meaningful only with a constraint
  - Constraint here is a resource constraint as we look at the economy as a whole
  - Capital evolves according to
$$\dot{K}\left(t\right)=Y\left(K\left(t\right),L\right)-\delta K\left(t\right)-C\left(t\right) \tag{3.7}$$
  - The change in the capital stock (net investment) is given by output minus consumption (gross investment) minus depreciation $\delta K\left(t\right)$

- Maximization problem
  - The planner chooses the path of consumption $C(\tau)$ between $t$ and infinity to maximize $U(t)$ from (3.5) subject to the constraint (3.7)
  - An optimal consumption path $C(t)$ fixes an optimal path of saving rates $s(t)$ which (in most cases) will vary over time
    * By definition $s(t)Y(t) = Y(t) - C(t)$ or $C(t) = (1 - s(t))Y(t)$
    * Hence, the optimal saving rate would be given by $s(t) = (Y(t) - C(t))/Y(t)$
  - The maximization problem per se focuses on optimal consumption
  - Optimal saving rate is only implicitly determined

- Solving the maximization problem
  - The maximization problem is an intertemporal maximization problem under a dynamic constraint
  - Standard method: Hamiltonians (Wälde, 2012, ch. 5.1)
  - Hamiltonian $H$ for our problem reads

$$H = u(C(t)) + \lambda(t)\left[Y(K(t), L) - \delta K(t) - C(t)\right]$$

- General idea for Hamiltonians
  - take the argument from objective function behind the discount factor and write it at the first place of Hamiltonian, then
  - add a "multiplier" $\lambda(t)$ (similar to Lagrange approach) and
  - multiply it by the right-hand side of the resource constraint
  - "Multiplier" $\lambda(t)$
    * Correct term is co-state variable (optimal control term) – corresponding to a
    * State variable (capital $K(t)$ in our case)
    * Shadow price is the economic term: price of a good in utility units (how does utility increase if the amount of a good is marginally increased)

- Optimality conditions
  - There are two optimality conditions

$$\frac{\partial H}{\partial C(t)} = u'(C(t)) + \lambda(t)[-1] = 0 \Leftrightarrow u'(C(t)) = \lambda(t) \tag{3.8}$$

$$\dot{\lambda}(t) = \rho\lambda(t) - \frac{\partial H}{\partial K(t)}$$

  - First optimality condition computes derivative of Hamiltonian with respect to control variable, which is consumption here (compare similar approach with Lagrange function)
  - It requires that marginal utility from consumption equals the shadow price of wealth (see app. A.3 for a detailed interpretation)
  - Second optimality condition requires the shadow price $\lambda(t)$ to follow this path
  - Why? Take it as granted or see Wälde (2012, ch. 6.2)
  - Second optimality condition takes time preference rate $\rho$ from objective function and subtracts derivative of Hamiltonian with respect to state variable (which is capital here)

- Optimality conditions (in more detail)

  – Let us compute

$$\frac{\partial H}{\partial K(t)} = \lambda(t)\left[\frac{\partial Y(K(t), L)}{\partial K(t)} - \delta\right]$$

  – Hence, second optimality condition reads

$$\dot{\lambda}(t) = \rho\lambda(t) - \lambda(t)\left[\frac{\partial Y(K(t), L)}{\partial K(t)} - \delta\right] \tag{3.9}$$

  – As an intermediate summary, the optimality conditions (3.8) and (3.9) determine two variables, $\lambda(t)$ and $C(t)$, given the path for capital $K(t)$ from the resource constraint (3.7)

  – This is a three-dimensional differential equation system for three variables, $\lambda(t)$, $C(t)$ and $K(t)$

- Simplifying the system
  - Now remove $\lambda(t)$ from (3.9) by replacing it by marginal utility from consumption as in (3.8)

$$\frac{d}{dt}u'(C(t)) = \rho u'(C(t)) - u'(C(t)) \left[\frac{\partial Y(K(t), L)}{\partial K(t)} - \delta\right]$$

  - Computing the time-derivative on the left-hand side

$$\frac{d}{dt}u'(C(t)) = u''(C(t)) \frac{d}{dt}C(t) = u''(C(t)) \dot{C}(t)$$

    and dividing by marginal utility, we get

$$\frac{u''(C(t))}{u'(C(t))}\dot{C}(t) = \rho - \left[\frac{\partial Y(K(t), L)}{\partial K(t)} - \delta\right]$$

- The Keynes-Ramsey rule
  - The Keynes-Ramsey rule then reads (multiply by $-1$), (see tutorial 5.3 for another example)
$$-\frac{u''\left(C\left(t\right)\right)}{u'\left(C\left(t\right)\right)}\dot{C}\left(t\right)=\frac{\partial Y\left(K\left(t\right),L\right)}{\partial K\left(t\right)}-\delta-\rho$$
  - This is one equation fixing the optimal path of consumption over time – joint with the equation for capital in (3.7)
  - We therefore ended up with a two-dimensional differential equation system for two variables, $C\left(t\right)$ and $K\left(t\right)$
    * This is the end of deriving necessary conditions for optimal behaviour
    * If we had two boundary conditions (for our two differential equations), the maximization problem fixing optimal consumption and (implicitly) optimal saving rates would have been solved
    * See below on where they come from

- How can we understand the Keynes-Ramsey rule?
  - The term $-\frac{u''(C(t))}{u'(C(t))}$ is the Arrow-Pratt measure of absolute risk aversion. It measures
    * the curvature of the instantaneous utility function, i.e.
    * how much individuals dislike risk
  - more capital $K(t)$ increases output by the marginal productivity of capital, $\frac{\partial Y(K(t),L)}{\partial K(t)}$, which can be called gross interest rate
  - Subtracting the depreciation rate $\delta$ gives the net interest rate or net return to an additional unit of capital
  - Consumption grows ($\dot{C}(t) > 0$) if and only if the right-hand side is positive. This is the case when net return from more capital is larger than the time preference rate
  - This difference captures the trade-off between the reward to less consumption today and the downside/ punishment/ disadvantage of less consumption today
    * Benefit is the net return to an additional unit of capital
    * Cost is captured by the time preference rate, i.e. by the discounting of consumption that is shifted to the future

- The specific Keynes-Ramsey rule
  - For our instantaneous utility function (3.6) from above, the rule reads

$$\frac{\dot{C}(t)}{C(t)} = \frac{\frac{\partial Y(K(t),L)}{\partial K(t)} - \delta - \rho}{\sigma}$$

  - This tells us that $\sigma$ is the measure of relative risk aversion and
  - that $1/\sigma$ is the intertemporal elasticity of substitution
  - An individual with a high $1/\sigma$ reacts more strongly (in terms of changes in consumption growth) to changes in returns to capital than an individual with a low intertemporal elasticity of substitution
  - A central planner that chooses consumption and thereby the savings rate optimally follows a consumption path that satisfies this Keynes-Ramsey rule

### 3.1.4 A phase diagram analysis

- How to proceed from the Keynes-Ramsey rule?
  - We need to analyse optimal consumption jointly with the evolution of the capital stock as described in the resource constraint (3.7)
  - Technically speaking, we face a two-dimensional differential equation system (which is non-linear)
  - (see tutorial 5.4 for a one-dimensional-differential-equation-system example)
  - qualitative method to understand its properties: phase diagram analysis (see Wälde, 2012, ch. 4.2 for more background)

- First step: Find a steady state (see tutorial 5.4, question 1, for a general definition of a steady state)
  - Definition of steady state here: Values of $K^*$ and $C^*$ for which the capital stock and consumption do not change over time
  - Formally

$$\dot{K}(t) = 0 \Leftrightarrow C^* = Y(K^*, L) - \delta K^* \tag{3.10}$$

$$\dot{C}(t) = 0 \Leftrightarrow \frac{\partial Y(K^*, L)}{\partial K^*} - \delta = \rho \tag{3.11}$$

  - These two (algebraic) equations pin down $K^*$ and $C^*$
  - We can plot them into a phase diagram, see figure 16
- Second step: How do $K(t)$ and $C(t)$ change when they are not in the steady state?
- Step 2a: Draw the zero-motion lines (curves on which consumption or capital do not change)
  - Zero motion line for capital comes from (3.10)
  - Zero motion line for consumption is a vertical line given by (3.11)
  - steady state is NOT at the point where consumption is highest (compare 'golden rule')

- Step 2b: Draw 'arrows of motion' into the phase diagram
  - Starting with the resource constraint, we know

    $$\dot{K}(t) \geq 0 \Leftrightarrow Y(K(t), L) - \delta K(t) \geq C(t)$$

    Capital rises, whenever consumption is below the zero motion line
  - This is intuitive: capital rises if output minus depreciation is larger than consumption
  - Capital falls, whenever consumption is above the zero motion line (consumption is larger than output minus depreciation, capital is 'eaten up')
  - The Keynes-Ramsey rule tells us

    $$\dot{C}(t) \geq 0 \Leftrightarrow \frac{\partial Y(K(t), L)}{\partial K(t)} - \delta \geq \rho$$

  - Consumption rises if net return is sufficiently large, or if individuals are sufficiently patient
  - Consumption rises, whenever we are to the left of the zero motion line for consumption, i.e. for $K(t) < K^*$ (as for $K(t) < K^*$, the marginal productivity is larger than at $K^*$ given concavity of the production function)
  - consumption falls, whenever we are at $K(t) > K^*$
  - Draw all of this into the phase diagram in figure 16 as well

- Step 2c: Draw trajectories into phase diagram
  - Starting at any point in the phase diagram, look at arrows of motion and describe changes over time by arrows on trajectories
  - Here, we can find a saddle path that leads to the steady state (which is a saddle point here)

- How does optimal consumption evolve over time?
  - Start with some initial (exogenous) capital stock $K_0$
  - Path $A$ implies too little consumption growth and hits (and crosses) the $\dot{C}(t) = 0$ line below the steady state
  - Path $C$ implies too much consumption growth and crosses the zero-motion line for capital vertically
  - Optimal consumption level is given by the consumption level that puts the economy on the saddle path $B$. In the figure, this is $C_0$
  - As of then, the economy grows and approaches the steady state

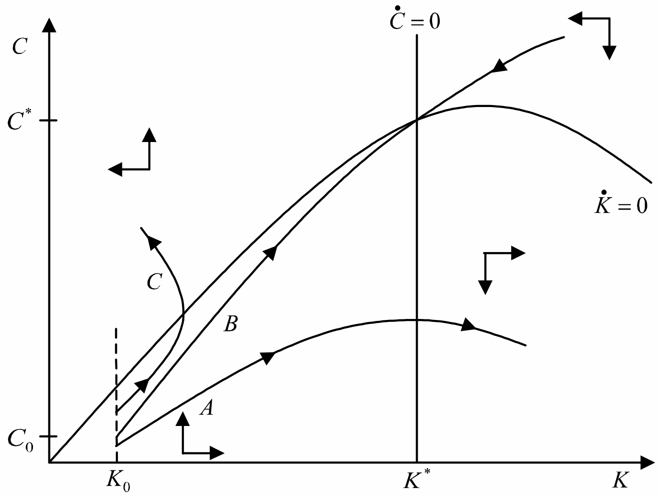



**Figure 16** *Phase diagram analysis for optimal consumption of a central planner*

### 3.1.5 What have we learned?

- Why do countries grow?
  - What does growth mean? Growth of GDP, GDP per capita or something else (happiness?)?
  - GDP: because of population growth and TFP growth
  - GDP per capita: because of TFP growth (technological progress)
  - is this a non-explanation for long-run growth? TFP growth is exogenous!
- Why are some countries richer than others (in terms of GDP per capita)?
  - In the short run: more capital, higher TFP level
  - In the long run (Solow): conditional convergence - the long-run capital stock $k^*$ depends on parameters (e.g. saving rate, depreciation). If they differ, GDP per capita differs in the long run
  - In the long run (optimal saving): endogenous saving rate depends, inter alia, on depreciation rate and time preference rate

- Why do some countries grow faster than others?
  - temporarily: there can be a catching-up process
  - in the long run: we do not know (given the Solow model)
- Why does growth come to an end (in the absence of technological progress)?
  - returns to capital fall with more capital
  - returns not high enough, given saving rate, depreciation rate and population growth (Solow model)
  - returns not high enough to compensate for forgone consumption (i.e. given time preference rate) and depreciation (optimal saving)

## 3.2 New growth theory: Incremental innovations

### 3.2.1 Some background on the "new" endogenous growth theory

Innovation and growth - main contributions and ideas

- Romer (1986)
  - The use of capital requires knowledge which is a public good
  - Capital accumulation goes hand in hand with knowledge accumulation
  - Technically, this implies constant returns to scale in factors of production that can be accumulated
  - Assume the production function reads $Y(t) = AK(t)$
  - The marginal productivity of capital is constant (and given by $A$), growth never comes to an end (even in the absence of technological progress)

- Romer (1990) “Endogenous Technological Change”
  - Presents an economic mechanism highlighting the economics behind innovation and growth
  - Again, knowledge arises as an externality in the process of innovation, of intentional R&D
  - Technically, constant returns in the R&D process imply a constant long-run growth rate
- Aghion and Howitt (1992) “A model of growth through creative destruction”
  - Innovations are no longer incremental but can have negative side-effects for competitors
  - Schumpetarian view of the growth process
  - Also highlights the downsides of technological progress

Further developments

- Non-scale growth models
  - Jones (1995)
  - Segerstrom (1998)
- Unified growth theory
  - Galor (2005) provides a survey
  - Literature studies the question of how an economy moves from subsistence activities to systematic economic growth
  - Would be a nice master thesis
- International trade and economic growth
  - Grossman and Helpman (1991)
  - Many others

### 3.2.2 The principle of endogenous growth theory (Shell, 1966)

- The idea
  - An economy needs resources for innovation and growth
  - Technological progress does not come costlessly (as in Solow growth model)
- A planner setup
  - Social welfare function reads

$$U(t) = \int_t^{\infty} e^{-\rho[\tau - t]} u(C(\tau))\, d\tau$$

where utility from aggregate consumption is given by

$$u(C(\tau)) = \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma}, \ \sigma > 0$$

- The objective function is maximized subject to two constraints

$$C(\tau) = A(\tau)[L - L_A(\tau)]$$
$$\frac{\dot{A}(\tau)}{A(\tau)} = L_A(\tau) \tag{3.12}$$

  where $A(\tau)$ is labour productivity in $\tau \geq t$ and labour $L$ is the only (fixed) factor of production. The number of workers in the research sector is given by $L_A(\tau)$

• Solution of maximization problem and the principle

  - The central planner chooses $L_A(\tau)$ and faces a classic trade-off
  - Few workers (low $L_A(\tau)$) in the R&D sector imply low growth but high consumption (at least contemporaneously at $\tau$). Many workers in R&D imply fast growth but low consumption
  - The principle of endogenous growth theory: some workers in R&D sector are needed. What are the determinants of $L_A(\tau)$?

- Problem 1 with Shell's approach
  - It is a central planner setup
  - Real world economies are decentralized
  - How can one imagine R&D in decentralized economy?
- Problem 2
  - Where does the $A(t)$ in the denominator of (3.12) come from?
  - What is the mechanism behind it and the economic idea?
- It took economics 30 years to come up with an answer: We now consider a classic model of the new growth theory that
  - uses the principle of Shell's setup
  - explains R&D via competitive firms in decentralized economy
  - explains where $A(t)$ in (3.12) comes from

### 3.2.3 The setup in the Grossman and Helpman model

- Questions
  - How can one imagine a decentralized growth process being driven by R&D?
  - How does "endogenous technological change" in the spirit of Romer (1990) work ("where does $A(t)$ come from")?
  - How is factor allocation between R&D and production (the principle of Shell's setup) competitively determined?
  - What are ultimately the determinants of the economy-wide growth rate?

- What is the problem with decentralized R&D?
  - How can a firm finance R&D if R&D is costly and only at some future point leads to success?
  - Firms under perfect competition would not work (but see models with "prototypes")
  - Answer: Firm must act under imperfect competition, thereby make profits which allow to repay the costs of R&D

- Approach
  - We follow Grossman and Helpman (1991, ch. 3), due to its conceptional clarity

- Technologies
  - Differentiated good setup: there is not "one car" but many different "varieties of cars"
  - Known as a "Dixit-Stiglitz framework", going back to Dixit and Stiglitz (1977), see tutorial 5.8
  - A variety $i$ is produced by employing labour $l(i,t)$

$$x(i,t) = l(i,t)$$

  - New varieties are developed in an R&D sector, using labour as well

$$\dot{n}(t) = \varphi L_R(t)$$

  where
  - $n(t)$ is the current number of varieties
  - $\varphi$ is labour productivity in the R&D sector
  - $L_R(t)$ is the number of researchers and
  - $\dot{n}(t) \equiv dn(t)/dt$ is the increase in the number of varieties

- Labour market
  - The economy is endowed with $L$ workers (fixed quantity)
  - Workers either work in R&D sector or in production sector, i.e. wage is the same in both sectors in equilibrium
  - Labour market clears if

$$\int_0^{n(t)} l\left(i,t\right)di + L_R(t) = L$$

    meaning that total employment in production sector (the integral $\int_0^{n(t)} l\left(i,t\right)di$) plus employment $L_R(t)$ in R&D equals supply $L$

- Why the integral?
  - Assume we had a discrete number of varieties $n\left(t\right)$, starting with the first variety $i = 1$ and going to $n\left(t\right)$
  - Employment in the production sector would be $\Sigma_{i=1}^{n(t)} l\left(i,t\right)$
  - Here, for economic and technical reasons (optimal behaviour of firms does not need to take strategic interactions into account), we assume a continuum of varieties
  - Hence, we integrate from 0 to $n(t)$ instead of computing a sum from 1 to $n\left(t\right)$

- Household preferences
  - We assume all households or individuals are the same
  - We therefore work with a representative agent (and ignore distributional issues)
  - Intertemporal (or overall) objective function of the representative household reads

$$U(t) = \int_t^\infty e^{-\rho[\tau - t]} u(c(\tau))\, d\tau \tag{3.13}$$

    where $u(c(\tau))$ is instantaneous utility from consumption $c(\tau)$ in $\tau$
  - The instantaneous utility function reflects the Dixit-Stiglitz structure with a continuum of varieties and a "love-of-variety" interpretation

$$u(c(\tau)) = \ln \left( \int_0^{n(t)} c^\theta(i, \tau)\, di \right)^{1/\theta} \tag{3.14}$$

    where $0 < \theta < 1$ implies decreasing marginal utility from consuming variety $i$
  - Elasticity of substitution $\varepsilon$ between varieties

$$\varepsilon \equiv \frac{1}{1-\theta} > 1$$

  - This elasticity must be larger than 1 as varieties must be substitutable in a sufficiently easy way. Otherwise firms (see below) would make profits too easily

- Budget constraint
  - The $r$espresentative households owns wealth $a_r(t)$
  - Intertemporal dynamic budget constraint

$$\dot{a}_r(t) = r(t)a_r(t) + w(t) - e_r(t)$$

  - Change in wealth $\dot{a}_r(t)$ is determined by the difference between capital income, labour income and consumption expenditure $e_r(t)$
  - Intratemporal budget constraint for expenditure $e_r(t)$

$$e_r(t) = \int_0^{n(t)} p(i,t)\, c(i,t)\, di$$

- Aggregate wealth $a(t)$ and interest rate $r\left(t\right)$
  - Wealth in the economy as a whole is given by the value $v(t)$ of a (representative) firm times the number of firms,

    $$a(t) = v(t)n(t),$$

    where the number of firms is also the number of varieties (see below for more)
  - The interest rate in the dynamic budget follows from deriving the budget constraint (see tutorial 5.6) and reads

    $$r(t) \equiv \frac{\pi(t) + \dot{v}(t)}{v(t)}$$

  - Profits by firms are denoted by $\pi(t)$, the change of the value of a firm is $\dot{v}(t)$. Their sum, relative to the price $v\left(t\right)$ of a firm, is the interest rate

### 3.2.4 Optimal behaviour

- Households ("static")
  - Households behave optimally at each point in time and also over time
  - Optimal choice between varieties leads to instantaneous demand function

$$c(i,t) = \frac{p(i,t)^{-\varepsilon}}{P(t)} E(t)$$

where $E(t)$ is total expenditure in this economy, $E(t) = e_r(t)L$, and

$$P(t) \equiv \int_0^{n(t)} p(i,t)^{1-\varepsilon} \, di$$

is the price index for all varieties

  - Optimal consumption of variety $i$ depends on the prices of all the other varieties as well (unlike in the more standard Cobb-Douglas case)

- Households ("dynamic")
  - Given this optimal instantaneous behaviour, how is total expenditure $E(t)$ optimally spread over time?
  - Expenditure follows (another) Keynes-Ramsey rule,

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho$$

  - Total consumption expenditure rises if the interest rate exceeds the time preference rate

- The number of firms
  - The number of firms and varieties is the same as, given costly R&D, firms have no incentive to (re) develop an existing variety
  - Hence, each firm develops its own, new variety
  - Same idea (here in dynamic setup) as in static international differentiated goods trade in the tradition of Krugman (1979) – who also uses Dixit-Stiglitz structure

- Differentiated goods
  - As there are as many firms as varieties, each firm has its own variety and can therefore act as a monopolist
  - Given competition from other monopolistic firms, that offer their varieties, this setup is called "monopolistic competition"
  - Price charged by firm $i$ is a markup $1/\theta > 1$ over marginal costs,

    $$p(i,t) = \frac{w(t)}{\theta},$$

    where $w(t)$ represents marginal costs from production
  - Pricing equation implies a symmetric equilibrium

    $$p(i,t) = p(t)$$

    where all firms charge the same price $p(t)$

- R&D firms
  - Research is undertaken ($L_R(t) > 0$) as long as

$$v(t) \geq \frac{w(t)}{\varphi}$$

    i.e. as long as payoff from R&D $v(t)$ exceeds (or just equals) the costs $\frac{w(t)}{\varphi}$
  - This equation comes from
    * profit maximization of R&D firms or from
    * free entry condition into R&D
  - In equilibrium (with ongoing innovation)

$$v(t) = \frac{w(t)}{\varphi}$$

### 3.2.5 Equilibrium

- After many steps (see tutorial 5.6), economy can by described by following reduced form

$$\dot{n}(t) = \varphi L - \frac{\theta}{v(t)}$$
$$\frac{\dot{v}(t)}{v(t)} = \rho - \frac{1-\theta}{n(t)\, v(t)}$$

- Its dynamics can be understood in phase diagram
- The steps are as always
  - Is there a steady state?
  - How do zero-motion lines look like?
  - In which regions of the phase diagram do $n(t)$ and $v(t)$ rise or fall?
  - How do trajectories look like?
  - Is there a saddle-path?

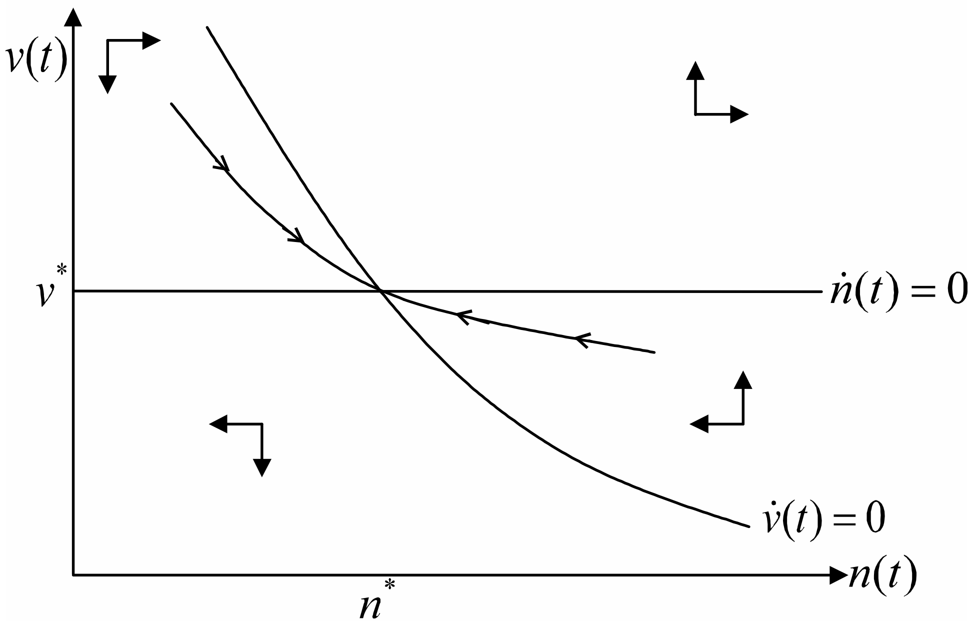



**Figure 17** *Phase diagram for the model with temporary innovation (due to absence of knowledge spillovers)*

Phase diagram shows that

- there is temporary innovation but
- no long-run growth
- the number of varieties increases up to some maximum level after which
- the economy comes to a halt – growth peters out

### 3.2.6 Knowledge spillovers yield long-run growth

- Idea
  - Researchers stand "on giants' shoulders"
  - Doing R&D does not only lead to a new variety, but it also creates knowledge
  - This knowledge $Kn(t)$ is a public good and available for others afterwards

$$\dot{n}(t) = \varphi L_R(t) Kn(t), \tag{3.15}$$
$$Kn(t) = n(t)$$

  - This is the answer to "where does $A(t)$ in (3.12) come from" – which is due to Romer (1990)

- Reduced form
  - Again, we obtain a system in number of varieties and value of a representative variety

$$\frac{\dot{n}(t)}{n(t)} = \varphi L - \frac{\theta}{n(t)v(t)}$$
$$\frac{\dot{v}(t)}{v(t)} = \rho - \frac{1-\theta}{n(t)v(t)}$$

- Long-run growth
  - First question (as always): is there some type of steady state or balanced growth path?
    - We guess that there is a constant growth rate $g$ with $\dot{n}(t)/n(t) = -\dot{v}(t)/v(t) = g$
    - Verify that this makes sense and compute $g$ by plugging guess into reduced form

$$g = \varphi L - \frac{\theta}{n(t)\, v(t)}$$

$$-g = \rho - \frac{1-\theta}{n(t)\, v(t)}$$

  - Next question: what is $g$ and what is $n(t)\, v(t)$ (which are constant on balanced growth path)?
  - Solving this two-equation system for $g$ and $n(t)\, v(t)$ gives our endogenous growth rate in this economy

$$g = (1-\theta)\, \varphi L - \theta\rho$$

- Determinants of endogenous growth rate
  - Growth rate depends on economic determinants – in (very strong) contrast to Solow growth model where $g$ in (3.1) is an exogenous parameter
  - The growth rate of the economy is the higher,
    * the higher productivity of R&D workers ($\varphi$)
    * the larger the economy ($L$) – the notorious 'scale-effect'
    * the more patient individuals (lower $\rho$) and
    * the lower the elasticity of substitution (lower $\theta$)
  - Low $\theta$ means high markups and profits for firms, i.e. high incentives to do R&D
  - Growth could
    * also be zero, there would be no growth
    * not be negative (as varieties do not disappear and cannot be consumed)

- What if the economy is not on the balanced growth path?
  - Look at a phase diagram (see next slide)
  - Construct zero-motion lines

$$\frac{\dot{n}(t)}{n(t)} \geq 0 \Leftrightarrow \varphi L \geq \frac{\theta}{n(t)v(t)} \Leftrightarrow v(t) \geq \frac{\theta}{n(t)\varphi L}$$
$$\frac{\dot{v}(t)}{v(t)} \geq 0 \Leftrightarrow \rho \geq \frac{1-\theta}{n(t)v(t)} \Leftrightarrow v(t) \geq \frac{1-\theta}{n(t)\rho}$$

  - Check under which conditions zero-motion line for $v(t)$ lies above zero-motion line for $n(t)$ (when $g > 0$)
  - Ask, where in phase diagram $n(t)$ rises and $v(t)$ falls
  - After having plotted pairs of arrows, we see that there are three (types of) trajectories for a given initial level $n_0$ of varieties
  - Which path is the only reasonable one? The one that lies on the balanced growth path
  - This gives unique $v_0$, i.e. the unique initial value for a representative firm

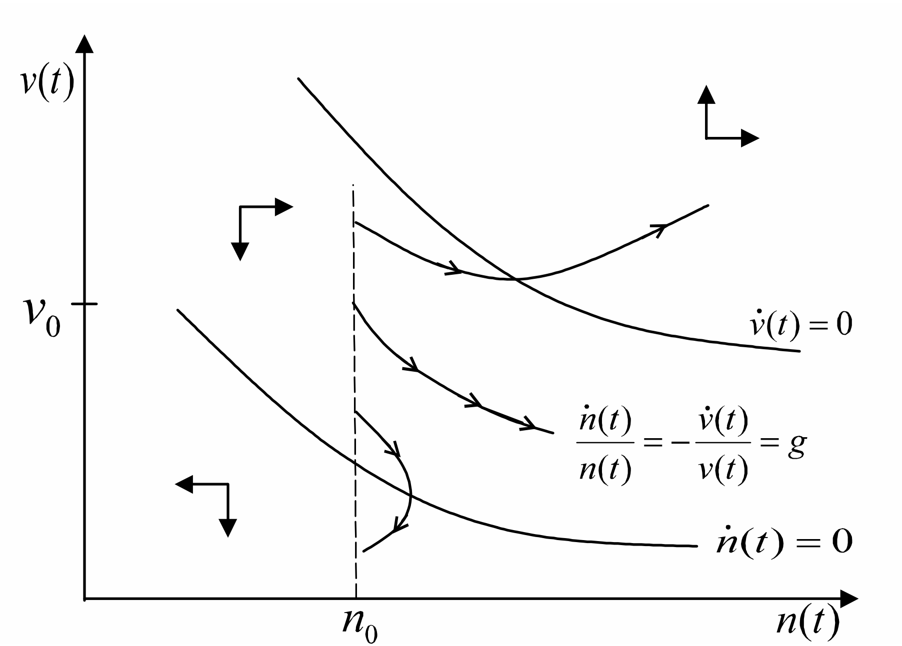



**Figure 18** *Phase diagram for the case of endogenous long-run growth*

- What is actually growing?
  - Number of varieties grows at endogenous rate $g$
  - Knowledge in R&D grows at $g$
  - Output per variety $i$ actually *falls* at rate $g$

$$x(i,t) = l(i,t) = \frac{L - L_R(t)}{n(t)} = \frac{L - g/\varphi}{n(t)} \equiv x(t) \tag{3.16}$$

where we used that by (3.15), $L_R(t) = g/\varphi$ on the balanced growth path

- Does the model predict consumption growth?
  - Apparently not for consumption per variety!
  - But model predicts growth of consumption index, i.e. of utility

$$u(c(\tau)) = \ln\left(\int_0^{n(\tau)} c(i,\tau)^\theta \, di\right)^{1/\theta}$$

  - (Take into account that utility grows if and only if $e^{\theta u(c(\tau))}$ grows)
  - To see this, use the property of a *symmetric* equilibrium where $x(t)$ from (3.16) is consumption of the *representative* variety, i.e. $c(i,\tau) = x(\tau)$ for all $i$ and write

$$\begin{aligned} e^{\theta u(c(\tau))} &= \int_0^{n(\tau)} x(\tau)^\theta \, di = x(\tau)^\theta \int_0^{n(\tau)} 1 di = x(\tau)^\theta n(\tau) \\ &= \left(\frac{L - g/\varphi}{n(\tau)}\right)^\theta n(\tau) = (L - g/\varphi)^\theta n(\tau)^{1-\theta} \end{aligned}$$

  - As $n(\tau)$ grows and the rest is constant, consumption grows
  - As consumption (index) grows, so does consumption per capita (constant population size here)

- Why does the consumption index grow while consumption per variety falls?
  - Negative effect of growth: $c(i,\tau)$ falls (as $x(i,t)$ falls)
  - Positive effect of growth: $n(\tau)$ rises
  - Overall, we gain, as there are decreasing marginal utilities from each variety ($0 < \theta < 1$)

- Interpretation of increase in consumption index via "love of variety"
  - Individuals enjoy consuming more varieties even at the cost of reduced consumption *per* variety
  - Imagine consumption per varieties reduces from $c_2$ to $c_1$ by $x\%$ (see next figure)
  - Utility per variety then reduces as well but by less than $x\%$
  - Overall, there is a gain in utility from *all* varieties taken together

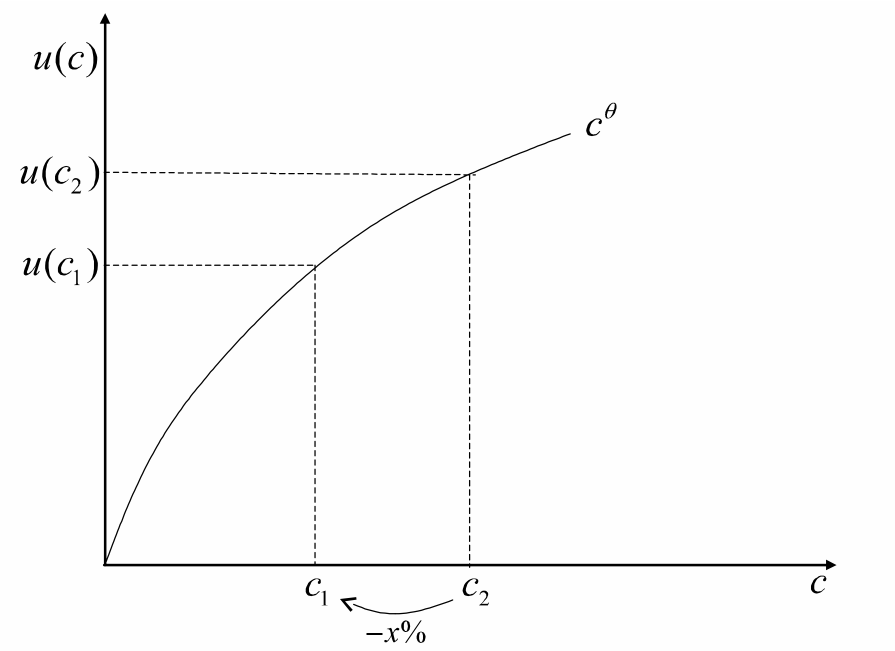



**Figure 19** *Concave utility function* $u(c) = c^{\theta}$ *with* $0 < \theta < 1$ *and gains in overall utility despite consumption drop per variety:* $c_1 = (1 - x\%)\, c_2$ *but* $u(c_1) > (1 - x\%)\, u(c_2)$

- Interpretation of increase in consumption index via “division of labour”
  - Original argument by Adam Smith on ’the making of pins’ (see http://oll.libertyfund.org/pages/adam-smith-and-j-b-say-on-the-division-of-labour)
  - mining, smelting/ melting, forging, splitting, heading
  - greater output can be achieved if workers specialize in one small task each instead of everybody doing all tasks

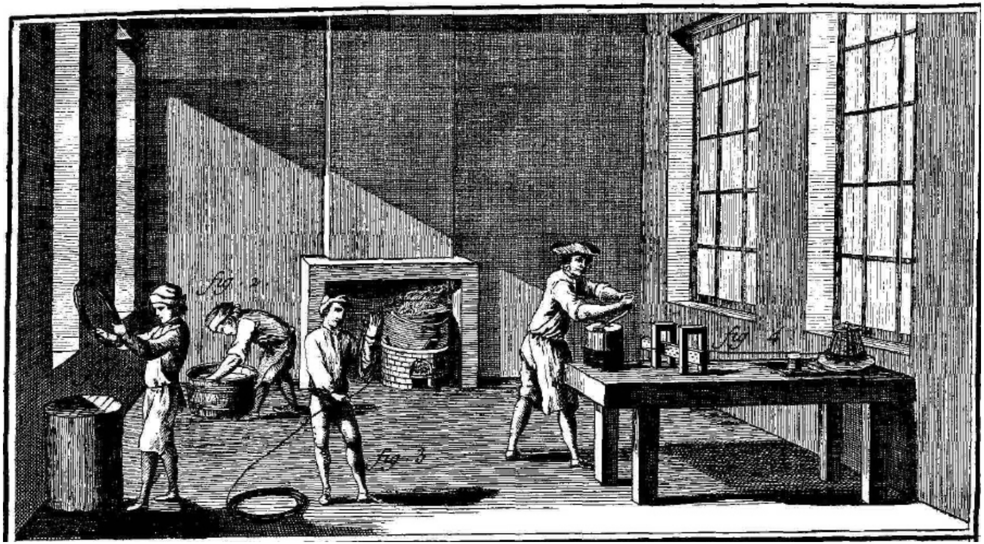



**Figure 20** *The making of pins*
*Source: http://oll.libertyfund.org/pages/adam-smith-and-j-b-say-on-the-division-of-labour*

- “Division of labour” in the model
  - Define aggregate output as

$$Y(t) \equiv \left( \int_0^{n(\tau)} c(i,\tau)^\theta \, di \right)^{1/\theta}$$

    i.e. rename utility index and call it output
  - More and more intermediate goods ($n(\tau)$ grows) enter a production (assembling) process
  - Input per intermediate good $c(i,\tau)$ goes down
  - The overall effect is positive as there are decreasing marginal *productivities* (instead of utilities) of intermediate goods

### 3.2.7 What have we learned?

- Why do countries grow?
  - Number of varieties grows because of positive knowledge externalities in the R&D process
  - Consumption per capita and GDP and GDP per capita grow for the same reason
- Why are some countries richer than others (in terms of GDP per capita)?
  - Catching up does not necessarily take place (strong difference to Solow model)
  - All countries jump immediately (compare phase diagram) on the balanced growth path
  - All countries always grow with $g$ (which is the same as long as parameters that determine $g$ are the same)
  - relative income differences can persist forever

- Why do some countries grow faster than others?
  - they do not, as long as $g$ is the same across countries
- Can countries grow faster temporarily or also permanently?
  - there is conditional catching-up, overtaking and falling behind
  - "conditional" means "it depends on parameters and policies"
  - if countries differ in their parameters, they can grow differently

- Is the growth rate optimal?
  - General principle: growth rate might not be optimal due to some inefficiency/ market-failure
  - Growth rate can be too high or too low
  - Growth rate here might not be optimal due to knowledge externality (one example of market failure)
  - In fact, it is too low because the externality is a positive externality
  - The growth rate could also be too high (for different externalities, see below)
- In summary, compared to neoclassical growth model, new growth theory offers fundamentally different (and much richer) views on
  - economic growth rate
  - welfare properties of growth

## 3.3 New growth theory: Major innovations

### 3.3.1 The question

- How does a growth process look like?
  - Is it just better technologies (as in Solow model) where everybody benefits?
  - Is it just reallocation of labour to new activities that also benefits everybody?
  - Or is there some sort of Schumpeterian "creative destruction" component in the growth process as well?
- Creative destruction is part of normal growth
  - Idea going back (at least to) Schumpeter (1943, Capitalism, Socialism and Democracy, p. 82 ff)
  - The growth process comes from "new methods of production, ..., new markets, new forms of industrial organization ... that capitalist enterprise creates"
  - "industrial mutation ... revolutionizes the economic structure from within, incessantly destroying the old one"

### 3.3.2 References

- Aghion and Howitt (1992) “A Model Of Growth Through Creative Destruction“
- Wälde (1999) allows for risk-aversion
- much more subsequent work

### 3.3.3 The production side

- First (out of three sectors) produces the consumption good $y(t)$
  - The firm uses the technology

    $$y(t) = \gamma^t x(t)^\alpha h(t)^{1-\alpha} \equiv \gamma^t x(t)^\alpha$$

    by employing an intermediate good $x(t)$ and some indivisible factor $h(t)$ (the entrepreneur) which we normalize to one
  - The technological level is given by $\gamma^t$ where $\gamma > 1$ and $t$ is the currently most advanced technology ($t$ is not time, this is the notation employed by Aghion and Howitt, 1992)
  - The technology comes with the intermediate good $x(t)$ (it is embodied in $x(t)$)
  - Firms are price takers and maximize profits. Profits amount to (see tutorial 5.7)

    $$\pi_y(t) = (1-\alpha)\,\gamma^t x^\alpha(t) \tag{3.17}$$

    (which would be the factor reward to $h(t)$)

- The second sector is the "Schumpetarian sector" where creative destruction takes place
  - There is a monopolist in this sector producing the intermediate good $x(t)$ employing labour $L(t)$
$$x(t) = L(t)$$
  - The monopolist's optimal price is a markup $1/\alpha$ over marginal cost $w$ (see tutorial 5.5)
$$p(t) = \frac{w(t)}{\alpha}$$
    where $1/(1-\alpha)$ is the price elasticity of demand resulting from the consumption good sector
  - Profits of the monopolist amount to
$$\pi_x(t) = \alpha \pi_y(t)$$
- The third sector undertakes R&D ...
  - Research is risky and does not necessarily lead to a successful end
  - Riskiness is modelled by a Poisson process

- What is a Poisson process? (see Wälde, 2012, Definition 10.1.6 for details)
  - A Poisson process $q(t)$ is a stochastic process in continuous time
  - A stochastic process is a collection of random variables $q(t)$ at points in time $t$
  - A Poisson process can be characterized by its increment over a short period of time $dt$

    $$dq(t) = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\} \text{ with probability } \left\{ \begin{array}{c} \mu \mathrm{dt} \\ 1 - \mu \mathrm{dt} \end{array} \right\}$$

    where $\mu$ is the arrival rate
  - Why called Poisson process? The number of jumps between $t$ and $\tau > t$ is Poisson-distributed with parameter $\mu [\tau - t]$

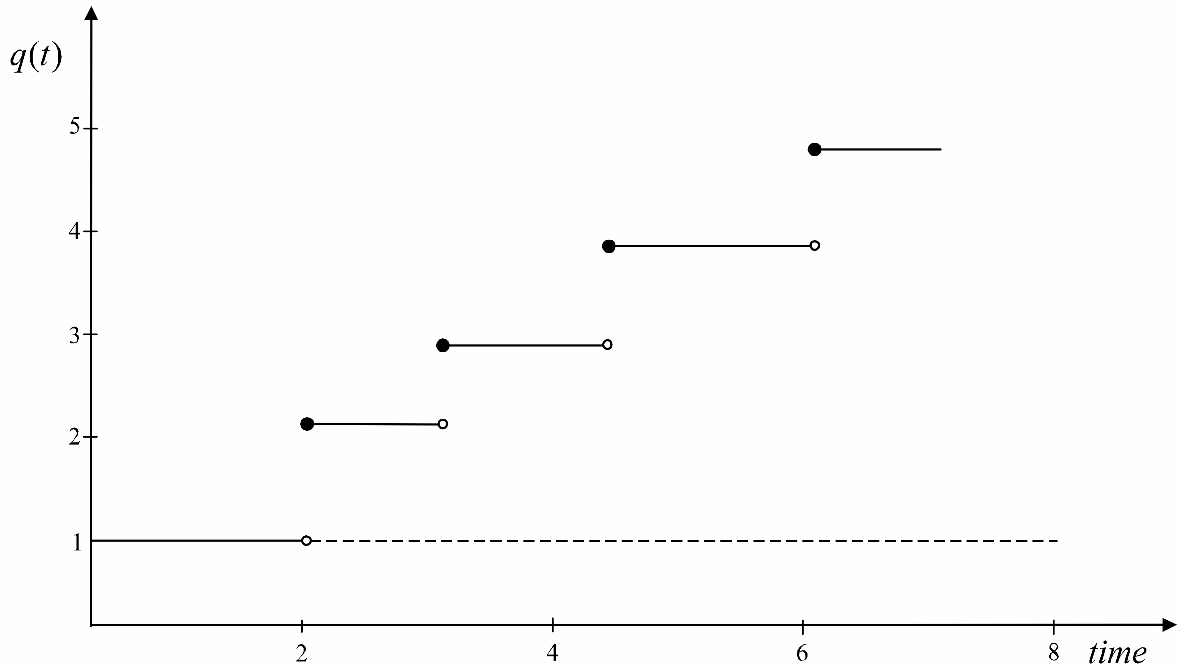



**Figure 21** *Illustration of Poisson process (also called counting process as it counts the number of jumps $dq(t)$)*

- [back to:] The third sector undertakes R&D
  - The arrival rate for success in R&D is
$$\mu = \lambda n(t)$$
    where $\lambda$ is a parameter and $n(t)$ is the number of engineers (workers) employed in this sector
  - When a firm has success in R&D, it knows how to produce an intermediate good that implies a productivity of $\gamma^{t+1}$
  - Firm drives intermediate monopolist (see above) out of market (Schumpeterian creative destruction) and earns profits $\pi_x(t)$
  - These profits provide incentives to do R&D
- How is R&D financed? (Wälde, 1999)
  - R&D firms sell shares to households at a certain price
  - Income from these sales allow to hire researchers
  - When R&D fails, shares are worthless
  - When R&D is successful, households own the new intermediate monopolists
  - As the latter makes profits in intermediate goods market, households are (in expectations, i.e. on average) rewarded for their buying of shares by these profits

### 3.3.4 Labour market

- Labour is the only factor that is mobile between sectors
- Total supply is $N$
- Wage rate $w(t)$ is determined on the labour market which is assumed to clear at every moment in time

$$n(t) + L(t) = N$$

### 3.3.5 Consumers

- Households maximize an intertemporal utility function (as in Cass-Koopmans-Ramsey model)
- Optimal behaviour is characterised by

$$C(t) = \delta\left[w(t) + \pi(t)\right]N$$

  where $\delta$ is the share of labour and capital income used for consumption

- Note that $\delta$ needs to be determined in equilibrium (see Wälde, 1999)

### 3.3.6 Equilibrium

- After many intermediate steps
  - beyond the scope of this lecture but of high interest for a seminar paper
  - we can describe equilibrium
- Schumpetarian equilibrium dynamics
  - There is a constant number of researchers active in equilibrium (independent of technological level $t$), $n(t) = \bar{n}$
  - The arrival rate for new technologies is constant and given by $\lambda \bar{n}$
  - The average waiting time between two innovations is $1/(\lambda \bar{n})$
  - With each new innovation, output increases by a factor $\gamma$
  - Dynamics of output, consumption, the wage etc follow a step function qualitatively identical to fig. 21

- Is the growth rate optimal?
  - Definition (general): Externalities are effects of consumption or production activity on agents other than the consumer or producer which do not work through the price system (based on J.J. Laffont, Externalities, New Palgrave Dict of Econ)
  - Examples: exhaust emissions of cars (negative), aircraft noise (negative), vaccination for a disease (positive), beekeeper helps pollination of crops (positive)
- Market failures in the creative destruction model
  - Standing on giants shoulders: Innovator does not take into account that his innovation increases output forever (too little investment)
  - Destructive effect of innovation, business stealing: innovator does not take into account that previous innovator is driven out of market (too much investment)
  - Monopolistic behaviour: distortive price setting
  - And more, see tutorial 5.6, and Aghion and Howitt (1992)
  - Growth rate can be too high or too low, depending on which market failure dominates

### 3.3.7 What have we learned?

(in addition to the answers we heard above)

- Is the growth rate optimal?
  - further externalities as just discussed show that there can be too much or too little growth
  - growth is not necessarily a very "peaceful process" where TFP rises smoothly (Solow) or more varieties are added to existing ones (Grossman and Helpman)
  - growth can be rather destructive (in the spirit of Schumpeter)

# 4 Dual selves and economic growth

- What does "advanced" in "Advanced Macroeconomics" mean (remember p. 1.1)? Lecture wants to
  - understand economic arguments based on economic models and
  - take psychological reasoning for decision making into account
- We now take this behavioural economic perspective
- Are there further determinants of a country's growth rate than interest rate, time preference rate, demand structure and the like?
- Can we envision psychological determinants?

## 4.1 A Dual-Self Model of Impulse Control

### 4.1.1 Motivation

- Individuals obviously face self-control problems
  - Go to a party or study for the exam?
  - Buy another pair of jeans or save for summer holidays?
- Self-control is harder when being distracted
  - Behaving politely is more tough when being stressed
  - Not eating a big piece of chocolate is harder when something failed

- Fudenberg and Levine (2006)
  - present a model of individual decision making with two “selves” and
  - argue that such a structure can explain these findings
- Goethes Faust (Heinrich Faust, a scholar) says:

  In me there are two souls, alas,

  and their division tears my life in two

  Zwei Seelen wohnen, ach! in meiner Brust

  Die eine will sich von der andern trennen
- Our motivation: Can we enrich existing growth theories through new non-standard (i.e. non-economic) determinants of growth?

### 4.1.2 Preferences

- Decisions of an individual are the outcome of a game between a short-run self and a long-run self
- Short-run self $u(y_t, r_t, a_t)$
  - $y_t$ : wealth
  - $r_t$ : action of long-run self (e.g. influence on preferences)
  - $a_t$ : action of short-run self (share of wealth not consumed, "saving rate")
  - action of long-run self reduces instantaneous utility: $u(y_t, r_t, a_t) < u(y_t, 0, a_t)$ for $r_t > 0$
  - utility function rises in $y_t$, falls in $r_t$ and in $a_t$ (as consumption equals $(1 - a_t)\, y_t$)
- Long-run self $U_t = \Sigma_{t=1}^{\infty} \delta^{t-1} u(y_t, r_t, a_t)$
  - $U_t$ : intertemporal utility function
  - $\delta$ : discount factor
  - $u(.)$ : instantaneous utility function of short-run self

- Game between short-run self and long-run self
  - Short-run self maximizes $u(y_t, r_t, a_t)$ by choosing $a_t$ in each period
  - Long-run self maximizes $U_t$ by choosing $r_t$ for each period
  - Result of game is outcome of a "reduced-form maximization problem" (see Fudenberg and Levine, 2006) which contains a cost-function for self-control

- Observed behaviour (reduced-form maximization problem)
  - Objective function

$$\tilde{U}_t = \Sigma_{t=1}^{\infty} \delta^{t-1} \left[ u\left(y_t, 0, a_t\right) - C\left(y_t, a_t\right) \right]$$

  - $0 < \delta < 1$ is discount factor – the higher $\delta$, the more patient the individual is
  - $u\left(y_t, 0, a_t\right)$ is the short-run utility from wealth $y_t$, no action of long-run self (no longer exists) and share $a_t$ of wealth not consumed
  - New cost $C\left(y_t, a_t\right)$ of self-control defined as

$$C\left(y_t, a_t\right) = \gamma \left[ \max_{a_t'} u\left(y_t, 0, a_t'\right) - u\left(y_t, 0, a_t\right) \right], \quad (4.1)$$

    the difference between (utility from) the best possible (short-run) action and the action $a_t$ actually chosen
  - $\gamma$ : preference parameter making costs comparable to utility $u\left(.\right)$

### 4.1.3 Optimal saving behaviour

- Budget constraint
  - There is only capital income (no labour income)
  $$y_{t+1} = R_t a_t y_t$$
  - $R_t$ gross return (equal to 1+interest rate)
  - Equation shows why $a_t$ has the somewhat unusual name "share of wealth" not consumed: it is the share of wealth, or "saving rate", used for investment

- Specification of utility function
  - Logarithmic structure
  $$u\left(y_t, 0, a_t\right) = \log\left(\left(1 - a_t\right) y_t\right) \tag{4.2}$$
  - Consumption is given by $\left(1 - a_t\right) y_t$ (as implied by budget constraint)

- Cost function
  - Cost of self-control from (4.1) reads, given utility function (4.2),
  $$C\left(y_t, a_t\right) = \gamma\left[\log y_t - \log\left(\left(1 - a_t\right) y_t\right)\right]$$
  - $\log y_t$ stands for best possible short-run action (consume entire wealth)

- Saving rate chosen by individual (see tutorial 5.9)

$$a = \frac{\delta}{1 + (1 - \delta)\,\gamma}$$

  - A more patient individual (higher $\delta$) has higher saving rate
  - Higher cost of self-control (higher $\gamma$) reduce saving rate
  - Cost of self-control is a new, psychological determinant of saving behaviour

## 4.2 Self-control and economic growth

How could one extend economic growth models and allow for self-control considerations?

- Start with the simplest model of consumption and saving decision
  - Simplest case is a two-period situation
  - Consumption and saving in first period, second period displays consumption only
  - An individual with this decision setup can then be studied jointly with other individuals
  - This leads to an overlapping-generations (OLG) model (see section 8.1.6)
- A growth model with cost of self-control
  - Replace standard saving decision in OLG model by saving decision of a dual-self individual
  - Then follow standard structure of OLG model

- Provide answers to questions about determinants of long-run GDP, GDP per capita
  - Are there new determinants?
  - Does the self-control problem affect GDP?
  - Do self-control problems of part of a society affect others (without self-control problems) in society?
  - How do regulations about self-control (tobacco taxation or deterrent pictures) affect output, health and growth?
- Let's talk about it in your seminar paper or Master thesis

## 4.3 What have we learned?

- What were our questions at the beginning of this part on growth?
  - Why do countries grow?
  - Why are some countries richer than others?
  - Why do some countries grow faster than others?
  - Can they grow temporarily faster or also permanently?
- Questions all center on one single question: What are the determinants of the growth rate of a country?
- We saw the answers of pure growth theory in previous chapters
- Behavioural economics tells us
  - Human behaviour is richer than behaviour of 'homo oeconomicus'
  - In our example, there are two souls, two conflicting motivations within an individual
  - The strength of self-control has a crucial impact on saving behaviour
  - If countries differ in their self-control abilities and they differ in their growth rates

# 5 Exercises

## 5.1 Solow growth model

Assume the production function is of a Cobb-Douglas form $Y = K^\alpha (AL)^{1-\alpha}$. Let $\tilde{k} = K/AL$ be capital per effective labour.

1. Compute $\frac{d\tilde{k}}{dt}$ and $\frac{d\tilde{k}/dt}{\tilde{k}}$ using the following results and identities to assist your analysis:

$$g \equiv \frac{1}{A}\frac{dA}{dt}, \text{ growth rate of technology (exogenously given)}$$
$$n \equiv \frac{1}{L}\frac{dL}{dt}, \text{ growth rate of labour (exogenously given)}$$
$$\dot{K} \equiv \frac{dK}{dt} = I - \delta K, \text{ net investment}$$
$$I = S, \text{ savings and investment equilibrium}$$
$$S = sY, \text{ constant savings rate (exogenously given)}$$

   Discuss the functional form of $\frac{d\tilde{k}/dt}{\tilde{k}}$.

2. Derive the long-run value of $\tilde{k}$ and discuss its meaning.

3. Derive the long-run growth rates of Output (Y) and Output per Capita (Y/L) defined as:

$$\frac{\dot{Y}}{Y} \equiv \text{ growth rate of output}$$
$$\frac{d(Y/L)/dt}{Y/L} \equiv \text{ growth rate of output per capita}$$

## 5.2 Empirical growth of GDP and GDP per capita

Consider the following three figures and relate them to the Solow growth model.

1. Which variables are the theoretical counterparts to these observations?

2. Do these figures represent short-run or long-run changes?

3. What do these figures tell you about the central determinants of long-run growth of GDP per capita?

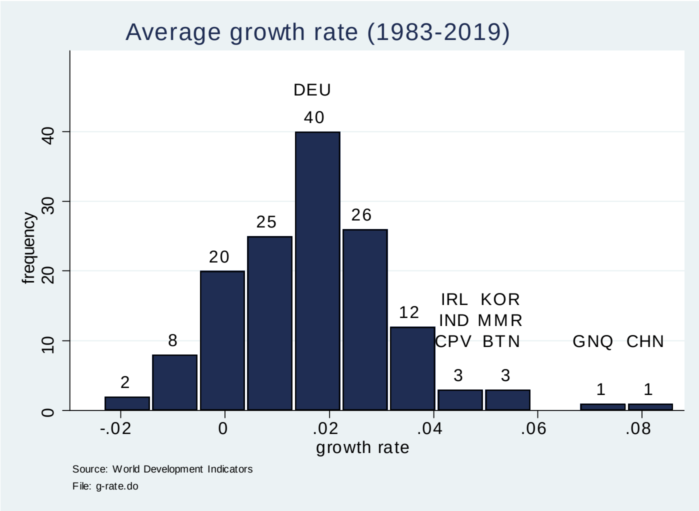



**Figure 22** *Frequencies of growth rates of GDP per capita (averages from 1983 to 2019)*
*Legend: For country codes see lecture or*
*wits.worldbank.org/wits/WITS/WITSHELP/Content/Codes/Country_ Codes.htm*

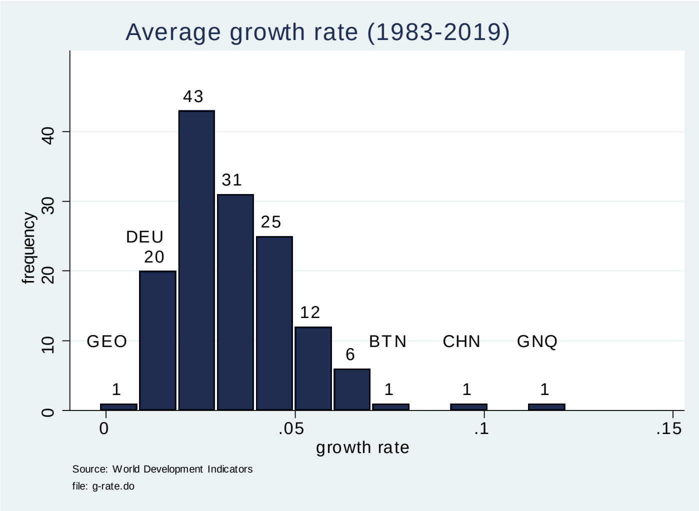



**Figure 23** *Frequencies of growth rates of GDP (averages from 1983 to 2019)*

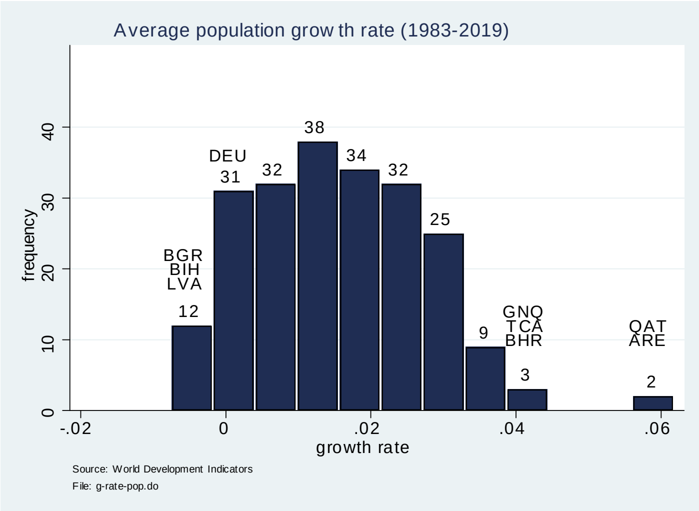



**Figure 24** *Frequencies of growth rates of population (averages from 1983 to 2019)*

## 5.3 Optimal Consumption

Consider the following objective function and dynamic budget constraint,

$$U = \int_t^\infty e^{-\rho(\tau - t)} u\left[c\left(\tau\right)\right] d\tau,$$

$$\dot{a}(\tau) = r(\tau)a(\tau) + w(\tau) - c(\tau).$$

1. Provide an interpretation of these equations.
2. Compute the Keynes-Ramsey rule for the individual's savings problem.
3. What is the Keynes-Ramsey rule if the instantaneous utility function takes a CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

## 5.4 Phase diagrams

1. General form - (first-order autonomous differential equation example)

   The first-order autonomous differential equation is give as

$$\dot{x}(t) = F(x(t)). \tag{5.1}$$

(a) What is an equilibrium state of the equation? Provide an interpretation of the equilibrium state.

(b) What are the properties of these equilibria?

2. Solow Growth model example

(a) Derive the differential equation for the capital-labour ratio (or capital per head), $k = K/L$ using the following

$$\begin{aligned} \dot{K} &= sF(K, L), \quad \text{capital accumulation} \\ \frac{\dot{L}}{L} &= \lambda, \quad \text{exogenous labour growth rate.} \end{aligned}$$

The production function is assumed to be strictly increasing and concave. There is no capital depreciation.

(b) Draw its phase diagram.

## 5.5 Differentiated goods

1. Interpret the following budget constraint

$$\int_0^n p_j c_j dj = w, \tag{5.2}$$

where $p$, $c$, and $w$ respectively stand for goods price, consumption and income. $j$ is good index.

2. Consider Dixit-Stiglitz preferences over $n$ varieties of a good and where $\theta$ represents tastes for different varieties with $0 < \theta < 1$,

$$U = \left( \int_0^n c_j^\theta dj \right)^{1/\theta}.$$

   How do households behave optimally when they face a budget constraint as in (5.2)?

3. Assume the technology is given as a function of labour only, with $a$ denoting the marginal cost of labour and $\phi$ are fixed costs,

$$x_j = al_j - \phi.$$

   Let firms maximise profits

$$\pi_j = p_j x_j(p_j) - wl_j$$

   where the wage rate is $w$. How do firms behave optimally and what does monopolistic behaviour mean here? Take the price elasticity of demand into account.

4. What is the elasticity of substitution between two goods, 1 and 2?

## 5.6 Innovation & Growth

1. Compute the optimal allocation of expenditure over time (i.e. $\dot{E}/E$) using the functional forms (3.13) and (3.14) from the lecture and the result in 5.5, question 2, for optimal consumption levels. (Hint: *Use the indirect utility function and Keynes-Ramsey rule.*)

2. Assets accumulation is given by the interest income from assets plus wage income minus expenditure. Derive the budget constraint

$$\dot{a} = ra + w - e$$

formally and and also the corresponding interest rate

$$r = \frac{\dot{v} + \pi}{v}.$$

The general procedure runs as follows.
*Step 1: de*fine savings, specifying wealth with all relevant prices and incomes
*Step 2*: compute the accumulation of wealth (i.e. the first difference) from $t$ to $t+1$
*Step 3*: relate current (i.e. at time $t$) savings to current changes (from $t$ to $t+1$) in capital
*Step 4*: rearrange to obtain a difference equation (or differential equation in continuous time), and define the interest rate $r$.

3. What is a reduced form of an equilibrium? Solve for $\dot{n}$ and $\dot{E}/E$. Use the following results from the lectures,
$$\dot{n} = \varphi L_R, \qquad v = w/\varphi,$$
where $\dot{n}$ is the accumulation of knowledge, $L_R$ is the share of the labour force engaged in R&D, $v$ is the price of a (representative) firm, and $\varphi$ is a parameter. Also employ standard optimality conditions for profits and consumption. The technology in the economy is equal to labour directed towards building a specific variety for a specific firm
$$x(i) = l(i).$$
Labour supply $L$ equals labour demand from production and research,
$$L = \int_0^n l(i)di + L_R.$$

4. Draw the phase diagram. Discuss the long-run implications of this model.

5. Derive the growth rate of the consumption index using Dixit-Stiglitz preferences.

## 5.7 Creative destruction

1. Final goods firms maximise profits $\pi_y = y - px$. Production takes the form of a Cobb-Douglas function
$$y = \gamma^t x^\alpha h^{1-\alpha} = \gamma^t x^\alpha$$

where $h$ is some indivisible production factor normalised to 1.

(a) What is the demand function by the (many) firms competing in the final good sector for the intermediate good?

(b) Using this demand function to rewrite the profit function of the final good producer such that it depends on $\alpha$ and $y$ only .

2. The monopolist maximises profits

$$\pi_x = px - wL.$$

Production of good $x$ depends linearly on labour and thus technology for the monopolist is given by

$$x = L.$$

What is the optimal price decision for the monopolistic firm?

## 5.8 Properties of the CRRA utility function

1. Intertemporal elasticity of substitution – Why is intertemporal elasticity of substitution constant in: $(c^{1-\sigma} - 1) / (1 - \sigma)$?

2. Logarithmic utility function – What is the limit of $\left(c^{1-\sigma}-1\right)/\left(1-\sigma\right)$ as $\sigma$ tends to 1? Use l'Hôpital's rule plot the function qualitatively for increasing levels of $\sigma$.

Reminder: l'Hôpital's rule says that if $\lim_{x\to c} f\left(x\right) = \lim_{x\to c} g\left(x\right) = 0$ and $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$.

## 5.9 Towards Behavioural Growth

1. A Dual-Self Model of Impulse Control (see Fudenberg & Levine, 2006)

   Consider an infinite-horizon consumer making a savings decision. Wealth $y$ may be divided between consumption and savings according to the savings rate $a \in [0,1]$. The consumer is represented by the short-run self and the long-run self. The parameter $\delta$ is the time preference rate.

   Preferences for the short-run self are

$$u\left(y_t, 0, a_t\right) = \log\left[\left(1-a_t\right) y_t\right]$$

   and savings yield a gross return $R \equiv 1 + r$,

$$y_t = R a_{t-1} y_{t-1}.$$

The short-run self wishes to spend all wealth on consumption. The choice of variable $a$ implies self-control costs

$$C(y_t, a_t) = \gamma \{\log [y_t] - \log [(1 - a_t) y_t]\} \tag{5.3}$$

Preferences for the long-run self are

$$U = \sum_{t=1}^{\infty} \delta^{t-1} [(1+\gamma) \log [(1 - a_t) y_t] - \gamma \log (y_t)]$$

and the savings rate $a$ is constant.

(a) How does the self-control function (5.3) relate to (4.1)?

(b) Find the optimal saving behaviour and give an interpretation to this result.

2. A deterministic OLG model

Consider an agent living for two periods. The constraint in the first period reads

$$w_t = c_t + s_t$$

where $w_t$ is wage at time $t$, $c_t$ is consumption and $s_t$ represents savings. In the second period, i.e. at $t+1$, the constraint reads

$$(1 + r_{t+1}) s_t = c_{t+1}$$

where $r_{t+1}$ is the interest rate, and consumption at $t+1$ is given by the value of savings at $t$ plus interests.

(a) Solve the maximisation problem

$$\max_{s_t} u(c_t) + \beta u(c_{t+1})$$

given the first- and second-period constraints.

(b) Assuming Cobb-Douglas preferences, find the optimal consumption and saving paths for

$$\max_{s_t} \gamma \ln c_t + (1-\gamma) \ln c_{t+1}.$$

(c) Let there be many firms, employing capital $K$ and labour $L$ to produce output $Y$ according to

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

where $A$ is a constant measure of total factor productivity. Capital stock in $t+1$ is a function of labour size (in each period) and savings,

$$K_{t+1} = Ls_t$$

Households behave exactly as in $b$ above. Using your results from $b$, find the expression for the capital stock at $t+1$ and draw its phase diagram. Give an interpretation of the graph.

## 5.10 Public solution key for ex. 5.8: Properties of the CRRA utility function

1. Solution to 1

   – see tutorial –

2. Solution to 2.

   First we need to note l'Hôpital's rule: If $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$, and $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$.
   Now, we have the following here:

$$
\begin{aligned}
f(\sigma) &= c^{1-\sigma} - 1 \\
g(\sigma) &= 1 - \sigma \\
\lim_{\sigma\to 1} f(\sigma) &= c^{1-1} - 1 = 1 - 1 = 0 \\
\lim_{\sigma\to 1} g(\sigma) &= 1 - 1 = 0
\end{aligned}
$$

   Thus the first condition is met. Let us now assume that the second condition, i.e. the existence of the limit of the quotient of the derivatives exists, so that we can apply l'Hôpital's rule.

$$\lim_{\sigma\to 1} \frac{f(\sigma)}{g(\sigma)} = \lim_{\sigma\to 1} \frac{f'(\sigma)}{g'(\sigma)} = \lim_{\sigma\to 1} \frac{-c^{1-\sigma}\ln c}{-1} = \lim_{\sigma\to 1} \left(c^{1-\sigma}\ln c\right)$$
$$= \ln c \left(\lim_{\sigma\to 1} c^{1-\sigma}\right) = \ln c \left(c^{1-1}\right) = \ln c$$

Thus the limit of the quotient of the derivatives of the functions does exist, and is equal to the natural log of $c$.

GUTENBERG SCHOOL OF MANAGEMENT & ECONOMICS

Johannes-Gutenberg Universität Mainz

Master in International Economics and Public Policy 1st Semester

# Advanced Macroeconomics

2023/2024 winter term

Klaus Wälde (lecture) and Motoaki Takahashi (tutorial)
www.macro.economics.uni-mainz.de

# Part II
# Business cycles

## 7 Some numbers on business cycles

## 7.1 GDP and economic growth theory

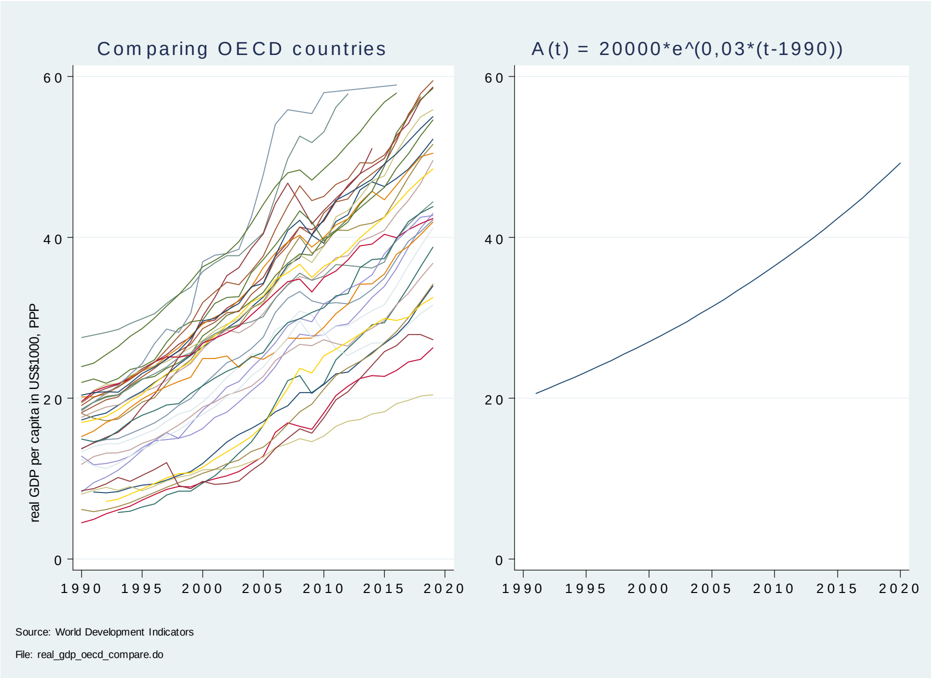



**Figure 25** *Real world GDP and average growth*

- Real world GDP and average growth
  - Real world GDP in left panel grows (and falls) irregularly
  - Yet, there is some overall tendency for a rise in GDP
  - Average growth can easily be estimated (as in right panel)
- There is an obvious difference between observed GDP and average growth
- Average growth is understood by growth theory
- Deviations in data are subject of business cycle analysis
- Central question: What is average growth, what is deviation from average growth?

## 7.2 Identifying "booms and recessions"

### 7.2.1 The idea

- Age-old question
  - Early systematic analysis by Burns and Mitchell (1946), "Measuring business cycles"
  - Ups and down in economic activity are reported for medieval markets and for much earlier times
- Central idea of modern approaches
  - Split a time series into a growth (or trend) component and a cyclical component
  - Cyclical component = time series - trend component
- Question: how find trend component?

### 7.2.2 An example

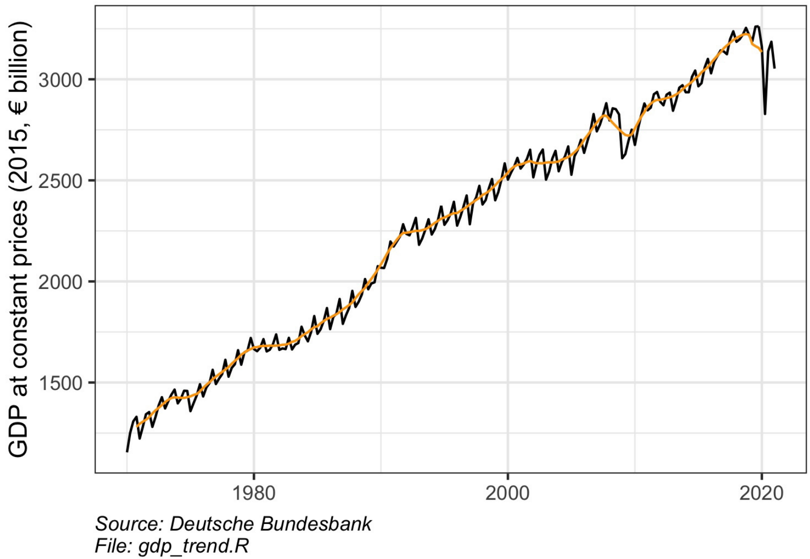



**Figure 26** *GDP in Germany (black line) and trend component (orange line). Source: Deutsche Bundesbank*

### 7.2.3 The methods

- Various so-called filters are used to obtain trend component
  - Hodrick-Prescott filter
  - Baxter-King filter
  - Beveridge-Nelson decomposition and other ...
- Simple methods
  - Moving averages
  - Linear regression on time
  - See tutorial 10.1 for an (Excel) example for moving averages
- Business cycle dating groups
  - Eurocoin (2014)
  - NBER's Business Cycle Dating Committee (2010)

### 7.2.4 The outcomes

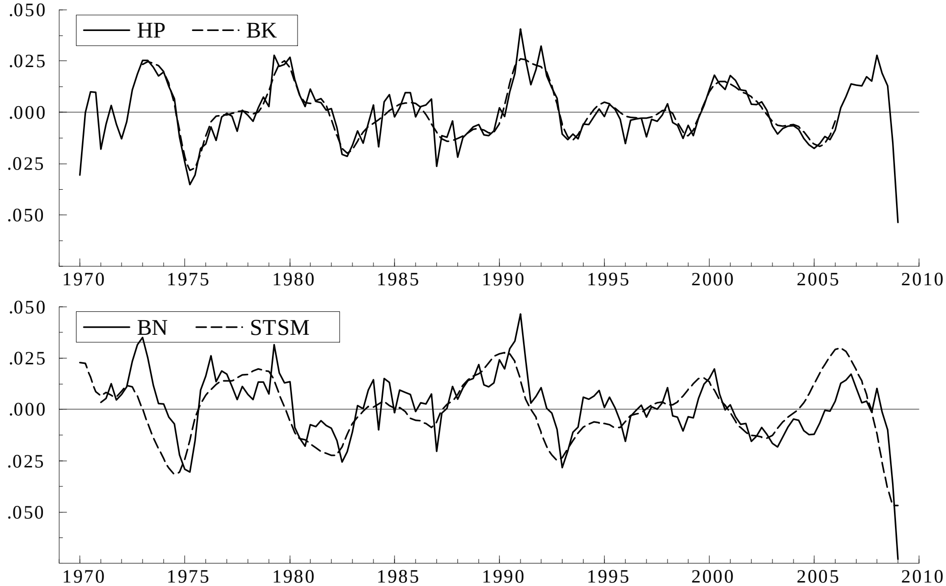



**Figure 27** *Cyclical components (=time series - trend) in Germany for 4 filters*
*Source: Marczak and Beisinger (2013, Fig. 1)*

- Business cycles are identified
  - as periods with above average growth and
  - as periods with below average growth and
  - by their peaks and troughs (maximum and minimum)
- Identification for Germany (Schirwitz, 2009, Table 3)

| Peak | Trough |
|---|---|
| 1974:1 | 1975:2 |
| 1980:1 | 1982:3 |
| 1992:1 | 1993:1 |
| 1995:3 | 1996:1 |
| 2002:3 | 2004:3 |

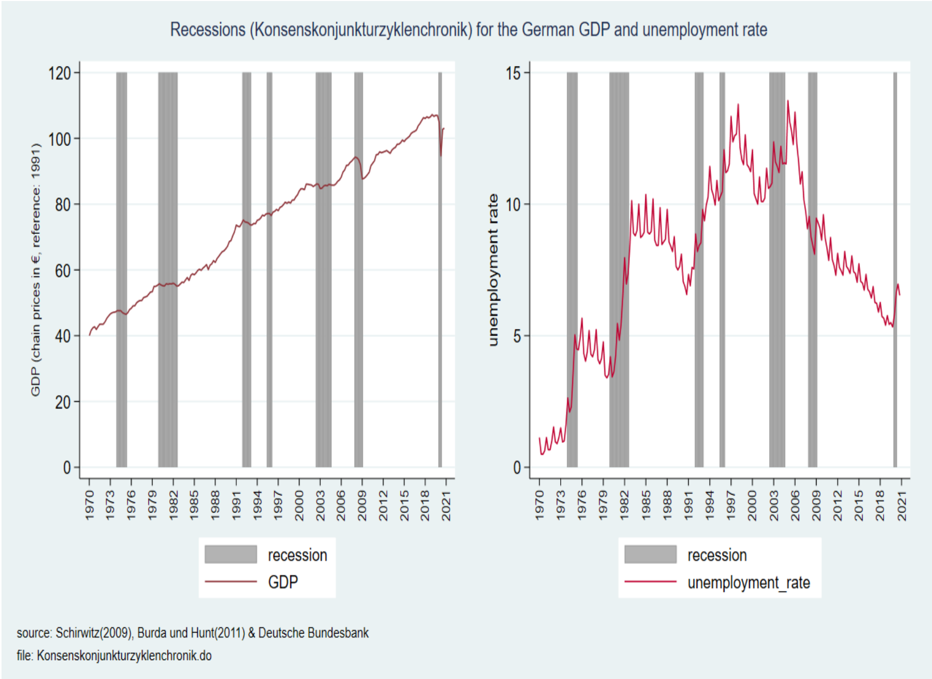



**Figure 28** *Recessions in Germany since 1970, GDP and the unemployment rate*

## 7.3 Questions for economic theory

- Where do business cycles come from?
  - Business cycles are a central feature of any economy
  - They are widely discussed in the public and academia
  - We therefore need answers
- What should economic policy do when there are business cycles?
  - One of the most discussed topic in economics
  - Stabilize or not?

- Leading answers I (origin)
  - Exogenous technology shocks
  - Endogenous technology shocks
  - Sunspot shocks (i.e. irrelevant events, animal spirits)
- Leading answers II (normative implications)
  - Nothing
  - A lot
  - Perfect example for understanding the interplay between theoretical and empirical economics
- Why business cycles are also important in this Masters programme
  - provide essential models and tools for the rest of the Master
  - go through OLG model
  - learn to work with uncertainty in a simple way
  - understand general equilibrium analysis

# 8 Business cycle theories and recent recessions

## 8.1 The real business cycle approach

### 8.1.1 Some references

- Kydland and Prescott (1980) “A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy“
- Kydland and Prescott (1982) “Time to Build and Aggregate Fluctuations“
- Romer (1996) “Advanced Macroeconomics” (ch. 4)
- Wälde (2012) “Applied Intertemporal Optimization“ (ch. 8.1)

### 8.1.2 Overview of a simple real business cycle (RBC) model

- Individuals live for 2 periods (e.g. young working and old retired)
- Rational expectations, all uncertainty is taken into account
- Firms act under perfect competition
- Closed economy in general equilibrium
- Time is discrete

### 8.1.3 Technology

- Aggregate technology
$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{8.1}$$
where $K_t$ is capital stock in $t$ and $L_t$ is employment and $0 < \alpha < 1$

- Crucial new aspect

  – Total factor productivity $A_t$ is uncertain and it can take $n$ different technological levels,
  $$A_t \in \{a_1, a_2, ..., a_n\}, \tag{8.2}$$
  where probability that TFP at time $t$ takes a value $a_i$ is given by $\pi_i$,
  $$\pi_i \equiv \text{Prob}\,(A_t = a_i), \tag{8.3}$$
  and where the "state" of the economy is denoted by $i$

  – Imagine this distribution implies a mean $A$ and variance $\sigma^2$

  – Drawing takes place from identical distribution for each $t$

  – TFP $A_t$ is i.i.d. (identically and independently distributed)

  – Implication: there is *no* growth in this model

  – Economic importance: TFP is random, there are technology shocks

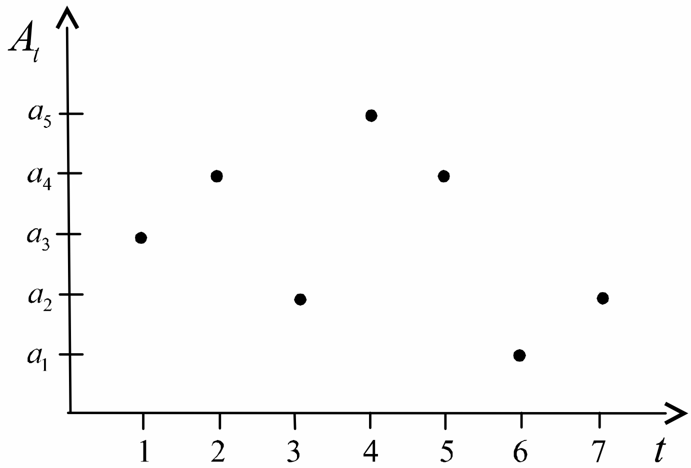
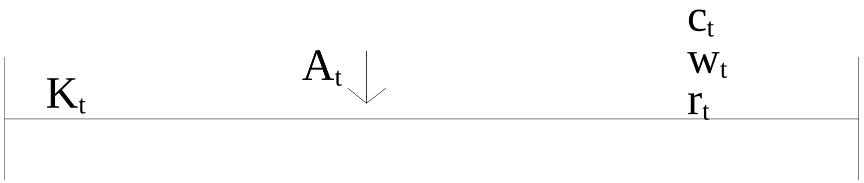



**Figure 29** *One example for values of TFP over 7 points in time (quarters, years ...)*

### 8.1.4 Timing

- As time is discrete and as there is uncertainty, we need to know when various "things" happen




- $K_t$ is inherited from last period
- $A_t$ is realized afterwards, i.e. realization of random variable TFP is known
  - like throwing a realization 4 (or 1 or 2 or 3 or 5 or 6) with a dice
  - here: knowing which $a_i$ determines the TFP level $A_t$ in this period
- Afterwards firms pay wage and interest rate and households choose consumption

### 8.1.5 Firms

- Is life of firms more complicated? Do their decisions need to take uncertainty into account?
- No, firms maximize profits in a deterministic fashion as
  - they rent factors of production ($K$ and $L$) in each period on spot markets
  - they know realization of TFP before making this decision

$$w_t = p_t \frac{\partial Y_t}{\partial L_t} \tag{8.4}$$
$$r_t = p_t \frac{\partial Y_t}{\partial K_t}$$

  - Firms equate value ($p_t$) marginal productivities ($\partial Y_t / \partial L_t$ and $\partial Y_t / \partial K_t$) to factor rewards ($w_t$ and $r_t$)
  - Firms do not bear any risk

### 8.1.6 Households and intertemporal optimization

- The OLG structure

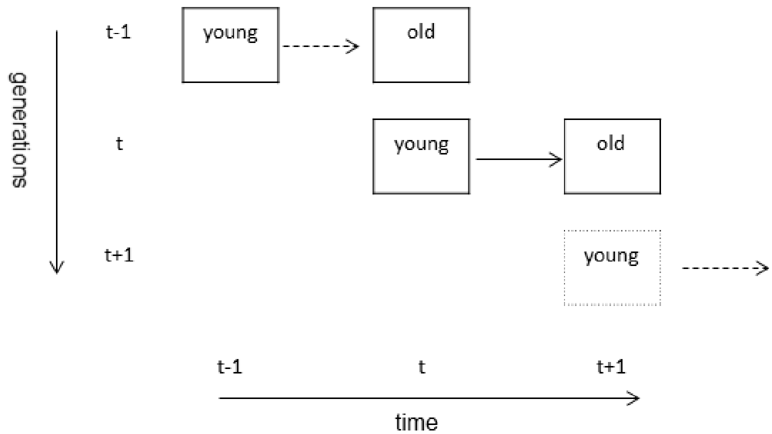



**Figure 30** *Overlapping generations*

- Preferences and constraints
  - Individual consumes in both periods

$$\max E_t \{u(c_t) + \beta u(c_{t+1})\}$$

    and needs to form expectations as consumption (via wage, via TFP $A_t$) is uncertain
  - Individual works only in period $t$
  - $E_t$ is the expectations operator saying that individual forms expectations in $t$ and takes all knowledge in $t$ into account

- Constraint in the first period (period $t$)

$$w_t = c_t + s_t$$

where $s_t$ is savings in $t$ and where we normalized the price $p_t$ of the consumption good (visible e.g. in the first-order conditions (8.4) of the firm to one)

- Constraint in the second period (individual is retired)

$$(1 + r_{t+1})\, s_t = c_{t+1}$$

where left-hand side is income in period $t+1$ (savings plus interest on savings) and right-hand side is consumption expenditure

- Solving the maximization problem

$$\max_{s_t} E_t \left\{ u\left(w_t - s_t\right) + \beta u\left(\left(1 + r_{t+1}\right) s_t\right) \right\}$$

  - First step: replace consumption levels by expressions from budget constraints, this gives nice trade-off when choosing $s_t$
  - Second step: what is uncertain?

$$\max_{s_t} \left\{ u\left(w_t - s_t\right) + \beta E_t u\left(\left(1 + r_{t+1}\right) s_t\right) \right\}$$

    only interest rate $r_{t+1}$ is unknown in $t$. We only need to form expectation about $u\left(.\right)$ in second period

- What is this expectations operator?
  - Consistent with optimal behaviour of firms from (8.2) and the discrete nature of TFP from (8.2), the interest rate $r_{t+1}$ is a discrete random variable
  - As a consequence, $u\left(\left(1+r_{t+1}\right)s_t\right)$ is a random variable and its mathematical mean is given by the sum of its realizations times their probabilites,

    $$E_t u\left(\left(1+r_{t+1}\right)s_t\right)=\Sigma_{i=1}^{n}\pi_i u\left(\left(1+r_{i,t+1}\right)s_t\right)$$

    where $r_{i,t+1}$ is the realization of the interest rate in $t+1$ when the economy is in state $i$, $\pi_i$ is the probability for this realization from (8.3) and $n$ is the number of states of TFP from (8.2)
  - Hence, the objective function reads

    $$\max_{s_t}\left\{u\left(w_t-s_t\right)+\beta\Sigma_{i=1}^{n}\pi_i u\left(\left(1+r_{i,t+1}\right)s_t\right)\right\}$$

  - This approach of forming expectations about future uncertainty originated from the 'rational expectations hypothesis' in economics by Robert Lucas (Chicago)

- An example
  - Consider a Cobb-Douglas utility function

$$E_t \{\gamma \ln c_t + (1-\gamma) \ln c_{t+1}\} \tag{8.5}$$

  - With constraints as above, this yields simple optimal behaviour (see tutorial 10.3)

$$\begin{aligned} c_t &= \gamma w_t \\ s_t &= (1-\gamma) w_t \\ c_{t+1} &= (1+r_{t+1})(1-\gamma) w_t \end{aligned} \tag{8.6}$$

- Is there any uncertainty left?
  - Yes, $r_{t+1}$ is unknown in $t$
  - Actual, realized consumption in $t+1$ differs from expected consumption

### 8.1.7 Aggregation over individuals and firms

- What is capital stock in $t+1$?
  - Remember that we have a model of overlapping generations
  - The capital stock in $t+1$ originates from savings in $t$ by 'the young'
  - These savings are invested when 'old' in $t+1$
  - Hence,
$$K_{t+1} = Ls_t$$
    where $L$ is the size of the labour force (in each period)

- With results from above

$$K_{t+1} = Ls_t = L\left[1-\gamma\right]w_t = \left[1-\gamma\right](1-\alpha)\,Y_t, \tag{8.7}$$

  where the last step

  - replaced wage by marginal productivity of labour and
  - used Cobb-Douglas assumption for technology
  - (see again tutorial 10.3 for details)

- Using Cobb-Douglas technology $Y_t = A_t K_t^\alpha L^{1-\alpha}$ from (8.1) again gives

$$K_{t+1} = (1-\gamma)(1-\alpha) A_t K_t^\alpha L^{1-\alpha}$$

- This equation describes the intertemporal evolution of the economy by linking period $t$ to period $t+1$ (by looking at the capital stock)
- It allows to understand the role of uncertainty as TFP $A_t$ is on the right-hand side

### 8.1.8 The dynamics of TFP, the capital stock and output

- The phase diagram

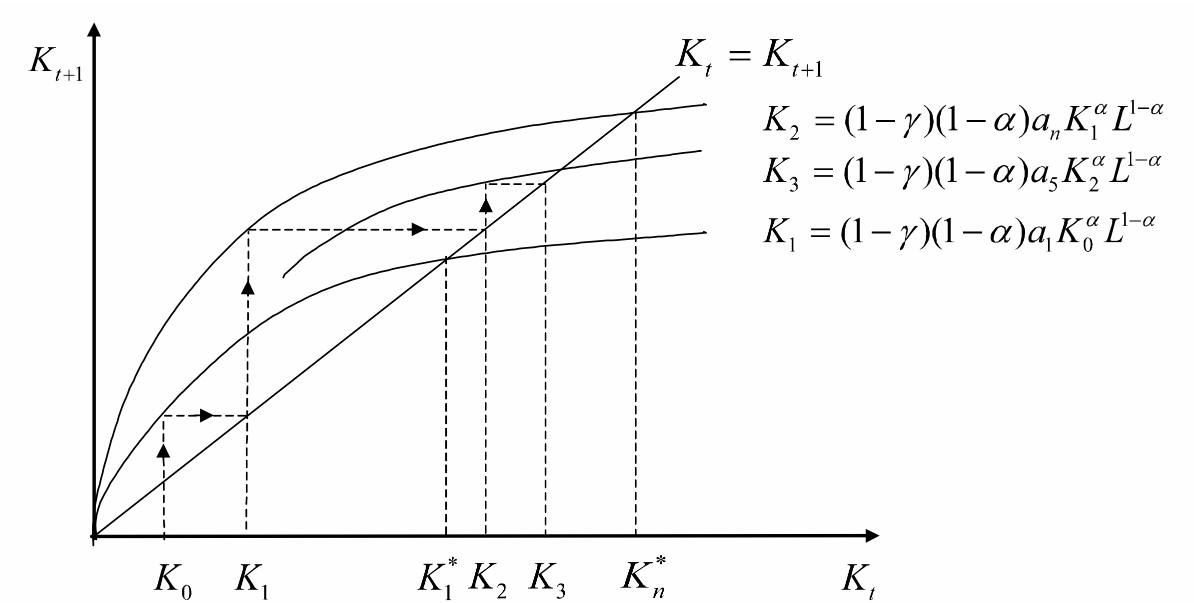



**Figure 31** *Convergence towards a "stochastic steady state"*

- The dynamics of $K_t$ for case of certainty
  - Consider first a case of certainty and study the evolution of $K_t$ (see tutorial 10.3)
  - Assume we are permanently in the worst realisation, i.e. the case of $A_t = a_1$ from (8.2). Then $K_t$ ends up in low steady state $K_1^*$
  - Assume we are always in the best of all (economic) worlds, i.e. $A_t = a_n$. Capital stock $K_t$ ends up at high steady state $K_n^*$
- The dynamics of $K_t$ for case of uncertainty (as illustrated in fig. 31)
  - In $t = 0$ we can predict $K_1$ but not $K_2$ as TFP $A_1$ is unknown
  - Once we are in $t = 1$, we can compute $K_2$ but not $K_3$ ... and so on
  - As we can compute the path of $K_t$ for a sequence of realizations of $A_t$, we can also compute the path of output or any other variable we are interested
- The long-run
  - In the long-run, the capital stock is distributed between $K_1^*$ and $K_n^*$
  - We do not get statements about capital stock or output in the long run but only about its distribution ("where will it probably be")
  - (see Wälde, 2012, ch. 7.4.1 for details)

### 8.1.9 First quantitative findings

- Let us now solve the model numerically to see the link to the cyclical components as e.g. in fig. 27
- We assume that $A_t \in \{90, 100, 110\}$ with probabilities $\pi_i = 1/3$. We set $\gamma = .8$ and $\alpha = .3$.
- We start from $k_0 = 0.5\ k^*$ where $k^*$ is the deterministic steady state for $A_t = 100$ for all $t$, i.e.

$$k^* = (1-\gamma)(1-\alpha)\,100\,(k^*)^\alpha \Leftrightarrow$$
$$k^* = [(1-\gamma)(1-\alpha)\,100]^{1/(1-\alpha)}.$$

- We compute

$$k_{t+1} \equiv \frac{K_{t+1}}{L} = (1-\gamma)(1-\alpha)\,A_t k_t^\alpha$$

  from $t = 0$ to $T = (2010 - 1970) * 4$

- The time-series for capital $\{k_t\}$ and output per capita $\{A_t k_t^\alpha\}$ are shown in the next figure
- This figure should be compared with empirical deviations from the trend in fig. 27
- At this point, empirical analysis, from calibration via reduced form analysis to structural estimation, would start ...

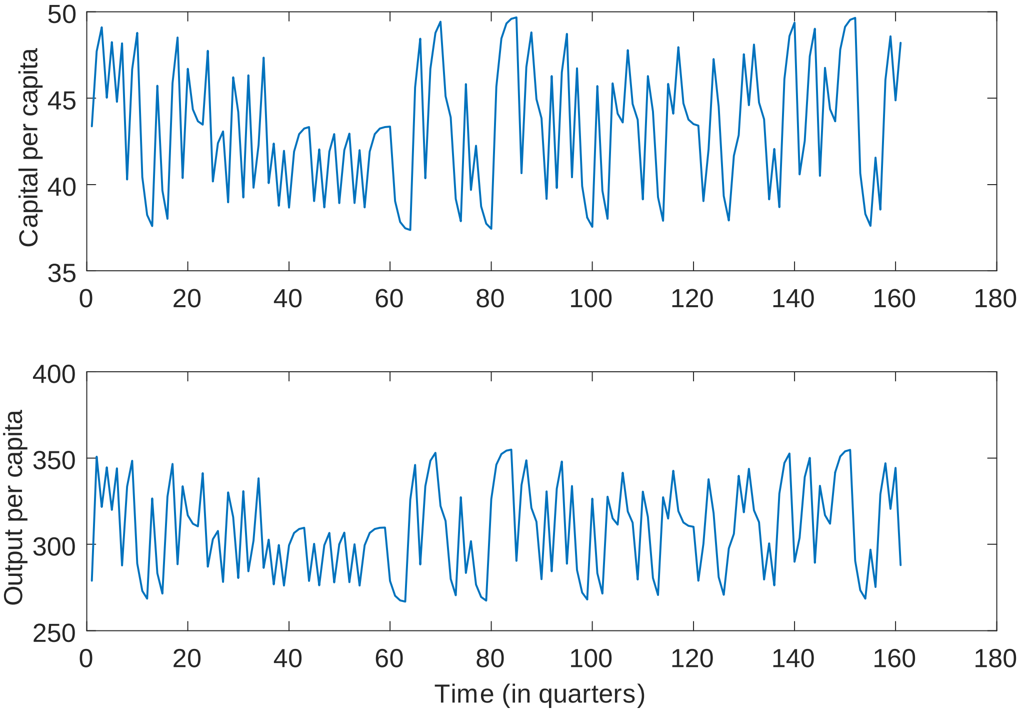



**Figure 32** *Capital and output for the OLG model with stochastic TFP*

### 8.1.10 What have we learned?

- The origins of business cycles
  - Business cycles are the results of shocks to technology
  - (One can just as easily imagine shocks to preferences, international prices, endowment)
  - These shocks are random and their realization can not be predicted
  - Agents know however that there are shocks (rational expectations)
  - Shocks in the real business cycle approach are as exogenous as technological growth in the Solow growth model
- Which of the observed business cycles can plausibly be explained by technology shocks?
  - Oil price shocks of the 1970s (see fig. 28)
  - Reunification of Germany (negative technology shock)
  - What about the financial crisis starting 2007 or the Covid-recession? → see below

## 8.2 Natural volatility

### 8.2.1 Some background

A definition

- Natural volatility is a view of
  - why growing economies experience
  - phases of high and phases of low growth
- The central belief is that both long-run growth and short-run fluctuations are jointly determined by economic forces that are inherent to any real world economy
- Long-run growth and short-run fluctuations are both endogenous and two sides of the same coin: They both stem from the introduction of new technologies

Comparison to other approches

- No exogenous shocks occur according to this approach
- Natural volatility models differ from
  - real business cycle (RBC) and
  - sunspot models and also from
  - endogenous growth models with exogenous disturbances
- Various models analyse endogenous growth jointly with endogenous volatility
  - An overview is provided at https://www.macro.economics.uni-mainz.de/teaching/master/ (see right column)
  - This section looks at a simple model that provides the basic intuition and follows Wälde (2012, ch. 8.4)

### 8.2.2 The basic idea

- Some measure of productivity (this could be labour or total factor productivity) does not grow smoothly over time (as in most models of exogenous or endogenous long-run growth)
- Productivity follows a step function

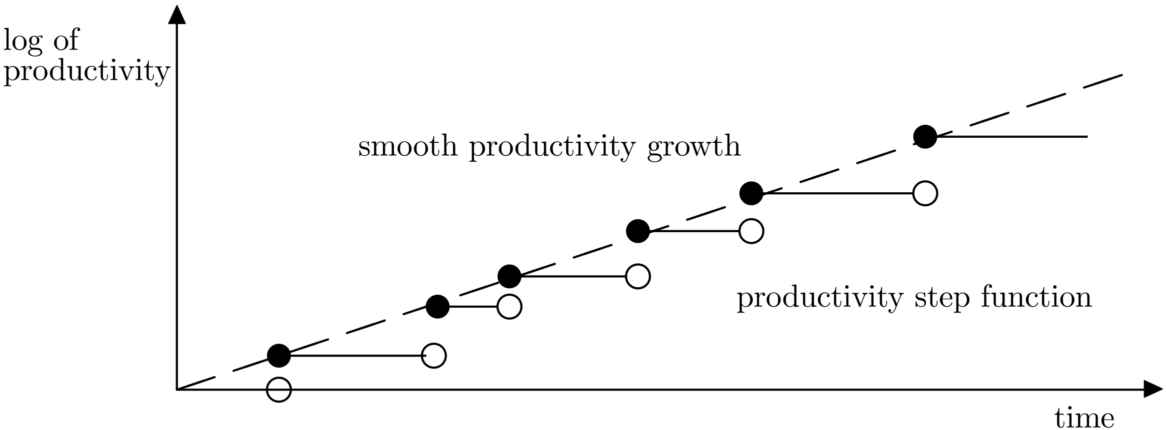



**Figure 33** *Smooth productivity growth in balanced growth models (dashed line) and step-wise productivity growth in models of natural volatility*

Understanding the figure

- Time on the horizontal and the log of productivity on the vertical axis
- Smooth productivity growth path as the dashed line
- In models of natural volatility, the growth path of productivity has periods of no change almost all of the time and some points in time of discrete jumps
- After a discrete jump, returns on investment go up and an upward jump in growth rates results
- Growth rates gradually fall over time as long as productivity remains constant
- With the next jump, growth rates jump up again
- Step function implies long-run growth and short-run fluctuations

The economic mechanisms

- Economic reasons for this step function follows from some deeper mechanisms
- A short list includes Fan (1995), Bental and Peled (1996), Freeman, Hong and Peled (1999), Matsuyama (1999, 2001), Wälde (1999, 2002, 2005), Li (2001) Francois and Lloyd-Ellis (2003,2008), Maliar and Maliar (2004), Phillips and Wrase (2006) and Posch and Wälde (2011)

### 8.2.3 A simple stochastic model

- Technologies
$$Y_t = A_t K_t^{\alpha} L^{1-\alpha},$$
where $A_t$ represents total factor productivity, $K_t$ is the capital stock and $L$ are (constant) hours worked

- Total factor productivity $A_t$ ...
  - ... can take discrete values as in the RBC model in (8.2)
$$A_t \in \{a_0, a_1, ...\} \tag{8.8}$$
  - ... can increase forever (in contrast to the RBC model)
  - ... starts at $a_0$

- Capital can be accumulated according to
$$K_{t+1} = (1 - \delta) K_t + I_t$$

- Resource constraint
$$C_t + I_t + R_t = Y_t,$$
where $C_t$, $I_t$ and $R_t$ are aggregate consumption, investment and R&D expenditure

- Total factor productivity follows

$$A_{t+1} = (1 + q_t) A_t \tag{8.9}$$

where

$$q_t = \left\{ \begin{array}{c} \bar{q} \\ 0 \end{array} \right\} \text{with probability} \left\{ \begin{array}{l} p_t \\ 1 - p_t \end{array} \right. .$$

The probability depends on resources $R_t$ invested into R&D,

$$p_t = p(R_t). \tag{8.10}$$

Clearly, the function $p(R_t)$ in this discrete time setup must be such that $0 \leq p(R_t) \leq 1$.

- Crucial difference to RBC type approaches: The probability that a new technology occurs is endogenous

- Obviously in the tradition of the "new growth theory" where the growth rate is endogenous

- Here, the source of growth and fluctuations all stem from one and the same source, the jumps in $q_t$

- Optimal behaviour by households
  - Assume that optimal behaviour by households implies constant consumption rate $s_C$ and R&D saving rate $s_R$

$$C_t = s_C Y_t, \quad R_t = s_R Y_t$$

  - This would be the outcome of a two-period maximization problem or an infinite horizon maximization problem with some parameter restriction
  - The literature uses optimal saving rates but the general insight can be obtained from this simplified approach as well

### 8.2.4 Equilibrium

- The reduced form
  - Given saving rates and conditional on the current technology, the capital stock evolves according to (see tutorial 10.4)

$$\begin{aligned} K_{t+1} &= (1-\delta)\, K_t + Y_t - R_t - C_t \\ &= (1-\delta)\, K_t + (1 - s_R - s_C)\, Y_t \\ &= (1-\delta)\, K_t + (1 - s_R - s_C)\, A_t K_t^{\alpha} L^{1-\alpha} \end{aligned}$$

  - This is a stochastic difference equation in $K_t$ where uncertainty enters via TFP $A_t$
  - In each period, the probability of a jump depends on investment in R&D

$$p_t = p\,(s_R Y_t) = p\left(s_R A_t K_t^{\alpha} L^{1-\alpha}\right)$$

  - With some initial capital stock $K_0$, these two equations describe how $K_t$ changes and with which probability $A_t$ increases

- Equilibrium dynamics
  - Assume we start with a technological level of $a_0$ from (8.8)
  - Let there be no technological progress for a while, i.e. $q_t = 0$ for a certain number of periods $t$
$$A_t = a_0 \text{ for } t < T$$
  - Then, capital stock converges to its "temporary steady state" defined by $K_{t+1} = K_t \equiv K$, see tutorial 10.4, question 1,
$$\left(\frac{K}{L}\right)^{1-\alpha} = a_0 \frac{1 - s_R - s_C}{\delta}$$
  - In this temporary steady state, variables are constant only temporarily until the next technology jump occurs

- The phase diagram

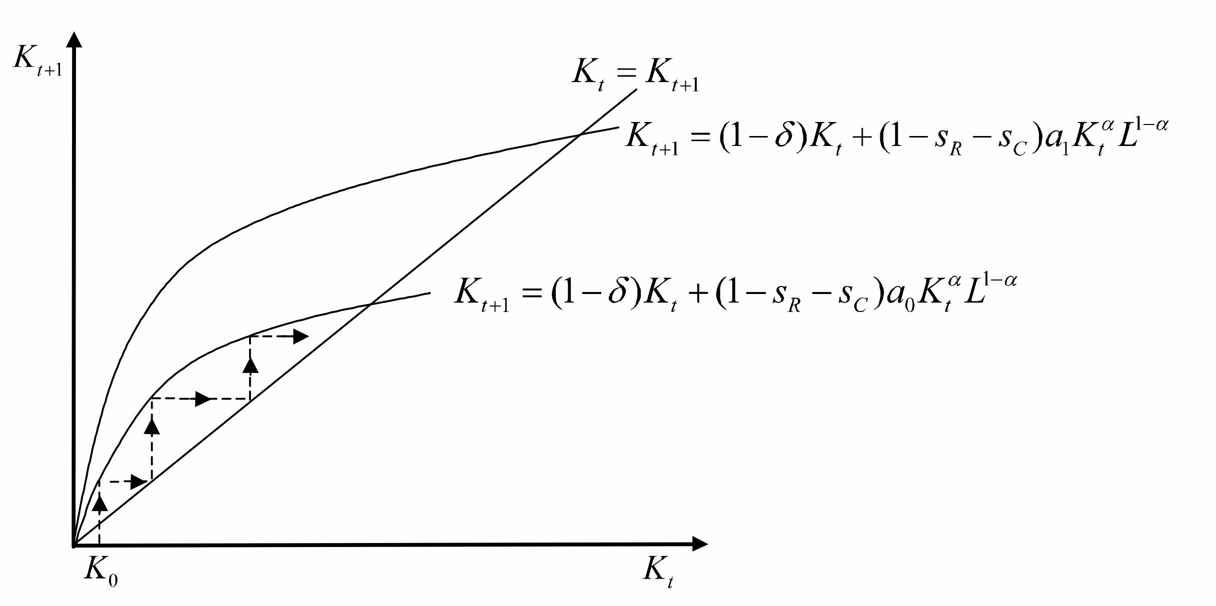
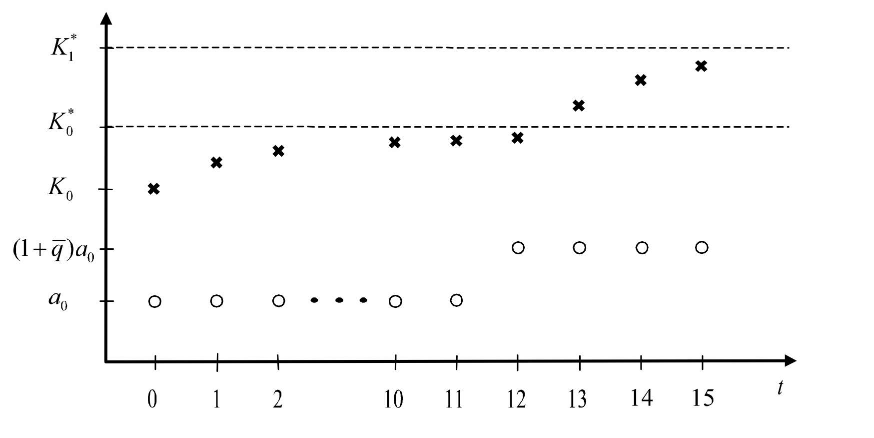



**Figure 34** *The Sisyphus economy - convergence to a temporary steady state*

Understanding the figure

- With a new technology in, say, period $T = 12$, total factor productivity increases from $a_0$ to $a_1 = (1 + \bar{q})\, a_0$
- The $K_{t+1}$ line shifts upwards and the (next temporary) steady state increases
- Subsequently, the economy approaches this new steady state
- The growth rate after the introduction of a new technology is high and then gradually falls
- Cyclical growth is therefore characterized by a Sisyphus-type behaviour
  - $K_t$ permanently approaches the current, temporary steady state
  - Every now and then, however, this steady state jumps outwards and capital starts approaching it again

How do variables evolve over time?




- The economy starts with TFP at $a_0$ and experiences an increase at $t = 12$ to $a_1 = (1 + \bar{q})\, a_0$
- The initial capital stock is $K_0$ and capital converges to $K_0^*$, the steady state corresponding to $a_0$
- With a TFP at $a_1$, the capital stock converges to new (temporary) steady state at $K_1^*$
- This continuous forever – the economy is (endogenously) growing in (endogenous) cycles

### 8.2.5 What have we learned?

- One can imagine a truly Schumpetarian explanation of both business cycles and growth
- Schumpetarian means that there
  - is not only endogenous long-run growth but there
  - are also endogenous short-run fluctuations
- Schumpetarian growth models allow to understand how the long-run growth rate is determined by economic incentives
- The natural volatility view stresses that short-run fluctuations are also "home-made" and therefore a function of economic policies

- Related empirical questions
  - What are the empirical counterparts of these new technologies?
    - In recent decades: PCs, Internet, mobile phones
    - Earlier: steam engine, combustion engine, solar energy generation ...
    - See literature on technological innovations and diffusions
  - What is the relationship between volatility and growth?
    - Do countries with higher volatility grow less quickly?
    - What is the causal link?
    - see Ramey and Ramey (1995) and the subsequent literature

## 8.3 Sunspot cycles

### 8.3.1 Central references

- Benhabib and Farmer (1994) “Indeterminacy and Increasing Returns“
- Farmer and Guo (1994) “Real Business Cycle and the Animal Spirits Hypothesis”
- Howitt and McAfee (1992) “Animal spirits”

### 8.3.2 The basic argument

- "Usual" structure of economic models
  - Unique equilibrium
  - Unambiguous policy recommendations
  - Example: Central planner problem studied in ch. 3.1.3

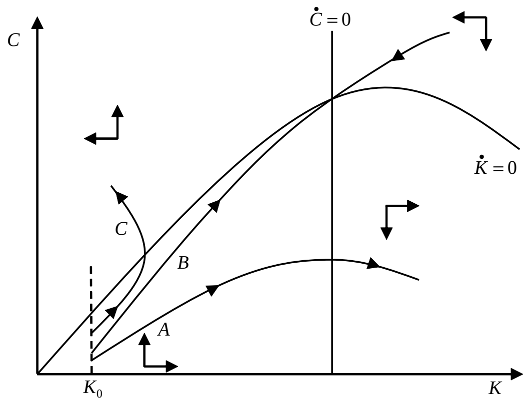



**Figure 35** *Unique equilibrium path for optimal consumption of a central planner (copied from fig. 16)*

- Phase diagram analysis showed that
  - there is exactly one $C_0$ such that the economy is on the saddle path to the unique steady state
  - unique equilibrium has advantage of unambiguous comparative static properties
- Further example: real business cycle models
  - Positive technology shock leads to boom
  - Negative technology shock leads to recession
  - Unambiguous relationship between parameters of the model (like TFP) and GDP, consumption, investment, wages ...

- The argument of “sunspot models”
  - Real world economies do NOT have unique equilibrium
  - There are many equilibria and there can actually be a continuum of equilibria
  - “Animal spirits” (Keynes, John Stuart Mill, Hayek, Thornton) can lead to business fluctuations
  - Fluctuations are due “to random waves of optimism and pessimism that are unrelated to fundamental conditions” (Howitt and McAfee, 1992)

- What are the causes of these indeterminacies?
  - Externality a la Romer (1986)
    - First model of new growth theory (see above on p. 3.31)
    - $Y = A(K) K^{\alpha} L^{1-\alpha}$ where TFP $A(K)$ is a public good resulting from knowledge
    - With $A(K) = A_0 K^{1-\alpha}$, there are constant returns to scale to accumulable factors
    - Marginal productivity of capital with $Y = A_0 K L^{1-\alpha}$ is constant and there is perpetual growth
  - Differentiated goods as in Dixit and Stiglitz (1977)
  - Standard structures lead to non-standard properties, i.e. to indeterminacies
  - For details see Benhabib and Farmer (1994)

### 8.3.3 A phase diagram illustration

- The phase diagram

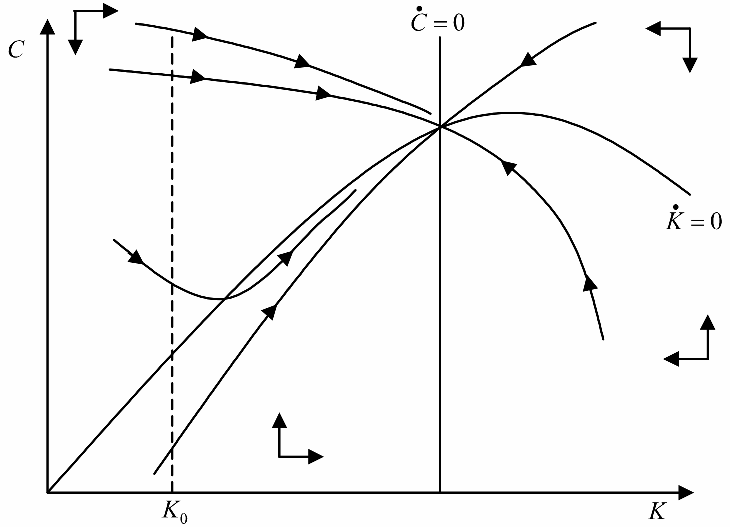



**Figure 36** *Multiple equilibrium paths*

Main point of this figure

- For a given $K(0)$ there are many (and in this case actually a continuum of) initial consumption levels $C(0)$ and they all lead to a unique steady state
- Exogenous events (mood, animal spirit, sunspots, non-fundamental issues) push economy between paths
- One can imagine an exogenous stochastic process which determines the occurrence of these non-fundamental events (like sunspots)
- Exogenous events lead to business cycles which have no fundamental meaning

### 8.3.4 What have we learned?

- Fluctuations of consumption, output and growth have no fundamental meaning
- Alternative to RBC view (with respect to fundamentals)
- Alternative to natural volatility view (with respect to origin of shocks)

## 8.4 Recent real-world business cycles

- Let us now look at real-world recessions
  - We will understand the anatomy of a recession in more detail than by looking at "technology shocks"
  - We will nevertheless understand that technology shocks are a useful shortcut to understand certain aspects of a recession
- We start with the great recession from 2007/08
- We also briefly look at the Covid-19 pandemic and its economic consequences

### 8.4.1 The great recession of 2007/2008

- The story
  - It all started with the bursting of the housing market bubble in the US in 2007
  - Bursting occurred as
    - there was a bubble in the first place (new market structure in mortgage market - "originate and distribute" - new assets)
    - (it had to burst as) the FED raised interest rates
  - Mortgage crisis led to banking crisis as
    - there was excessive maturity transformation
    - systemic risk caused by shadow banks
  - Banking crisis led to economic crisis as
    - banks would almost stop lending to each other
    - banks would provide less credit to firms (rational and irrational/ emotional component)

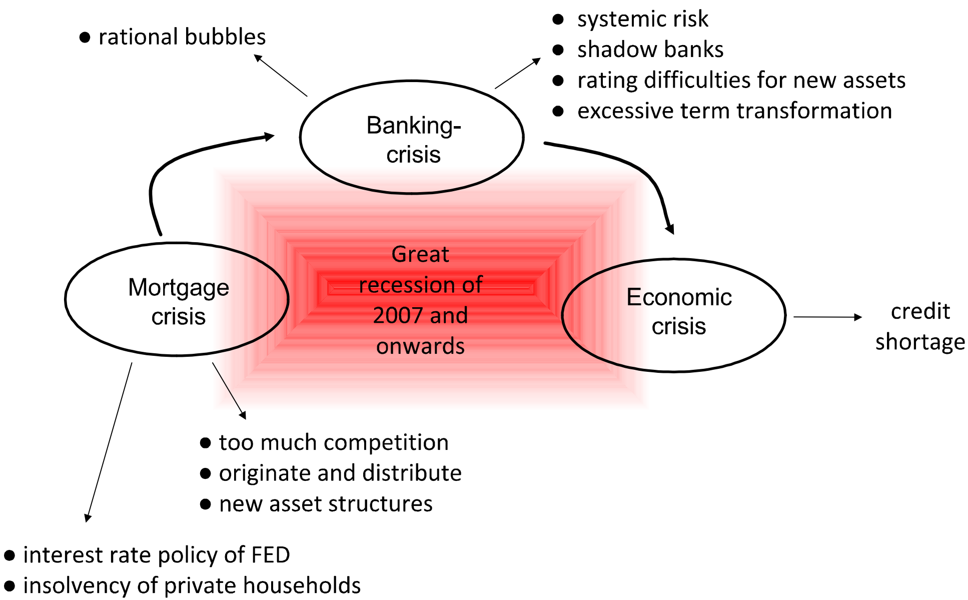



**Figure 37** *The crises, the crisis and potential explanations*

- Background
  - Brunnermeier (2009), Dodd (2007), Hellwig (2009, section 3 and 3.8)
  - Lecture notes Macro I (Bachelor) (see “Teil 2” of “Konjunkturzyklen”, in German)
- Can the great recession be understood by RBC shocks?
  - Not its origin
  - Maybe its effect on output and employment
- The analysis
  - Do it yourself
  - Seminar paper
  - Master thesis

### 8.4.2 The Covid-19 pandemic

The SIR model - a classic framework to understand epidemics

- We can understand data only with some theoretical concepts in mind
- Evaluations (e.g. of the 'severity' of a pandemic) also require abstract ideas about (dynamics) of a pandemic
- Let us therefore look at some concepts on epidemics
- SIR (susceptible, infectious, removed) model
- A *quantitative* version of this model allows us
  - to understand existing and non-existing time series
  - to study the effect of public health measures (PHM)
  - to be able to recommend PHM

- The classic SIR model (Kermack and McKendrick, 1927)
  - Three types of individuals: susceptible $S(t)$, infectious $I(t)$, removed $R(t)$ (recovered or deceased)
  - Arrival rates determine stochastic transitions
  - Note the conceptual link (Poisson processes) to growth (e.g. fig. 21 related to Aghion and Howitt model) or, especially to search & matching models (see ch. 13 below)

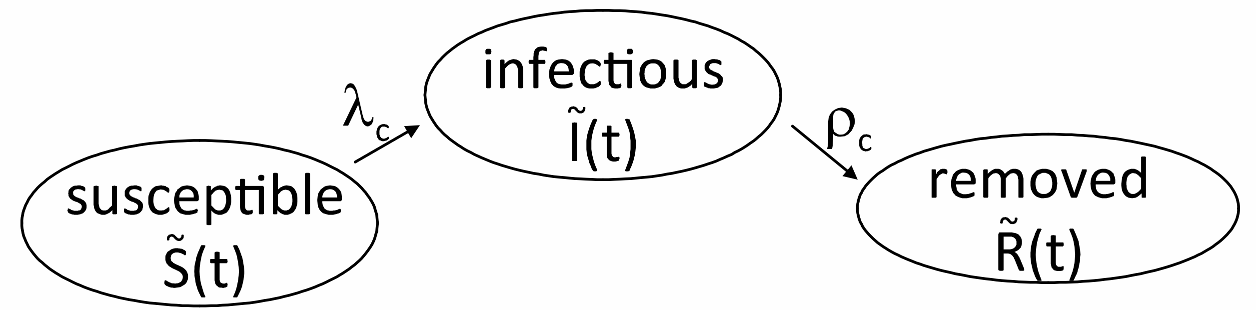



**Figure 38** *Transition between states in the classic SIR model*

- The algebra
  - The number of susceptible individuals falls according to

$$\frac{d}{dt}S(t) = -\lambda_c(t)\, S(t)\,,$$

    where $r$ is a constant and

$$\lambda_c(t) \equiv rI(t) \tag{8.11}$$

    called the individual infection rate
  - Infection rate captures idea that risk of becoming infected is the greater, the higher the number $I(t)$ of infectious individuals
  - Merging individual recovery rate and death into one constant $\rho$, the number of infectious individuals changes according to

$$\frac{d}{dt}I(t) = \lambda_c(t)\, S(t) - \rho I(t)$$

  - As a residual, the number of removed individuals rises over time according to

$$dR(t)/dt = \rho I(t)$$

CoV-2 and the economy

- What were the economic effects of the pandemic?
- What do we learn about reasons for recessions?
- Concerning the anatomy of a recession, where do we (do we?) find a negative technology shock?

Economic consequences of the pandemic

- Covid-19 offers rare example of shock to both demand and supply
  - Demand for goods has decreased (e.g. restaurants and concerts, as individuals try to avoid large gatherings)
  - Supply has also decreased as supply chains got disrupted around the world
- Shock also impacted labour market
  - Sudden increase in unemployment as firms laid off workers
  - Reduction in vacancy openings as firms are unsure of future demand

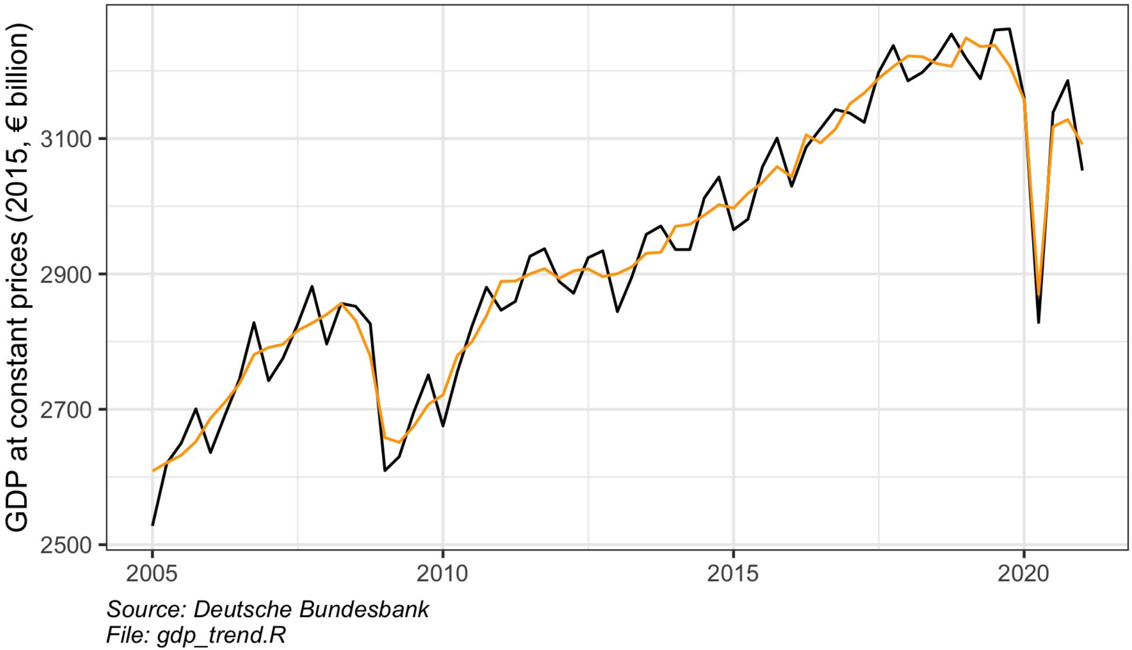



**Figure 39** *Index of GDP in Germany, quarterly data from Statistisches Bundesamt – see Bundesbank for anual data*

- What does figure 39 tell us?
  - More pronounced decline in 1st quarter than after financial crisis
  - More rapid recovery in the subsequent quarter
  - Economic impact of financial crisis lasted for (at least) 3 years
  - Future will show duration of economic consequences of pandemic
- Sectoral employment shifts are visible in the next figure 40
  - Average employment dropped by around 20% (relative to 2020)
  - IT and banking/ insurance hit similarly
  - construction and healthcare hit the least (max minus 10%)
  - hospitalities are unambigous losers with a decline of more than 40%

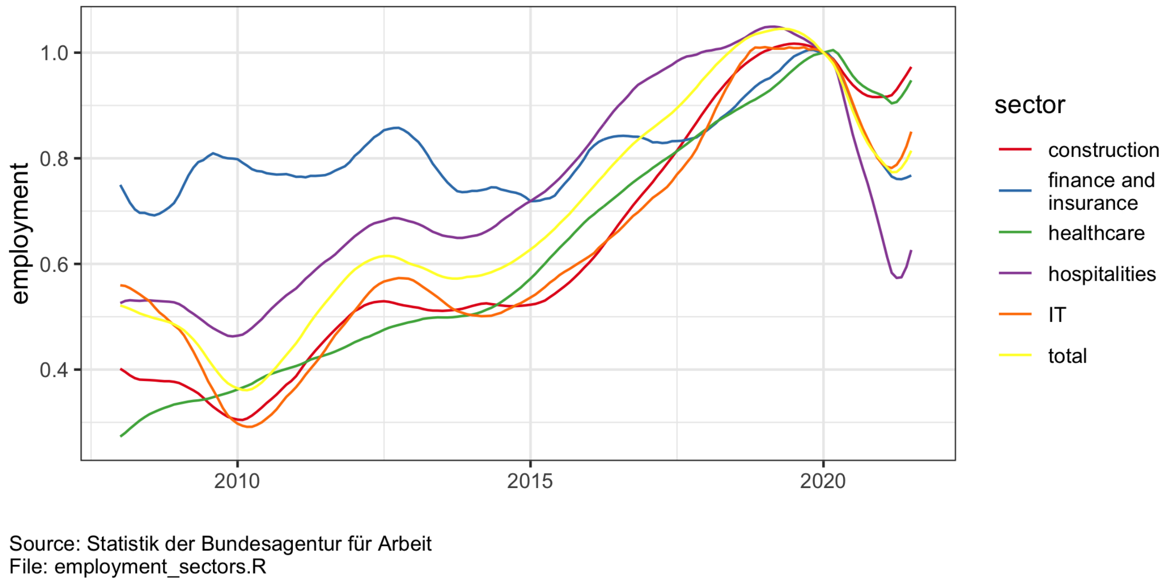



**Figure 40** *Sectoral employment effets of the Corona pandemic*

### 8.4.3 What have we learned?

- ... about the Covid-business cycle and TFP shocks
- Models offer a detailed picture of the various channels from the pandemic to the economy
- Models also make clear that it is not a pure ’technology shock’ that causes the recession
- Yet, the recession and the distributional effects across sectors could be understood by sector-specific technology shocks
- The RBC approach to business cycles can be seen as a useful shortcut to understanding certain features of a business cycles
- When we want to understand potential policy measures against a recession, a more detailed understanding of the origin of the recission is needed
- A similar point can be made about the great recession from 2007/08

# 9 Time inconsistency and business cycles

- Let us again – for the second time in this lecture – go beyond standard mainstream macroeconomic thinking
- Could one imagine more human-made business cycles?
- Is there "typical" and "everyday" human behaviour that might explain business cycles?
- One such candidate is time-inconsistent behaviour

## 9.1 Background on time inconsistency

- General idea
  - A plan is made today about current and future behaviour
  - This plan is executed for today's behaviour
  - When tomorrow has come, the plan that was made for tomorrow is revised rather than executed
- A definition
  - Behaviour is time-inconsistent when a plan made in $t$ about some future point in time $T > t$ is revised at some point after $t$
  - Behaviour in $T$ differs from what was planned in $t$, even though no new information became available

- Literature
  - Time inconsistency: Strotz (1955), Laibson (1997), O'Donoghue and Rabin (1999), Benhabib and Bisin (2005)
  - Survey/ general interest papers
    * Prelec (2004)
    * Bryan Karlan and Nelson (2010, footnote 6)
    * Frederick Loewenstein and O'Donoghue (2002)
  - Caplin Leahy (2006) argue that solution method to be used is dynamic programming, not subgame perfection
  - Textbook: Dhami (2016)

## 9.2 Time-consistent behaviour

[if possible – remember macro II in 3rd year Bachelor]

### 9.2.1 Preferences

- Intertemporal utility function

$$U_t = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) \tag{9.1}$$

- This utility function employs
  - instantaneous utilities $u(c_\tau)$
  - discounted at discount factor $0 < \beta < 1$
  - to give $U_t$ as intertemporal utility
- Planning period starts in $t$ and runs up to $t + 2$

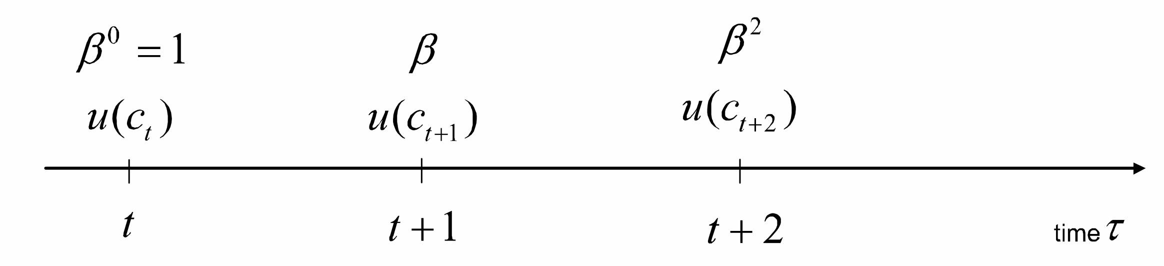



**Figure 41** *Illustrating time, discounting at $\beta < 1$ and instantaneous utility from consumption*

- Instantaneous utility function
  - Instantaneous utility for points in time $\tau = t, t+1, t+2$ is specified to be logarithmic,

$$u\left(c_t\right) = \ln c_t$$

  - This functional form simplifies the analysis but is not essential

- Budget constraint
  - Using $r$ for the interest rate (which is constant across periods)
  - $\tau$ as the time index
  - $p_\tau$ as the price of the consumption good in $\tau$ and
  - $a_t$ as wealth in $t$
  - the budget constraint reads

$$\Sigma_{\tau=t}^{t+2} \left( \frac{1}{1+r} \right)^{\tau-t} p_\tau c_\tau = a_t \tag{9.2}$$

  - In words: the present value of consumption expenditure must equal current wealth
  - (apparently, there is no labour income here – which also simplifies the analysis)

- Budget constraint (again)

$$\Sigma_{\tau=t}^{t+2}\left(\frac{1}{1+r}\right)^{\tau-t}p_\tau c_\tau = a_t$$

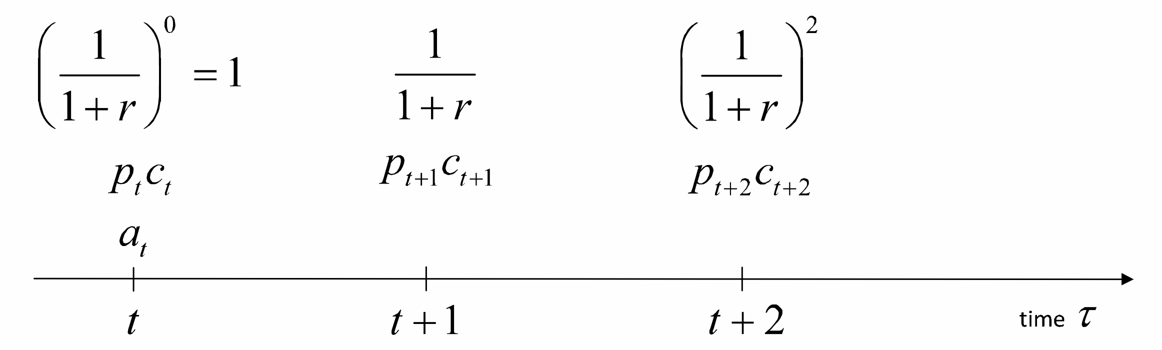



**Figure 42** *Illustrating the budget constraint*

### 9.2.2 Optimal behaviour ...

- Consumption in periods $t$ to $t+2$
  - All consumption levels are proportional to initial real wealth $a_t/p_t$
  - which is corrected for the interest rate and price level (see tut. 10.7)

$$c_t^* = \frac{1}{1+\beta+\beta^2}\frac{a_t}{p_t} \tag{9.3}$$

$$c_{t+1}^* = \frac{\beta}{1+\beta+\beta^2}(1+r)\frac{a_t}{p_{t+1}} \tag{9.4}$$

$$c_{t+2}^* = \frac{\beta^2}{1+\beta+\beta^2}(1+r)^2\frac{a_t}{p_{t+2}} \tag{9.5}$$

- Note the parallel to optimal consumption in the two-period model in (8.6)
  - There is an initial endowment: $w_t$ in (8.6), $a_t$ here
  - A certain share is used for consumption in different periods
  - Shares $\gamma$ and $1-\gamma$ in (8.6), shares $\frac{1}{1+\beta+\beta^2}$, $\frac{\beta}{1+\beta+\beta^2}$ and $\frac{\beta^2}{1+\beta+\beta^2}$ here
  - Consumption in subsequent periods rises in savings and interest rate $r$

### 9.2.3 ... is independent of calender time

- Now let the individual revisit her plans one period later
- Preferences
  - Consider the utility function in (9.1) moved forward into $t+1$

$$U_{t+1} = \beta^{-1} u\left(c_t^*\right) + u\left(c_{t+1}\right) + \beta u\left(c_{t+2}\right)$$

  - We use $c_t^*$ to denote that consumption in $t$ is already chosen as in (9.3) and therefore fix
  - (it is hard to change yesterday's consumption today)
- The budget constraint
  - Given that we are in period $t+1$, we write (9.2) as

$$p_{t+1}c_{t+1} + \frac{p_{t+2}c_{t+2}}{1+r} = \left(a_t - p_t c_t^*\right)(1+r) \tag{9.6}$$

  - The right-hand side stands for endowment left in period $t+1$
  - Endowment in $t+1$ is given by wealth in $t$ minus consumption expenditure in $t$ times $1+r$ (as $a_t - p_t c_t^*$ is saved and interest is paid)

- Optimal consumption
  - Consumption now reads (see tutorial 10.7)

$$c^{**}_{t+1} = \frac{\beta}{1+\beta+\beta^2}(1+r)\frac{a_t}{p_{t+1}}$$

$$c^{**}_{t+2} = \frac{\beta^2}{1+\beta+\beta^2}(1+r)^2\frac{a_t}{p_{t+2}}$$

  - These are the same expressions as in (9.4) and (9.5)
  - It does not matter when the decision about consumption in $t+1$ or $t+2$ is made
  - Whether made in period $t$ or in period $t+1$, the same behaviour results
  - This is called time-consistent behaviour

## 9.3 Time-inconsistent behaviour in a 3-period setup

### 9.3.1 The setup

- Preferences
  - We now generalize preferences of our individual to

    $$U_t = \Sigma_{\tau=t}^{t+2} D(\tau - t)\, u(c_\tau)$$

    where
  - $U_t$ is present value of current and future instantaneous utilities $u(c_\tau)$ (as before)
  - planning period starts in $t$ and runs up to $t+2$ and where the time index is $\tau$ (as before)
  - instantaneous utilities are discounted at discount function $D(\tau - t)$ (new)

- Preferences in the structure as above
  - In a three-period structure as in (9.1), this reads

$$U_t = D(0)\,u(c_t) + D(1)\,u(c_{t+1}) + D(2)\,u(c_{t+2}) \tag{9.7}$$

  - Instead of $\beta^0 = 1$, $\beta^1 = \beta$ and $\beta^2$, we have $D(0)$, $D(1)$ and $D(2)$, i.e. more general functions of time

- Budget constraint
  - ... is identical to before in (9.2), i.e.

$$\Sigma_{\tau=t}^{t+2}\left(\frac{1}{1+r}\right)^{\tau-t} p_\tau c_\tau = a_t$$

### 9.3.2 Optimal behaviour ...

- Optimal consumption
  - Let us consider optimal behaviour from the perspective of period $t$
  - Optimal consumption is given by (see tutorial 10.8)

$$c_t^* = \frac{D(0)}{D(0)+D(1)+D(2)} \frac{a_t}{p_t}$$

$$c_{t+1}^* = \frac{D(1)}{D(0)+D(1)+D(2)} (1+r) \frac{a_t}{p_{t+1}} \tag{9.8}$$

$$c_{t+2}^* = \frac{D(2)}{D(0)+D(1)+D(2)} (1+r)^2 \frac{a_t}{p_{t+2}} \tag{9.9}$$

- Interpretation
  - Similar structure to setup with discount factor $\beta$ as in e.g. (9.4)
  - But: Discount factor $D(t+1)/D(t)$ from one period to the next is not necessarily constant

### 9.3.3 ... can depend on calender time

- Preferences one day later
  - Discount function now moves to period $t+1$ where behaviour is chosen

$$U_{t+1} = D(-1)\,u(c_t^*) + D(0)\,u(c_{t+1}) + D(1)\,u(c_{t+2})$$

  - Again, consumption in $t$ is cannot be adjusted and is given by $c_t^*$
- Budget constraint
  - as above in (9.6), i.e. expressed for one period later

$$p_{t+1}c_{t+1} + \frac{p_{t+2}c_{t+2}}{(1+r)} = (a_t - p_t c_t^*)(1+r)$$

- Optimal behaviour
  - Consumption levels chosen in $t+1$ are given by (see tutorial 10.8)

$$c_{t+1}^{**} = \frac{D(0)(D(1)+D(2))}{(D(0)+D(1)+D(2))(D(0)+D(1))}\,(1+r)\,\frac{a_t}{p_{t+1}} \tag{9.10}$$

$$c_{t+2}^{**} = \frac{D(1)(D(1)+D(2))}{(D(0)+D(1)+D(2))(D(0)+D(1))}(1+r)^2\frac{a_t}{p_{t+2}}$$

- Does individual behave in a time-consistent way?
  - The basic structure of (9.10) is the same as for (9.8) and (9.9)
  - But: Are consumption levels (9.8) and (9.9), chosen in $t$ for $t+1$ and $t+2$, the same as those chosen in $t+1$?
  - Formally, under which conditions does

$$c_{t+1}^{*} = c_{t+1}^{**} \quad \text{and} \quad c_{t+2}^{*} = c_{t+2}^{**}$$

    hold?
  - It holds for (see tutorial 10.8)

$$\frac{D(1)}{D(0)} = \frac{D(2)}{D(1)} \tag{9.11}$$

- What does this condition tell us about time consistency?
  - When this condition (9.11) holds, optimal behaviour does *not* depend on calendar time
  - Behaviour is then time consistent
  - When the condition is violated, time behaviour is time inconsistent and *does* depend on calender time

- Examples for time-inconsistent behaviour
  - Present bias parameter $\delta$, also called "$\delta\beta$-" or "Laibson-preferences"

$$D(0) = 1, \; D(1) = \delta\beta, \; D(2) = \delta\beta^2$$

  - Hyperbolic discounting

$$D(t) = \frac{1}{1 + \alpha t}$$

### 9.3.4 What does condition (9.11) tell us about exponential discounting?

- The necessary condition (9.11) for time-consistency is satisfied by geometric discounting, i.e. by $D(i) = \beta^i$
  - Geometric discounting is sufficient for time-consistent behaviour
  - But: geometric discounting is not necessary for time-consistency
- In plain words: geometric discounting implies time consistency, but it is not needed
- There are many other examples for discounting that yields time-consistent behaviour
- see next figure for examples

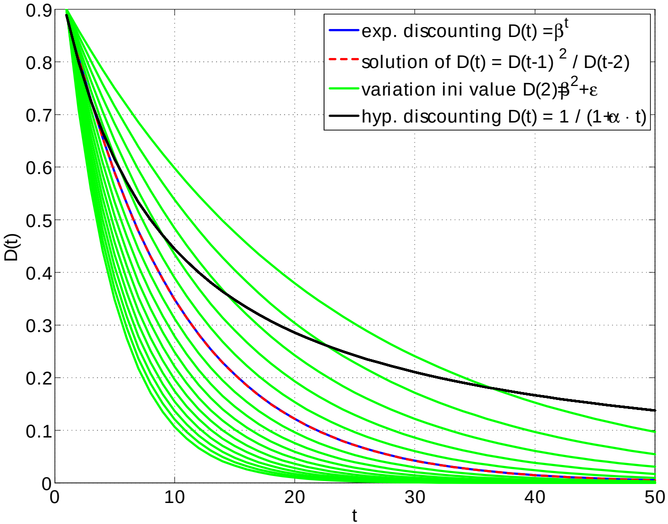



**Figure 43** *Non-exponential discount functions which imply time-consistent behaviour (all satisfying (9.11))*

### 9.3.5 Where does time-inconsistency come from?

- Time consistent behaviour occurs in section 9.2 because
  - moving forward one period implies a multiplication of all terms in utility function by $\beta^{-1}$
  - this does not change relative weight attached to instantaneous (marginal) utilities
  - moving back and forth in time does not change intertemporal preferences
- Time inconsistent behaviour occurs in section 9.3 because
  - moving forward one period can NOT be represented by a multiplication with a constant
  - relative weight attached to instantaneous (marginal) utilities changes
  - intertemporal preferences changes when time goes by
- Most simple example
  - Present bias parameter $\delta$ in
  - "$\delta\beta$-" or "Laibson-preferences" as just seen

$$D(0) = 1, \; D(1) = \delta\beta, \; D(2) = \delta\beta^2$$

- The idea can be generalized
  - The discount function is only one example for changes in relative marginal utilities over time
  - Imagine instantaneous utility is given by $v(c_\tau, \tau)$, $v(c_\tau, k_\tau)$ or $v(c_\tau, x_\tau)$ and discount is exponential at $\beta$
  - In the first case, there is a pure time effect, in the second, there is a status effect depending on wealth $k_\tau$ and the third case captures anxiety $x_\tau$ (an anticipatory emotion, see Caplin and Leahy or Wälde, 2016)
  - In all of these cases, time-inconsistent behaviour can result (to be investigated in more detail in a seminar paper or a great Master thesis)

## 9.4 Business cycles via time-inconsistency?

- We started this part with an analysis of business cycles which is based on real shocks (to TFP)
- Can one imagine business cycles that occur because of time inconsistency?
  - This would be very much in the spirit of "animal spirits" where (see appendix ch. **??**) fluctuations are due "to random waves of optimism and pessimism that are unrelated to fundamental conditions" (Howitt and McAfee, 1992)
  - If the resolution of time inconsistency is idiosyncratic, i.e. if one individual revises plans independently of other individuals, then time inconsistency does *not* lead to aggregate fluctuations
  - If a revision of plans takes place according to some aggregate event, time inconsistency can lead to business cycles
- Big political events and news in the media could play such a role (civil wars, terrorist attacks, pandemic waves etc.)
- Overreactions to small fundamental events can magnify them (Hartz reforms in Germany and subsequent reduction in unemployment)
- A theory fixing points in time when plans are revised is needed

## 9.5 What have we learned?

- We got to know the concept of time inconsistency
  - The empirical relevance of time inconsistent behaviour is obvious (and known by many from everyday life)
  - Time inconsistent behaviour occurs, by definition, when an individual does not stick to a plan made earlier, even though no new information is available and fundamentals have not changed
- The economic view of time inconsistency postulates that individuals do not fully understand changes in their preferences
  - The relative weight between two periods changes when time goes by
  - This implies that optimal behaviour changes over time
  - When individuals have the chance to revise their plans, they will do so

- Can time inconsistency explain business cycles?
  - Not if revisions of plans occur independently among individuals
  - Yes if some “small” aggregate effects cause a revision of plans by many individuals at the same time
  - Research is waiting for you!

# 10 Exercises

## 10.1 Empirics of business cycles

1. Using the Excel file provided, detrend the time series and identify the cyclical components, given the concept of detrending presented in the lecture. Plot quarterly GDP data for the entire period.

2. Compute leading, lagging and centred moving averages for each quarter (using 9 and then 17 quarters).

3. Compute the cyclical component of GDP and plot it.

4. When are the peaks and troughs of business cycles in Germany?

## 10.2 The economics of intertemporal saving

Let us return to a deterministic saving problem for a moment to understand the economics behind intertemporal saving. This example is taken from Wälde (2012, ch. 3.1). Consider the utility function of an individual,

$$U_t = \Sigma_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c_\tau\right), \tag{10.1}$$

where $\beta \equiv (1+\rho)^{-1}$ is the discount factor and $\rho > 0$ is the positive time preference rate. The utility function is to be maximized subject to a budget constraint. The individual chooses an entire path of consumption. The budget constraint can be expressed in the intertemporal version by

$$\Sigma_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} e_\tau = a_t + \Sigma_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} w_\tau, \tag{10.2}$$

where $e_\tau = p_\tau c_\tau$. It states that the present value of expenditure equals current wealth $a_t$ plus the present value of labour income $w_\tau$. Labour income $w_\tau$ and the interest rate $r$ are exogenously given to the household, its wealth level $a_t$ is given by history. The only quantity that is left to be determined is therefore the path $\{c_\tau\}$.

1. Maximizing (10.1) subject to (10.2) by employing a Lagrange function.
2. Provide an economic interpretation of the intertemporal optimal consumption rule and compare it to static optimal consumption rules when consuming two goods.

## 10.3 A two period model under uncertainty

Consider an agent living for two periods. The constraint in the first period is given by

$$w_t = c_t + s_t$$

where $w_t$ is wage at time $t$, $c_t$ is consumption and $s_t$ represents savings. In the second period, i.e. at $t+1$, the constraint reads

$$(1 + r_{t+1})\, s_t = c_{t+1}$$

where $r_{t+1}$ is the interest rate, and consumption at $t+1$ is given by the value of savings at $t$ plus interests.

1. Rewrite the maximisation problem using the constraints and explicitly showing which part of the expression is uncertain. Write out the full form of the problem.

$$\max_{s_t} U = E_t \left\{ u\,(c_t) + \beta u\,(c_{t+1}) \right\}$$

2. Solve the maximisation problem in its full form.

3. Given the Cobb-Douglas preferences in (8.5), find the optimal consumption and saving paths.

4. Go through the intermediate steps and understand them that lead to (8.7).

5. Illustrating dynamics in a phase diagram

   (a) Draw a phase diagram for the deterministic case, i.e. where $A_t$ is constant.

   (b) Explain how this relates to the phase diagram in fig. 31 in the stochastic case.

10.2

## 10.4 A stochastic model of natural volatility

Let technology follow a Cobb-Douglas specification (*with Harrod-neutral technical progress*),

$$Y_t = K_t^{\alpha} \left( A_t L \right)^{1-\alpha},$$

where $A_t$ is total factor productivity, $K_t$ the capital stock and $L_t$ represents hours worked. The accumulation of capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t$$

with $\delta$ denoting the depreciation rate. The technological level (or total factor productivity) grows stochastically,

$$A_{t+1} = (1+q_t) A_t,$$

as the increase in the technological level can take two values in random fashion,

$$q_t = \left\{ \begin{array}{c} \bar{q} \\ 0 \end{array} \right\} \text{ with probability } \left\{ \begin{array}{c} p_t \\ 1-p_t \end{array} \right\}.$$

Growth and cycles are endogenous as this probability depends on resources $R_t$ invested into R&D as in (8.10).

At each point in time $t$, the resource constraint in this economy equates output $Y_t$ with consumption $C_t$, investment $I_t$ in capital accumulation and R&D expenditure $R_t$,

$$C_t + I_t + R_t = Y_t.$$

Assuming optimal behaviour from the household, aggregate consumption and R&D expenditure are determined by

$$C_t = s_C Y_t \quad \text{and} \quad R_t = s_R Y_t$$

where $s_C$ and $s_R$ are constant saving rates.

1. Show that after a jump in the technological level, the growth rate of capital is higher at first and then progressively decreases over time. Draw a diagram showing this process.

2. What is the equilibrium for capital per effective labour ($\frac{K_t}{A_t L_t} = k_t$)? Find the temporary steady-state.

3. Explain why the steady-state is called temporary and what happens to it after a technological jump.

4. Draw the implication of technological jumps on production over time and give an interpretation.

## 10.5 Oil price shocks in the economy

Consider an economy which utilises oil as an intermediate good, but which affects its entire production process:

$$Y_t = AK_t^{\alpha}O_t^{\beta}L^{1-\alpha-\beta}$$
$$\alpha, \beta \in (0,1) \;\; ; \;\; \alpha + \beta < 1$$

1. Consider a representative firm in the economy. Specify its profit schedule as a function of the factors of production and find the first-order conditions. What is the price of oil equal to?

2. Use your result above to rewrite the overall production function, i.e. rewrite $O_t$ using your result above, and insert this result into $Y_t$.

3. What would the outcome of a rise in oil prices be on productivity?

## 10.6 Infinite horizon RBC setup

1. Consider the following infinite horizon problem. Output is given by the usual Cobb-Douglas process,

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}.$$

The household maximizes according to

$$\max_{\{c_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

subject to a wealth constraint

$$a_{t+1} = \left(1 + r_t\right) a_t + w_t - c_t$$

saying that the difference between capital income $r_t a_t$ plus labour income $w_t$ and consumption expenses $c_t$ gives the change in the household's wealth level.

(a) Provide a formal expression for the last sentence.

(b) What is the optimality rule for the household?

2. Consider now a central planner, seeking to maximise the following function

$$\max_{\{C_\tau\}} E_t \left[ \sum_{\tau=t}^{\infty} \beta^{(\tau-t)} u\left(C_t\right) \right]$$

subject to

$$K_{t+1} = \left(1 - \delta\right) K_t + Y_t - C_t.$$

(a) Provide an intuitive comparison between the objective function and constraint of the planner with the same objects of a household.

(b) Using these two expressions and a production function, find the optimality conditions for the central planner.

(c) Compare the optimality conditions of the planner with the one of the household.

## 10.7 Time-consistent behaviour

1. Compute optimal consumption levels from (9.4).

2. Compute optimal consumption levels one period later when $c_t$ is fixed at $c_t^*$.

## 10.8 Time-inconsistent behaviour

1. Compute optimal consumption levels when in period $t$ under generalized discounting as in (9.7)

2. Derive optimal consumption levels (9.10) when in period $t+1$.

3. Derive condition (9.11) for time-consistency.

4. Solve it numerically and discuss the difference to exponential discounting (if time permits).

Johannes-Gutenberg Universität Mainz

Master in International Economics and Public Policy 1st Semester

# Advanced Macroeconomics

2023/2024 winter term

Klaus Wälde (lecture) and Motoaki Takahashi (tutorial)
www.macro.economics.uni-mainz.de

# Part III
# Unemployment

## 12 Facts about unemployment

### 12.1 Definitions

- Splitting a population into economic sub-groups

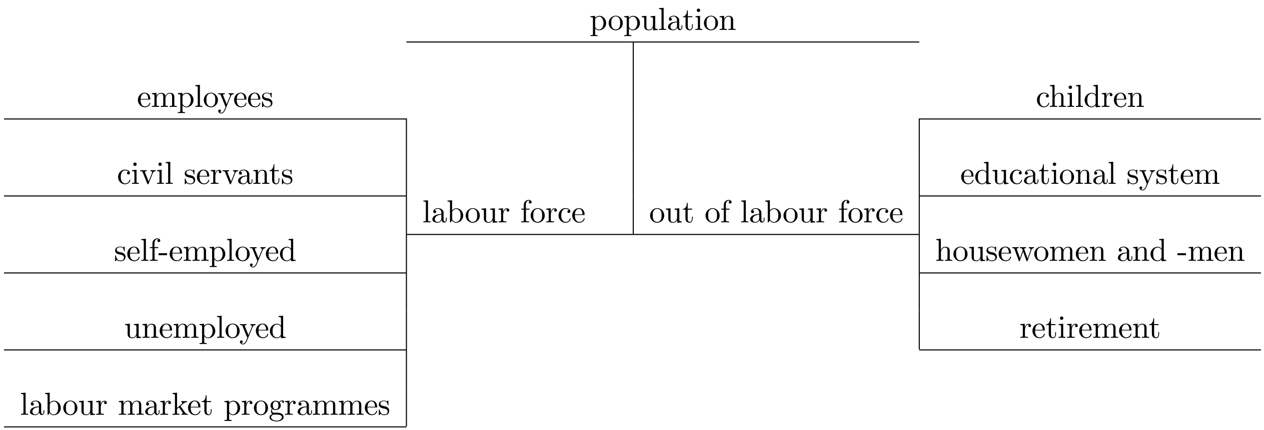



**Figure 44** *Classifying a population by economic status*

- Definition of unemployment (OECD-ILO-Eurostat): A worker is unemployed if s/he is

  1. without work, that is, were not in paid employment or self employment during the reference period;
  2. available for work, that is, were available for paid employment or self-employment during the reference period; and
  3. seeking work, that is, had taken specific steps in a specified recent period to seek paid employment or self-employment

## 12.2 Unemployment stocks

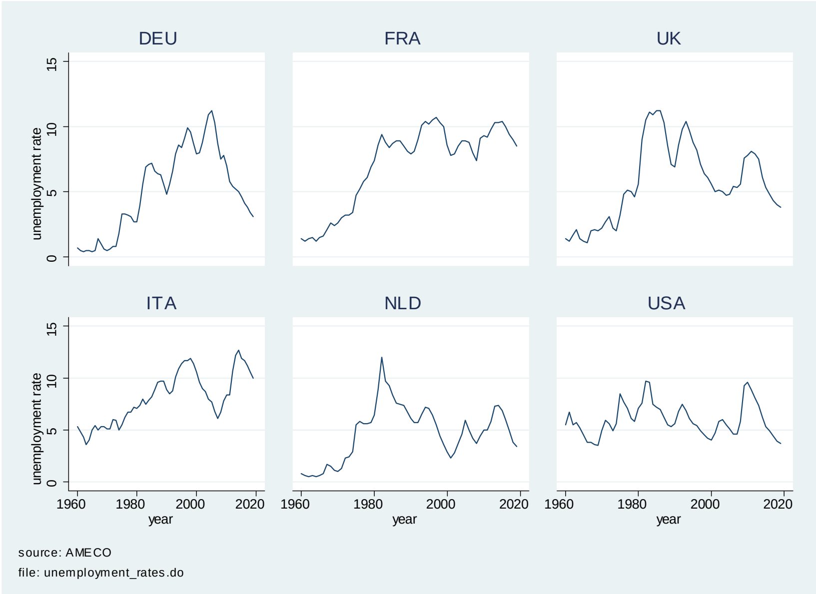



**Figure 45** *Unemployment rates from 1960 to today. Source: Slides of Launov and Wälde (2013)*

## 12.3 Unemployment flows

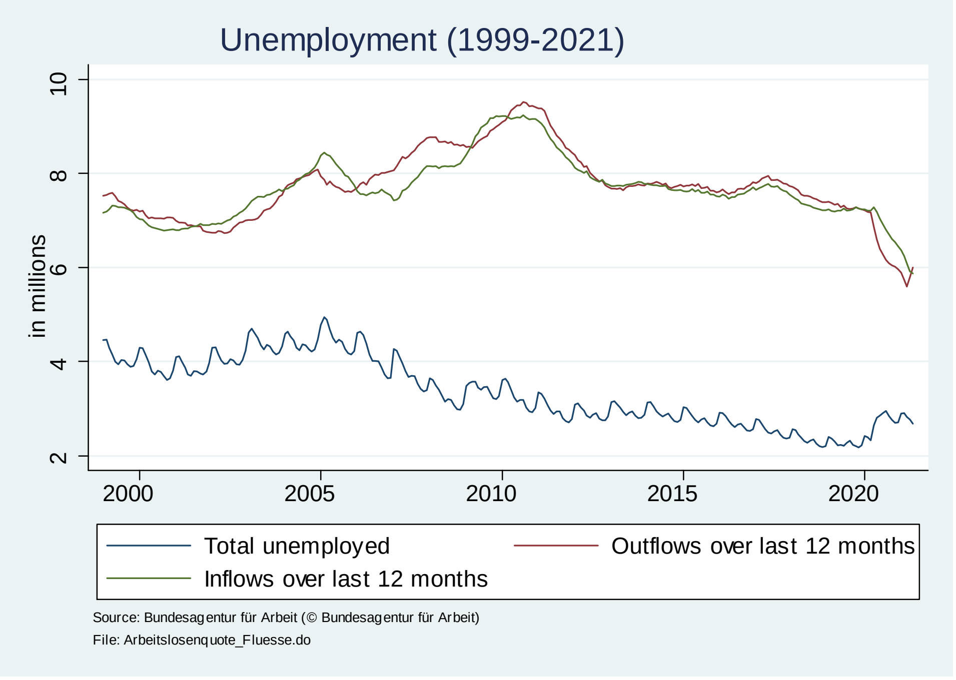



**Figure 46** *Stocks and flows on the German labour market*

## 12.4 Questions for economic theory

- Why do we have unemployment?
- How can we understand both a stock of unemployed and the contemporaneous turnover on the labour market?

# 13 Matching models of unemployment

## 13.1 The literature

- Diamond-Mortensen-Pissarides models (Nobel prize in 2010)
- Pissarides (1985) "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages"
- Pissarides (2000) "Equilibrium Unemployment Theory", ch 1
- Rogerson, Shimer and Wright (2005) "Search-Theoretic Models of the Labor Market: A Survey"

## 13.2 Basic structure

- There is a fundamental job separation process going on in any economy (which is unrelated to the real wage). These separations capture the result of
  - reorganisations of production processes or bankruptcy of firms being caused by
  - new technologies, aggregate business cycle effect, globalisation or other
- Once a worker is unemployed and once a firm has a vacancy
  - there is no such thing as a spot market in the real world
  - finding a job and finding a worker takes time
  - the fundamental reason for this is incomplete information (Stigler, 1961)
- Search processes play an important role on the labour market

## 13.3 The model

(most closely related to Pissarides, 1985, ch. 1)

### 13.3.1 An illustration

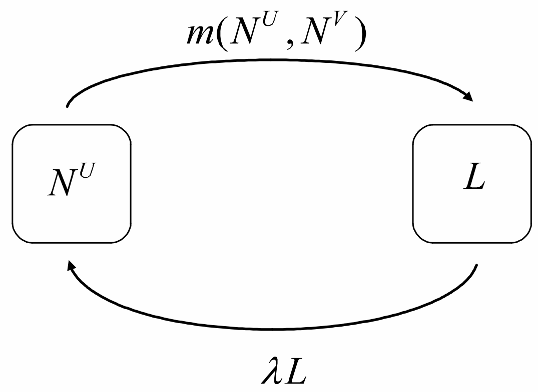



**Figure 47** *Labour market flows from employment $L$ to unemployment $N^U$ given a separation rate $\lambda$ and a matching rate (matching function) $m\left(N^U, N^V\right)$ where $N^V$ is the number of vacancies*

### 13.3.2 Notation

- Economy consists of a fixed labour force (no labour-leisure choice) $N$
- A firm either has one vacancy or employs one worker
- Workers are either unemployed or employed

$$N = N^U(t) + L(t)$$

- Unemployment rate

$$u(t) = N^U(t)/N \tag{13.1}$$

  This implies an employment rate of $1 - u(t)$ - note that all individuals are in the labour force, compare fig. 44

- Vacancy rate

$$v(t) = N^V(t)/N \tag{13.2}$$

- Job finding rate is the rate with which an unemployed worker finds a job

$$p(\theta(t)) = \frac{m\left(N^U(t), N^V(t)\right)}{N^U(t)} = m(1, \theta(t)) \tag{13.3}$$

  - where the last equality employs the property of constant returns to scale of the matching function $m(.)$ and where
  - the variable

$$\theta(t) = \frac{N^V(t)}{N^U(t)} = \frac{v(t)}{u(t)} \tag{13.4}$$

    denotes 'labour market tightness'

- Job filling rate is the rate with which a vacancy is filled

$$q(\theta(t)) = \frac{m\left(N^U(t), N^V(t)\right)}{N^V(t)} = m\left(\frac{1}{\theta(t)}, 1\right) \tag{13.5}$$

- Once a firm and a worker have met, they produce output $y$ (a fixed quantity identical for all firm-worker pairs)

### 13.3.3 What do we know about vacancies?

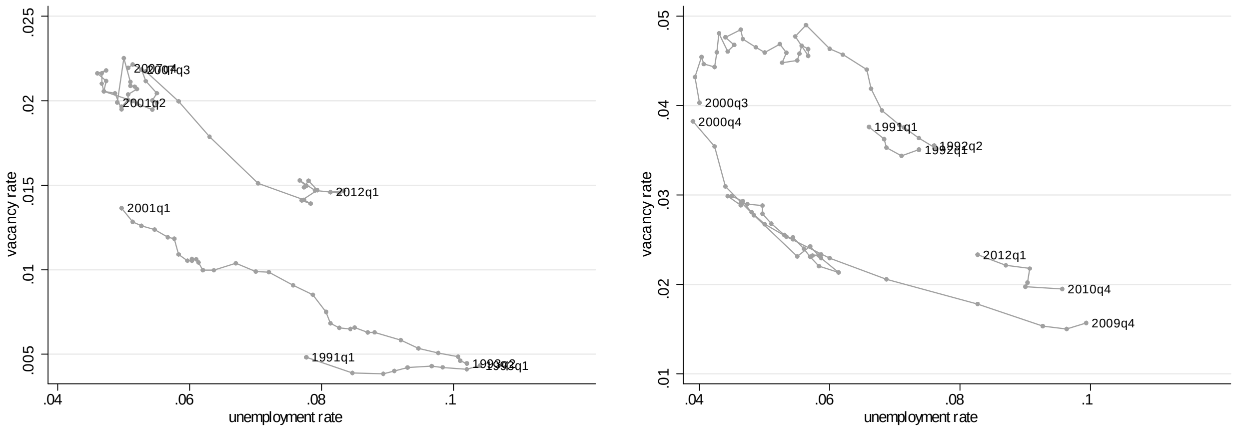
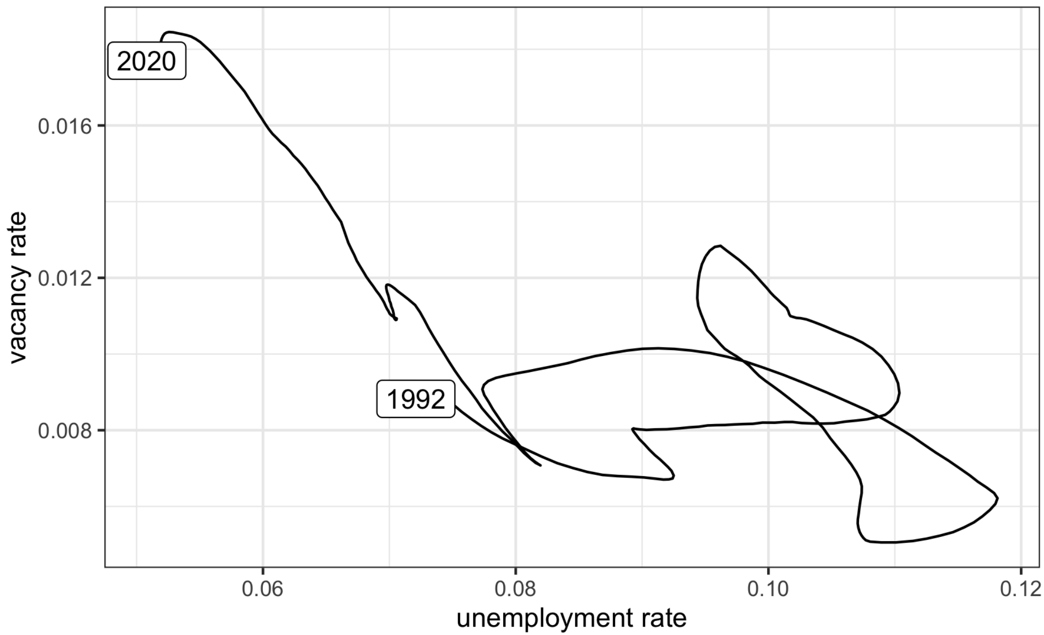



**Figure 48** *The Beveridge curve for the UK (left) and USA (right) – see (13.1) and (13.2) for definitions of unemployment and vacancy rates used here*




Source: Bundesagentur für Arbeit, BMBF
File: beveridge.R

**Figure 49** *The Beveridge curve for Germany.*

### 13.3.4 The dynamics of the unemployment rate

- The number of unemployed follows (by deriving some mean or by intuition building on figure 47)
$$\frac{d}{dt}N^U(t) \equiv \dot{N}^U(t) = \lambda L(t) - m\left(N^U(t), N^V(t)\right)$$

- A remark on the derivation: All the rates we work with here are rates of Poisson processes. As a Poisson process implies uncertainty, the model predicts distributions and not deterministic outcomes. The above equation is therefore, strictly speaking, an equation on the mean of a distribution. In most practical senses, this is of no importance for the analyses. We therefore speak of all quantities as if they were deterministic quantities

- Using the definitions from above, we obtain (see tutorial 15.1)
$$\dot{u}(t) = \lambda[1 - u(t)] - p(\theta(t))\, u(t) \tag{13.6}$$

- The change of the unemployment rate is determined by
  - its current level $u(t)$ and
  - labour market tightness $\theta(t)$

- Intermediate illustration - What if $\theta(t)$ was a constant?
  - The time-varying $\theta(t)$ would be replaced by some constant $\theta$
  - The job-finding rate could be denoted by $\mu \equiv p(\theta)$
  - We would write (13.6) as
$$\dot{u}(t) = \lambda - (\lambda + \mu) u(t) \tag{13.7}$$
  - The unemployment rate would have a steady state value at
$$u^* = \frac{\lambda}{\lambda + \mu}$$
  - The unemployment rate rises in the separation rate $\lambda$ and falls in the matching rate $\mu$
  - Next step: Consider the dynamics – e.g. by looking at the following phase diagram

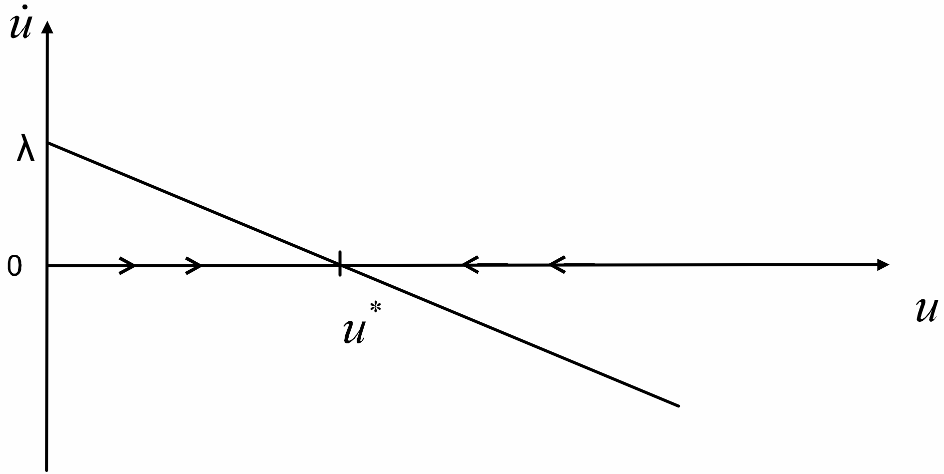



**Figure 50** *Phase diagram analysis for* $\dot{u}(t) = \lambda - (\lambda + \mu)\, u(t)$ *where* $\mu \equiv p(\theta)$

- Intermediate illustration - What if $\theta(t)$ was a constant? (cont'd)
  - As the above figure shows, the unemployment rate rises below and falls above this steady state value $u^*$
  - When we solve the linear ordinary differential equation (13.7), we obtain

    $$u(t) = u^* + (u_0 - u^*)\, e^{-(\lambda+\mu)t}$$

    where $u_0$ is some initial unemployment level (say, after a shock)
  - (verify this by computing the time derivative of $u(t)$ – it must satisfy (13.7))
  - This solution tells us that
    * the unemployment changes only slowly over time
    * the unemployment rate approaches the steady state value at a rate of $\lambda + \mu$
    * the unemployment rate never reaches the steady state
      · This shows a property of all saddle-path stable systems
      · Another example (without closed-form solution) is the Ramsey-Cass-Koopmans central planner optimal consumption system illustrated in fig. 16

- Comparison with static models of unemployment
  - The equation for $u(t)$ can be used to compute how much time it takes to reduce an economy-wide unemployment rate from, say, 10% to 8%
  - Necessary basis for analysing or predicting the effects of policy reforms
  - Static models would be unable to make such predictions
- The next question: How is $\theta(t)$ being determined?
- In order to understand the determination of the number of vacancies (and thereby $\theta(t)$), we need to understand
  - optimal behaviour of workers,
  - optimal behaviour of firms and
  - wage setting

### 13.3.5 Optimal behaviour of workers

- Value $U$ of being unemployed is described

$$\rho U(t) = b + \dot{U}(t) + p(\theta(t))[W(t) - U(t)]$$

- This equation has a simple intuitive meaning

  - The value of being unemployed is given by $U(t)$
  - When $\rho$ is the time preference rate, $\rho U(t)$ is like an instantaneous payment, a flow of utility, out of this overall value (compare wealth $a(t)$ and interest payments $ra(t)$)
  - This flow of utility for the unemployed worker is given by unemployment benefits $b$ on the one hand, by value changes in $U$ on the other, and by the last term $p(\theta(t))[W(t) - U(t)]$
  - This last term captures the expected change in utility from finding a job: The rate with which a job is found is given by $p(\theta(t))$, the gain in overall utility from accepting the job is $W(t) - U(t)$ (where $W(t)$ is the value of being employed)

- Where does this equation come from more formally? (see Wälde, 2012, ch. 6.1.2)
  - The objective function of a worker that is always unemployed is

$$U(t) = \int_t^\infty e^{-\rho[\tau - t]} u(b(\tau))\, d\tau$$

  - When we compute the time derivative of the intertemporal utility function $U(t)$ with respect to time $t$, we obtain (use Leibniz rule, see Wälde, 2012, ch. 4.3.1 or tutorial)

$$\begin{aligned} \dot{U}(t) &= -e^{-\rho[t-t]} u(b(t)) + \rho \int_t^\infty e^{-\rho[\tau - t]} u(b(\tau))\, d\tau \\ &= -u(b(t)) + \rho U(t) \end{aligned}$$

  - When we assume linear utility and a constant $b$, we get

$$\rho U(t) = b + \dot{U}(t)$$

  - We can therefore look at the equation $\rho U(t) = b + \dot{U}(t) + p(\theta(t))[W(t) - U(t)]$ as describing how intertemporal utility of the individual changes over time
  - Technically, this equation is a Bellman equation (which is the basis of maximization using dynamic programming)

- Value $U$ of being unemployed is described (as just seen above)

$$\rho U(t) = b + \dot{U}(t) + p(\theta(t))[W(t) - U(t)]$$

- Value $W(t)$ of being employed

$$\rho W(t) = w(t) + \dot{W}(t) + \lambda[U(t) - W(t)]$$

  - Same intuitive interpretation as Bellman equation for unemployed worker
  - $\rho W(t)$ is the payoff from being employed
  - It consists of the wage $w(t)$, the change $\dot{W}(t)$ in the value and the expected loss from losing the job

- Given the wage, benefits and parameters, this system describes how the values of being in both states evolves over time

- (It is a linear differential equation system)

### 13.3.6 Optimal behaviour of firms

- We still need to understand how vacancies are being created
- To this end, we now turn to optimal firm behaviour
- Generally speaking, firms open vacancies as a function of (their expectations about)
  - speed of finding a worker
  - joint output and implied profit (i.e. the value of a job)
  - cost $k$ of finding a worker
- More precisely speaking, the value $V(t)$ of a vacancy is given by

$$\rho V(t) = -k + \dot{V}(t) + q(\theta(t))[J(t) - V(t)]$$

- Value $J(t)$ of a filled job

$$\rho J(t) = y - w(t) + \dot{J}(t) + \lambda[V(t) - J(t)]$$

where $y - w(t)$ is profit of firm (output minus wage)

- Free entry into vacancy creation
  - The value of a vacancy equals zero: $V(t) = 0$
  - The Bellman equation for vacancy implies

$$J(t) = \frac{k}{q(\theta(t))} \tag{13.8}$$

  - The Bellman equation for jobs becomes

$$\rho J(t) = y - w + \dot{J}(t) - \lambda J(t) \tag{13.9}$$

- Do we not have two equations for $J(t)$?
  - No, as the first gives $\theta(t)$ as a function of $J(t)$
  - The second continues to fix $J(t)$
- Have we found an equation for labour market tightness $\theta(t)$?
  - Yes, in principle: (13.9) fixes $J(t)$ and (13.8) fixes $\theta(t)$
  - But what determines the wage $w$ in (13.9)?

### 13.3.7 Wages

- Standard static way to understand wage setting: real factor rewards equal marginal productivity as firms instantaneously hire and fire on spot market
- Here both workers and firms have temporary/ local market power once a firm and worker met
- Wage is determined by some bargaining process – here Nash bargaining
- Technically, we need the contribution of worker and firm to total surplus of match
- Total surplus of match is

$$\begin{aligned} S(t) &= W(t) - U(t) + J(t) - V(t) \\ &= W(t) - U(t) + J(t) \end{aligned}$$

  i.e. the sum of the gain for the worker and the gain for the firm

- Nash product is

$$(W(t) - U(t))^{\beta} J(t)^{1-\beta}$$

  where $\beta$ is bargaining power of workers

- Nash bargaining with a bargaining power parameter $\beta$ (of the worker) implies

$$\begin{aligned} W(t) - U(t) &= \beta S(t) \\ &= \beta \left[ W(t) - U(t) + J(t) \right] \end{aligned} \tag{13.10}$$

- The worker receives a share $\beta$ of the surplus of the match (see tutorial 15.1)
- Rewriting this equation gives (after several steps - see tutorial 15.1)

$$w(t) = (1 - \beta) b + \beta \left[ y + \theta(t) k \right] \tag{13.11}$$

- The wage $w(t)$ amounts to the sum of (i) a share $\beta$ of joint production $y$, (ii) a share $\beta$ of expected costs of vacancy of firm (as the firm does not need to pay them) and (iii) a share $1 - \beta$ of his alternative income $b$
- Much simpler version than sharing rule above (no intertemporal variables)
- Nash bargaining takes place
  - initially when firm and worker meet and
  - continuously (at each point in time)
  - see this as technical simplification compared to bargaining a wage path

### 13.3.8 Equilibrium and dynamic adjustment of the unemployment rate

- All variables fixed now!
  - Wage equation (13.11) with (13.8) and (13.9) fix $w(t)$, $J(t)$ and $\theta(t)$
  - We can describe equilibrium
- Equilibrium (see tutorial) ...
  - ... is described by one equation fixing tightness $\theta(t)$

$$\frac{q'(\theta(t))}{q(\theta(t))}\dot{\theta}(t) = \frac{1-\beta}{k}(y-b)\,q(\theta(t)) - \rho - \lambda - \beta p(\theta(t))$$

  - and one equation determining dynamics of unemployment rate $u(t)$

$$\dot{u}(t) = \lambda - [\lambda + p(\theta(t))]\,u(t)$$

- What do these equations tell us?
  - Second equation is familiar from (13.7)
  - First equation looks more involved – but it makes model rich
    * Output $y$ (and therefore potentially technological progress) enters as a determinant of unemployment
    * Labour market policy $b$ plays a role
    * Turnover on labour market affects the economy via $\lambda$
    * Preferences of households (via $\rho$) plays a role
    * Bargaining power of workers matters
    * All of these determinants of the unemployment rate enter via the determination of the number of vacancies (i.e. via $\theta(t)$)

- How to understand these equations?
  - We proceed in steps and are heading towards an illustration via a phase diagram
  - Step 1: Zero-motion line for $\theta$ and zero-motion line for $u$
  - Step 1b: Steady state
  - Step 2: Arrow-pairs determine dynamics outside the steady state

- Step 1: Zero-motion lines
  - Zero-motion lines are given by

$$\frac{q'(\theta(t))}{q(\theta(t))}\dot{\theta}(t) = 0 \Leftrightarrow \frac{1-\beta}{k}(y-b)\,q(\theta) - \rho - \lambda - \beta p(\theta) = 0$$
$$\dot{u}\,(t) = 0 \Leftrightarrow \lambda - [\lambda + p\,(\theta)]\,u = 0$$

  - These lines (i.e. curves) can be expressed as

$$\frac{1-\beta}{k}(y-b)\,q(\theta^*) - \beta p(\theta^*) = \rho + \lambda$$
$$p\,(\theta) = \frac{\lambda}{u} - \lambda$$

  - The first equation directly reveals that $\dot{\theta}(t) = 0$ for some fixed value $\theta^*$
  - The second equation shows (anticipating the tutorial which shows that $p\,(\theta)$ rises in $\theta$) that $\dot{u}\,(t) = 0$ on a falling curve (see next figure)

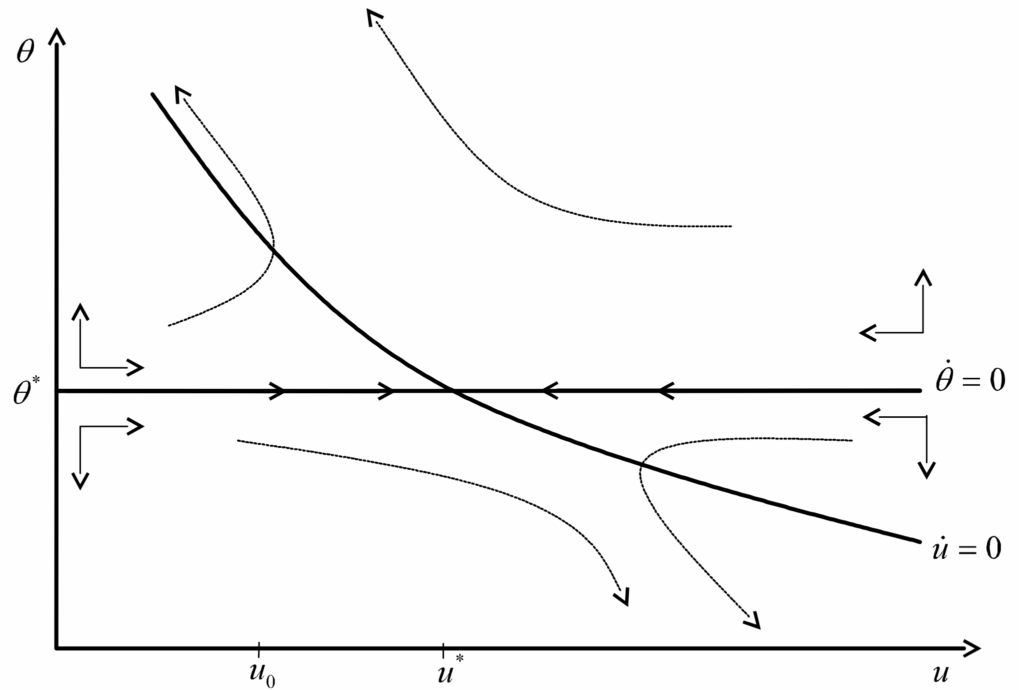



**Figure 51** *Phase diagram for the Pissarides textbook matching model*

- Step 1b: Steady state
  - Steady state is given by intersection point of zero-motion lines
  - In figure, it is point $(u^*, \theta^*)$
  - The aggregate unemployment rate $u$ and $\theta$, i.e. the number of vacancies relative to number of unemployed workers, do not change
  - Yet, at the microeconomic level, there is a lot of turnover
    * Workers lose their jobs
    * Firms lose their workers
    * Workers find new jobs
    * Firms find new workers
    * We have a stock of unemployment and – at the same time – turnover on the labour market

- Step 2: Dynamics outside the steady state
  - As always for phase diagrams, we need to understand how variables change when the system is not in the steady state
  - As tutorial shows, we obtain the arrow-pairs as depicted in the phase diagram
  - Interestingly, $\theta(t)$ falls when below $\theta^*$ and rises above,

$$\dot{\theta}(t) \gtrless 0 \Leftrightarrow \theta(t) \gtrless \theta^*$$

    * It does not seem to be a reasonable prediction of a model to have an ever-changing $\theta(t)$ when the model describes a stationary environment
    * The only empirically plausible prediction of the model is $\theta(t) = \theta^*$ for all $t$
  - Unemployment evolves as in (13.7)
    * For the analysis in (13.7) we assumed *some* (arbitrary) $\theta$
    * Here the $\theta$ is given by the equilibrium value $\theta^*$
    * Yet, the qualitative dynamics of $u$ is the same
    * The unemployment rate falls when $u(t)$ is too large (i.e. above $u^*$)
    * The unemployment rises when $u(t)$ is too small (i.e. below $u^*$)

- Equilibrium dynamics of unemployment rate
  - Let us assume we start with some relatively low initial unemployment rate at $u_0$
  - The number $N^V$ of vacancies adjusts such that the economy finds itself on the zero-motion line for $\theta$
  - This zero-motion line for $\theta$ is actually also the saddle-path in this economy
    * Compare the saddle path in figure 16 for the central-planner optimal saving problem
    * The saddle path there is *not* on some zero-motion line
    * The finding here that saddle-path and zero-motion line coincide is a special case
  - The number of vacancies or labour market tightness $\theta$ is the 'jump variable' in our system

- Equilibrium dynamics of unemployment rate (cont'd)
  - Once the economy is at $u_0$ and $\theta_0 = \theta^*$, the unemployment rate $u$ rises and approaches $u^*$
  - There is a constant $v$ to $u$ ratio (i.e. a fixed $\theta$) for the entire transition path
  - As the unemployment rate rises, the vacancy rate $v$ also rises
    * Put differently, when there are more and more unemployed workers, the number of vacancies increases
    * Firms find it profitable to open more vacancies as the rate with which they find a worker is faster when there are more unemployed workers
  - As emphasized after (13.7), the unemployment rate approaches $u^*$ asymptotically (i.e. it never reaches it)

### 13.3.9 Response to an output shock

- How does economy react to business cycles?
  - Original motivation of Pissarides (1985)
  - How do wages respond?
  - How does employment respond?
  - What happens to vacancies?
  - Research question up to today ("Shimer puzzle", Shimer, 2005)
- Incarnation of question
  - What is the effect of a rise in average productivity?
  - We study
$$y^{\text{new}} = (1 + \Delta)\, y$$
where $\Delta$ is the percentage increase of $y$ after a positive technology shock

- Procedure
  - Understand changes in phase diagram
  - Understand dynamic reactions in new phase diagram
- Changes in phase diagram
  - Consider system known from above

$$\frac{q'(\theta(t))}{q(\theta(t))}\dot{\theta}(t) = \frac{1-\beta}{k}(y-b)\, q(\theta(t)) - \rho - \lambda - \beta p(\theta(t)) \tag{13.12}$$

$$\dot{u}(t) = \lambda - [\lambda + p(\theta(t))]\, u(t) \tag{13.13}$$

  - Zero-motion line for unemployment does not change
  - Zero-motion line for $\theta(t)$ changes as $y$ changes
  - There is a new steady state level for $\theta(t)$, denoted by $\theta^*_\Delta$

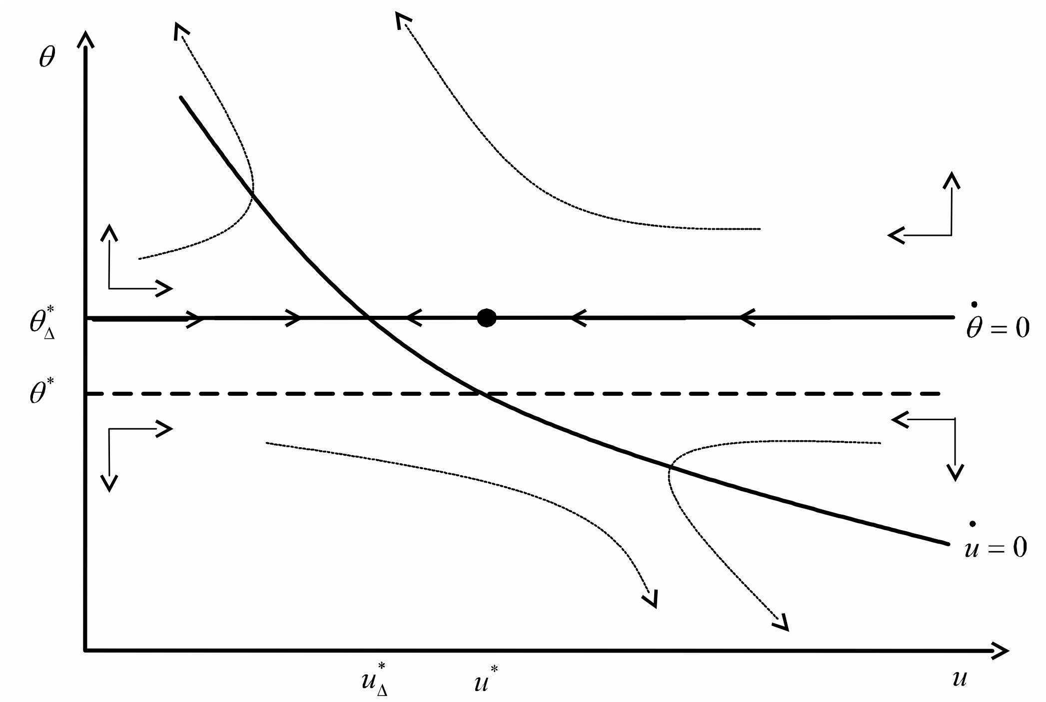



**Figure 52** *Unemployment and tightness after a positive technology shock (increase in y)*

- The dynamic reaction of the economy
  - The shock leaves unemployment initially unchanged ($u$ is a state variable that changes only slowly)
  - The shock implies a jump in labour market tightness
    * Remember that $\theta = v/u$
    * As $u$ does not change instantaneously, $v$ jumps upwards
    * The number of vacancies therefore immediately reacts positively to more output
  - The unemployment rate then adjusts slowly
    * The unemployment rate follows the differential equation in (13.13) for the new constant $\theta$
    * It converges to the new $\theta = \theta^*_\Delta$
  - The number of vacancies also adjusts slowly
  - The long-run unemployment rate is lower with higher output $y$

## 13.4 What have we learned?

... returning to learning objectives

- Why are individuals unemployed?
  - A separation rate, capturing anything from reorganisation of firms to firm bankruptcy (which can be the result of technological change, globalization, business cycle shocks and other), implies that individuals lose their job
  - Finding a job (and a worker) takes time
  - A search process is required due to incomplete information about job and worker characteristics
  - These inflows into unemployment joint with outflows lead to an equilibrium stock of unemployed workers

- How can we understand both a stock of unemployed and the contemporaneous turnover on the labour market?
  - Basically the same answer for why individuals are unemployed
  - There is a flow of workers losing their jobs
  - Finding a job takes time
  - The equilibrium between inflows and outflows implies a constant stock of unemployed workers

- What is this good for?
  - Framework to understand unemployment
  - Framework to understand the effect of international trade and business cycles on unemployment
  - Framework to understand labour market policies and their reforms
  - Much much more (the approach got the Nobel prize for a reason ...)

# 14 Stress and (Un-) Employment

## 14.1 Background on stress

### 14.1.1 Why think about stress?

- What is stress?
  - A leading definition from psychology is by Lazarus and Folkman (1984)
  - Stress occurs when there are *too many demands given available resources*
- How relevant is stress?
  - There are stress reports (e.g. "Stressbericht 2012" by Bundesanstalt für Arbeitsschutz und Arbeitsmedizin)
  - A rise of psychological diseases in overall diseases (as reported by health insurances)
  - The WHO identifies stress as one of the most important health risks of the 21st century
  - (to be worked out – Eurobarometer and NPR/Harvard dataset)

- A (short) history of stress research
  - Stress has been a (disturbingly) popular concept ever since it was introduced in medical science in 1936 by Selye
  - "Nowadays, everyone seems to be talking about stress" (Selye, 1982)
  - "It is virtually impossible today to read extensively in any of the biological or social sciences without running into the term stress" (Lazarus and Folkman, 1984)
  - Up to today, stress is a topic gaining more and more academic attention
- Why in an *economic* and even *macro*economic lecture?
  - Stress is of economic importance via its effect on well-being (happiness literature) and health effects
  - It is of macroeconomic importance as stress is not just an individual problem: a stressed person affects colleagues, partners and families and potentially society at large (e.g. via criminal activity)
  - It is (also) the consequence of stressors at work such that the design of labour markets and work relationships via governmental regulation affects the economy as a whole

### 14.1.2 The open issue

- Why do economists not work on stress?
- Economic world hosts a large group of stress-inducers
  - (Biased) Technological change
  - Globalisation
  - Unemployment
  - Financial and Euro crisis
  - ... are all "good" sources of stress
- A conceptual framework is missing for economic model building
- We need to bring more psychology into economics (Rabin, 2013)

### 14.1.3 The plan

- Provide (in the next section) a conceptual framework that allows to understand stressors – appraisal – stress – coping
  - Stressors: Anything that puts demand on resources of an individual
  - Appraisal: Process of evaluating a stressor concerning its implication for well-being of a person
  - Stress: Subjective feeling resulting from current and past appraisals of stressors
  - Coping: Behaviour aimed at reducing stress
- Apply this framework to understand optimal reaction to stress
  - Which coping strategies are chosen, i.e. which reactions to stress can be observed?
  - Beyond stressors and appraisal, understand the effect of (theory consistent) personality on coping

### 14.1.4 Questions asked

Questions on coping

- How does stress translate into more or less aggressive coping patterns (smooth stress regulation vs. "emotional outbursts")?
- What are conditions under which emotional outbursts occur?
- Should emotional outbursts be avoided or suppressed?
- How frequent are emotional outbursts?

Questions on stress

- How well do various coping strategies help in regulating stress?
- Can (temporary) positive or negative surprises have permanent effects on stress?
- What is the role of personality in stress regulation?
- Can we quantify the model predictions joint with an estimation of personality?

### 14.1.5 How important are outbursts quantitatively?

- Family disputes
  - USA: 75% of couples report verbal aggression (Stets, 1990, USA, 'random' sample)
  - Germany (GSOEP with weighting factors): 44% (women) to 52% (men) report "having arguments or conflicts"
  - conflict is with partner (45%), parents (14%), children (13%), siblings (7%), hardly with colleagues or outside family
- Incivility and bullying at work
  - Pressure for productivity ... leads to an increase in aggressive workplace behaviour (Baron and Neuman, 1996)
  - Is verbal aggression the precursor of more violent aggression? (Andersson and Pearson, 1999)
  - Verbal aggression is common (experienced by 1/3 of workers, Bjorkqvist et al, 1994)
- Domestic violence
  - USA: 10% of couples report physical aggression (Stets, 1990, USA)
  - much higher numbers for (biased) samples among students

## 14.2 A model of stress and coping

(see Wälde, 2017, for more background)

### 14.2.1 The origins of stress

Stress is usually understood to result from

- stressors and
- an appraisal process (evaluation of stressors)

Many events can be understood as stressors

- ... some of which occur rarely, some occur daily
- Social Readjustment Rating Scale (Holmes and Rahe, 1967) captures life-time to rare events like 'death of spouse', 'divorce', 'jail term', 'fired at work' ... 'vacation, 'christmas' and 'minor violations of law'
- Daily hassles and uplifts (Kanner, Coyne, Schaefer and Lazarus, 1981) captures everyday life like 'losing things', 'don't like fellow workers, 'too many obligations' ...

Modelling rare events

- Rare events imply positive or negative surprises $g(t)$
- Random variable $h(t)$ and subjective expectation $\mu$ yield surprise

$$g(t) = h(t) - \mu$$

(Bell, 1985, Loomes and Sugden, 1986)

- surprises occur at a certain arrival rate
- (dynamic continuous time model with Poisson uncertainty)

Modelling daily hassles and uplifts

- Flow of demand $p(t)$ paired with
- abilities $a(t)$ of individual yields
- intensity $p(t)/a(t)$ of stressor

The appraisal process

- "... an appraisal is an evaluation of a situation in terms of its relevance for oneself, specifically one's goals or well-being (e.g., Lazarus 1968)"
- "... appraisal as a temporal and causal antecedent to emotion (Scherer 1993b; 1999; Roseman and Smith 2001)" (all from Lewis, 2005)

Modelling appraisal

- a known function $f\left(\frac{p}{a}, .\right)$ for daily hassles
- a known function $G\left(g\left(t\right), .\right)$ for surprises
- both functions are specific to individual (personality)

### 14.2.2 The impact on the individual

How to model emotional tension and well-being?

- Direct channel affects well-being (utility) directly (Stress symptoms like headache, dizziness, sweating, sleeplessness ...)
- Indirect channel affects labour income of the individual via "cognitive load"
- Both channels affect instantaneous utility $u\left(c\left(t\right), W\left(t\right)\right)$

The indirect channel of cognitive load

- Stress is the result of appraisal of stressors, which can be "costless" or resource consuming
- Reflections on how to react to stressors is resource consuming ("high-level appraisals", e.g. Kalisch et al., 2006)
- Both processes lead to "cognitive load" (Sweller, 1988, Eysenck and Calvo, 1992, Hoffman, von Helversen and Rieskamp, 2013)
- Cognitive load stands for all the thoughts and worries, constructive or not, related to stressors and strategies on how to best react to stressors

Modelling cognitive load by a mental resource constraint

- An individual is endowed with a certain amount of working memory $M$ (see Smith and Kosslyn, 2007, esp. ch. 6 as a starting point)
- Stressors and coping use up resources of the working memory
- Memory/ resource constraint in the case of "stress" and "effort"

$$M\left(W\right)+M\left(e\right)=M$$

- Higher stress levels imply cognitive load and leave less working memory for other purposes
- If effective labour input rises in effort, consumption falls in stress

$$c=wl\left(e\right) \tag{14.1}$$

### 14.2.3 Strategies for coping with tension

Huge variety of coping strategies

- Coping seems so heterogenous that psychologists even disagree on how to classify strategies ... (Skinner et al. 2003)
- Frequent strategies: problem solving, support seeking, ... , emotional expression, aggression, ... , wishful thinking, worry
- Categories
  - functional vs. dysfunctional approaches
  - problem-focused vs. emotion-focused (Lazarus and Folkman, 1984)
  - automatic vs. controlled processes (e.g. Conner-Smith et al. 2000, Skinner and Zimmer-Gembeck, 2007)

Here: emotion-focus and automatic vs. controlled processes

- controlled process
  - talking to a friend, a colleague, a therapist
  - reduces tension by "sorting things out", i.e. by rationalizing events
  - practice some (endurance) sport
  - take a break and enjoy leisure
  - stress reduces gradually due to depreciation function $\delta\left(m\left(t\right),.\right)$
- automatic process – emotional outbursts
  - individuals feel overwhelmed by stressors
  - emotional tension rises to much, they "can't help" but explode
  - individuals start crying, shout at others, call other people names
  - relatively short event
  - outburst reduces tension by a fixed amount $\Delta$

$$W\left(\tau\right)=W\left(\tau_{-}\right)-\Delta$$

### 14.2.4 Formal modelling (functional forms)

- Emotional tension $W(t)$ is a state variable

$$dW(t) = \left\{ \phi \frac{p}{a} W(t) - \delta_0 W(t) - \delta_1 m(t) \right\} dt - \chi \left[ h(t) - \mu \right] dq(t)$$

- Deterministic part displays
  - stressors $p$ and ability $a$, both are exogenous and fixed
  - $\phi$ as appraisal parameter of stressor
  - $\delta_0$ as autonomous stress reduction ability
  - coping $m(t)$ that leads to
  - smooth reduction of tension given productivity $\delta_1$
- Stochastic part displays
  - surprises $h(t) - \mu$, exogenous and random in level
  - appraisal of surprises captured by $\chi$
  - Poisson process $q(t)$ with exogenous arrival rate
- “Outburst technology”

$$W(t) = W(t_-) - \Delta$$

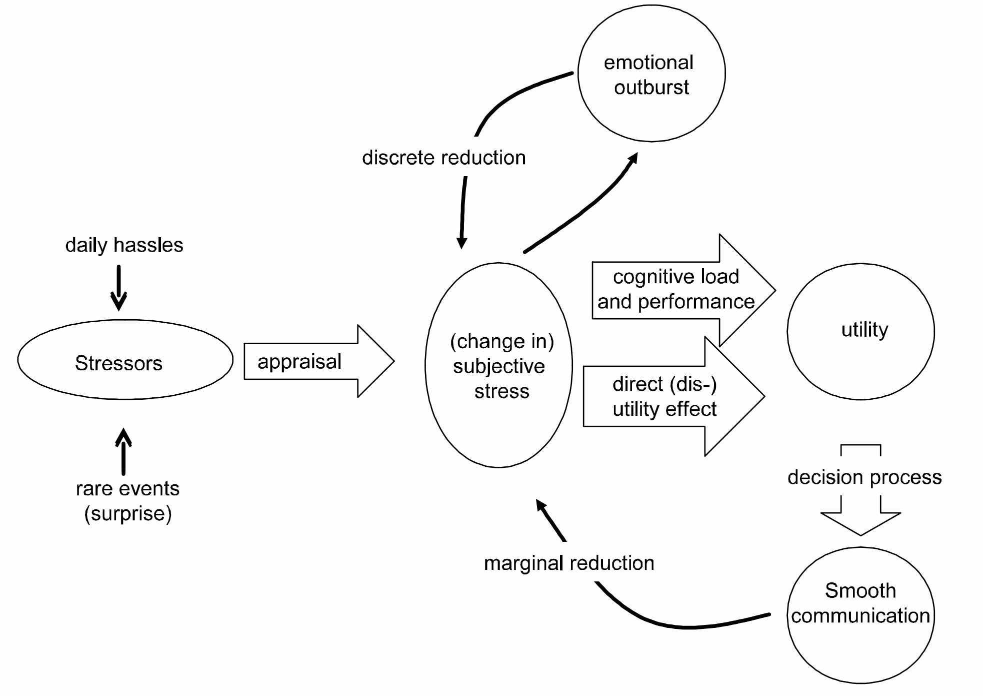



**Figure 53** *A graphical illustration of the economic stress and coping model*

## 14.3 Optimal coping

### 14.3.1 The maximization problem

How does individual behave optimally?

- How does s/he choose smooth coping $m(t)$?
- How does s/he choose (does s/he?) outbursts?

Costs of coping

- Cost of smooth coping $v(m(t))$, $m(t)$ is chosen optimally
- Cost of emotional outbursts $v^M \equiv \int_s^{s+\Lambda} e^{-\rho[\tau-s]} v(\bar{m}(\tau))\, d\tau$
- Outbursts occur when tolerance level $\bar{W}$ is hit (behavioural, exogenous, automatic ...)

Formal structure

- Optimal stopping problem with exogenous stopping

$$E_t \int_t^\infty e^{-\rho[\tau-t]} \left[u(c(\tau), W(\tau)) - v(m(\tau))\right] d\tau - \Sigma_{i=1}^n e^{-\rho[\tau_i - t]} v^M$$

- Choosing a path $\{m(\tau)\}_t^\infty$ anticipating outbursts at $\bar{W}$ and taking constraints on $W(t)$ into account

### 14.3.2 Optimal coping style

Properties of the closed-form solution

- Optimal coping level

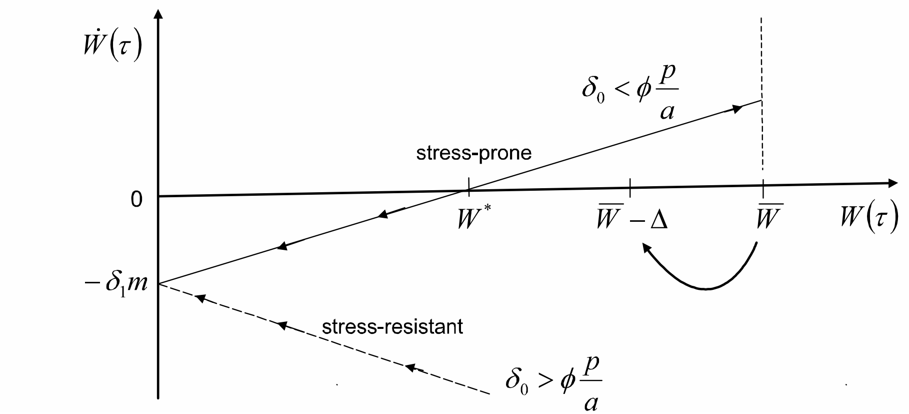
$$m = \left(\frac{\delta_1}{v_0}\frac{v^M}{\Delta}\frac{1}{1+\zeta}\right)^{1/\zeta}$$

- Assume convex cost function $v(m)$ for smooth coping, i.e. $\zeta > 0$
- Amount of talking
  - increases in $\delta_1$ – reflecting productivity of smooth coping
  - decreases in $v_0$ – reflecting costs of smooth coping
  - increases in $v^M$ – reflecting costs of emotional outburst
  - decreases in $\Delta$ – reflecting productivity of emotional outburst
- Amount of talking is independent of current tension level $W(t)$

## 14.4 Stress and coping patterns

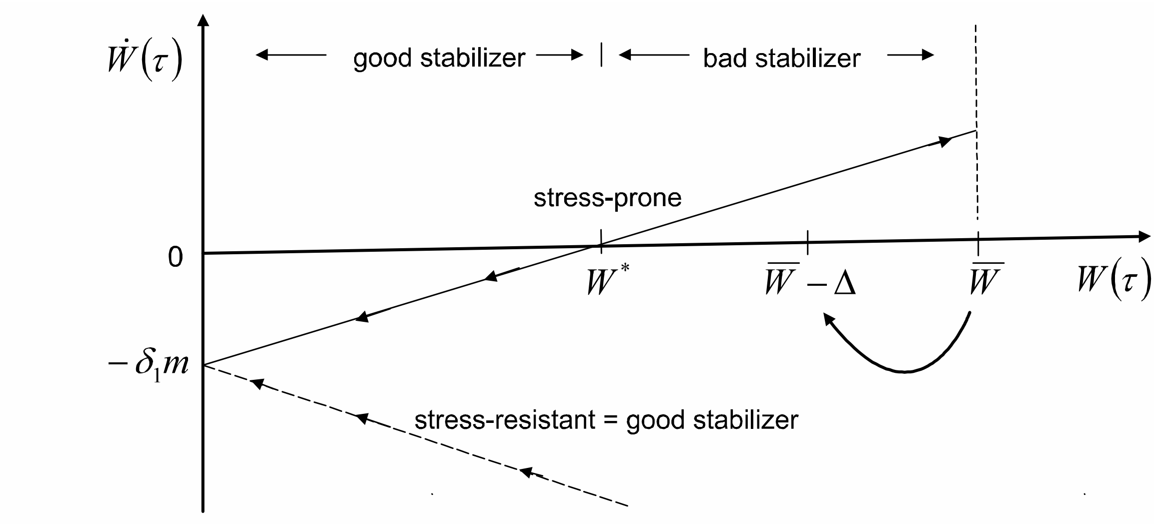
### 14.4.1 Dynamics of stress and coping and personality

- How does stress translate into more or less aggressive coping patterns (in a world without surprises)?



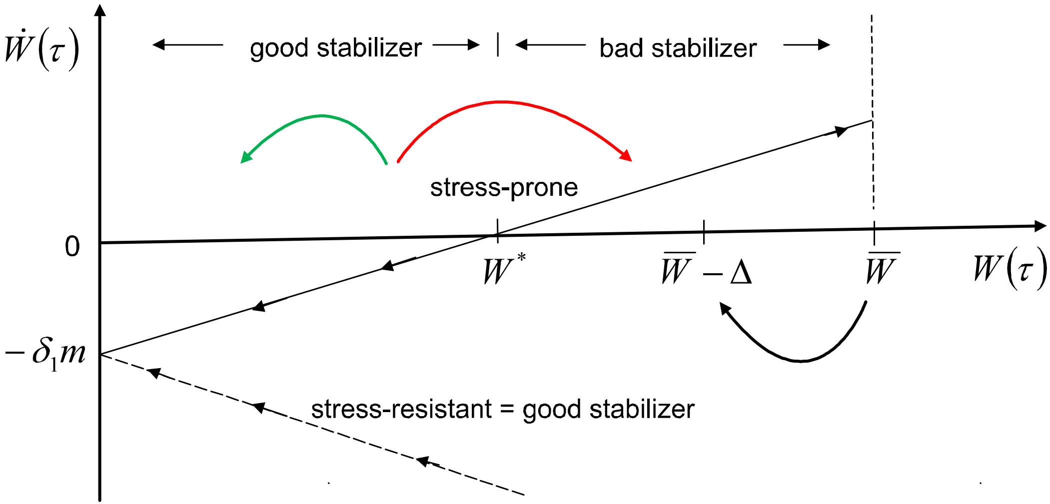

$$\dot{W}(t) = \Phi W(t) - \delta_1 m, \quad \Phi \equiv \phi \frac{p}{a} - \delta_0 \text{ “growth rate of stress”}$$

- How does stress translate into more or less aggressive coping patterns (in a world without surprises)?




$$\dot{W}(t) = \Phi W(t) - \delta_1 m, \quad \Phi \equiv \phi \frac{p}{a} - \delta_0 \text{ “growth rate of stress”}$$

- How does stress translate into more or less aggressive coping patterns (in a world *with* surprises)?



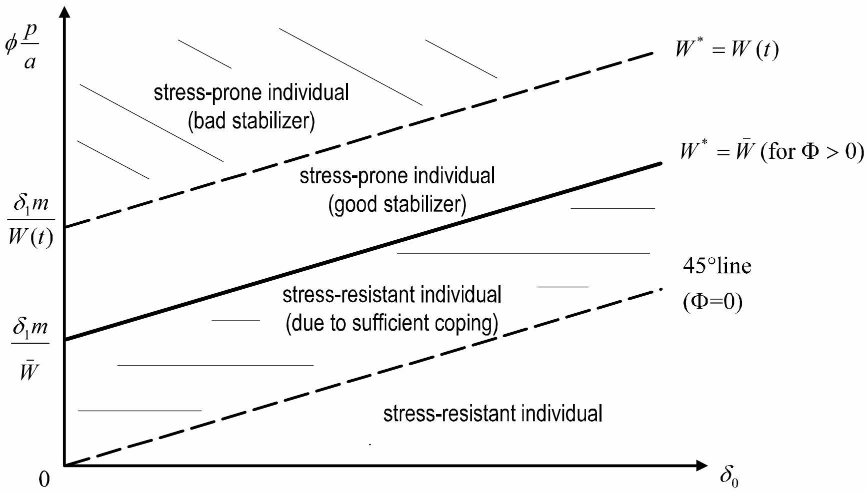

$$dW(t) = \{\Phi W(t) - \delta_1 m\}\, dt - \chi\,[h(t) - \mu]\, dq(t)$$

### 14.4.2 The outburst theorem

Questions

- What are conditions under which "emotional outbursts" occur?
- Of interest also for psychology: "one intriguing puzzle is why people use one emotion regulation strategy rather than another" (Gross, 2008, p. 505)
- Would outbursts ever occur in a world *without surprises* – and thereby in a predictable way?
- Or would the individual exclusively employ the smooth coping channel to reduce tension?

Findings




$\delta_0$ – autonomous stress-reduction potential
$\phi p/a$ – appraisal $\phi$ of intensity $p/a$ of stressors (daily hassles)
$\Phi = \phi p/a - \delta_0$ – growth rate of stress
$W^*$ – threshold level (beyond which stress rises)
$\bar{W}$ – tolerance level (beyond which outbursts)

### 14.4.3 Can surprises have permanent effect on stress?

Back in a stochastic world with surprises, how important are positive or negative surprises?

- Does a surprise have a transitory or permanent effect on stress?
  - Imagine a big clash in the department, can this have permanent effects?
  - Imagine a big team building effort in a firm, can this have permanent effects?
- We can understand all this by returning to distinction between stress-prone and stress-resistant individual

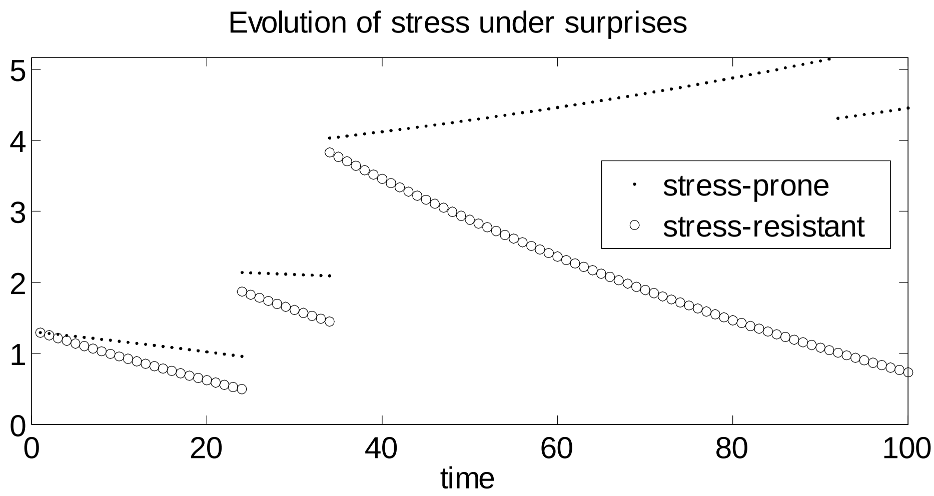



**Figure 54** *The evolution of stress after negative surprises for a stress-prone and a stress-resistant individuals*

Identical sequence of shocks pushes

- stress-prone individual to outburst while
- stress-resistant individual stays calm (remains a good stabilizer)

Can a single negative event have a permanent effect on an individual?

- No: if we look at stress-resistant individual
- Yes: if we look at stress-prone individual
- Stress-prone individual can remain permanently stressed by a unique negative event

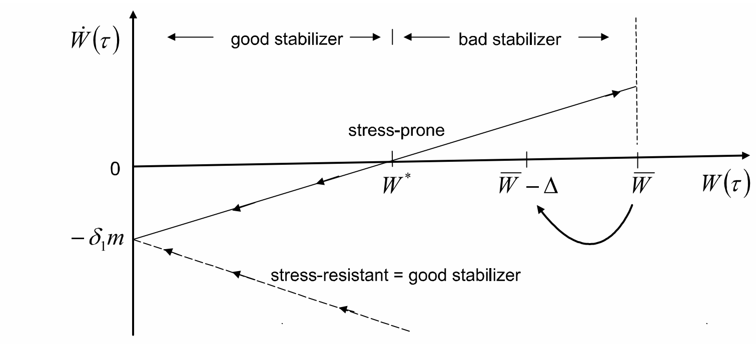
What about positive events?

- Crucial difference between stress-resistant and stress-prone individual here as well
- Stress-prone individual can permanently reduce stress level by a unique positive event

### 14.4.4 Should outbursts be suppressed?

Should outbursts be suppressed by increasing the tolerance level $\bar{W}$?

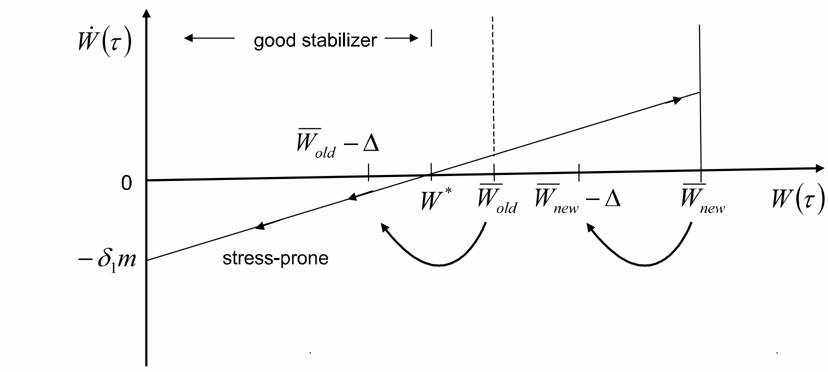
- The setup (a world without surprises)




- What happens when $\bar{W}$ rises?
  - sounds good: outburst at least comes later
  - but what about: “let it out”, “do not bottle your anger up inside“, “air-cleaning quarrels” (Bushman, Baumeister and Phillips, 2001)

Should outbursts be suppressed by increasing the tolerance level $\bar{W}$?

- It might actually *not* be such a good idea!




- While higher outburst level $\bar{W}$ postpones next outburst ...
- ... higher $\bar{W}$ might also make the permanent stress-reduction effect obsolete
- The individual might be caught in an outburst cycle

## 14.5 Employment, unemployment and stress

- Given this psychological formal model of stress and coping, what do we learn from it beyond 'stress and coping'?

- We can (eventually) learn something about

  - labour-supply decisions and
  - working conditions and labour market policy

### 14.5.1 Standard labour-supply choices

- Let an individual enjoy $c$ and leisure $l$ ($l$ does *not* stand for labour supply here)
- With a CES utility function characterized by a Constant Elasticity of Substitution, we get

$$U(c,l) = \left[\gamma c^{\theta} + (1-\gamma) l^{\theta}\right]^{1/\theta}, \quad \theta < 1 \tag{14.2}$$

- The weight attached to utility from consumption relative to utility from leisure is denoted by $\gamma$
- The elasticity of substitution between $c$ and $l$ is given by

$$\varepsilon = \frac{1}{1-\theta} > 0$$

- Remember the Dixit-Stiglitz structure in eq. (3.14)
  - The CES utility function is a discrete version of it (sum instead of integral)
  - The CES utility function has two arguments (and not a continuum as above)
- The budget constraint of a household with an hourly real wage of $w$ and a time-endowment of $\bar{l}$ units of time reads

$$c = \left(\bar{l} - l\right) w \tag{14.3}$$

- Optimal amount of leisure (see tutorial 15.2)

$$l(w) = \frac{1}{1 + \left(\frac{\gamma}{1-\gamma}\right)^{\frac{1}{1-\theta}} w^{\frac{\theta}{1-\theta}}} \bar{l} \tag{14.4}$$

- A simple parametric example

  – Let parameters be given by $\gamma = 1/2$, $\theta = 1/2$

  – Time endowment and the wage are $\bar{l} = 16$ hours, $w = 1$

  – The optimally chosen amount of leisure is $l(1) = 8$ hours

- Remember that the income and substitution effect determine whether hours worked rise in the real wage or not

$$\frac{dl}{dw} \gtreqless 0 \Leftrightarrow \theta \lesseqgtr 0 \Leftrightarrow \varepsilon \lesseqgtr 1$$

### 14.5.2 Labour supply choices in the presence of stress

- Hours worked and consumption (the good side)
  - Instead of $c = wl(e)$ from (14.1), we let the individual also choose hours worked $z(t)$
  - Consumption then equals
$$c(t) = wz(t)\, l(e(t))$$
  - With the functional forms from Wälde (2015), we obtain
$$c(t) = wz(t)[M - \kappa W(t)]$$
- Hours worked and stress (the downside)
  - The number of stressors now increases and is given by $pz(t)^{\xi}$ where $\xi > 1$
  - The growth rate of stress now becomes
$$\Phi(t) = \phi \frac{pz(t)^{\xi}}{a} - \delta_0$$

- Balancing the trade-off
  - There is a choice between
    - a lot of consumption via high hours worked $z(t)$ and a high stress level and
    - less consumption via fewer hours worked and a lower stress level
  - This is an additional channel which influences the standard labour-leisure choice
  - This can be linked to findings from industrial and organizational psychology on "spillover" of stress from professional to private life (see e.g. Bakker and Demerouti, 2013)
  - This also shows that stress is – to some extent – a choice

### 14.5.3 (Un-)Employment and mental health

- It is well-known that
  - unemployment is bad for well-being for workers (Clark and Oswald, 1994, Di Tella, MacCulloch, and Oswald, 2001, Ohtake, 2012)
  - mental health is a badly neglected issue in many OECD countries (OECD, 2014)
  - stressors at work and uncertain working condition (non-standard work, OECD, 2015) can easily be imagined to lead to poor mental health
- OECD (2014, ch. 2.2)
  - estimates costs of lost earnings due to (mild and moderate forms of) depression at around GBP 5.8 billion (for the UK in 2007)
  - suggests gains from reducing the 'treatment gap' (the share of individuals who need treatment but do not get one)

- Rising evidence on mental health consequences of unemployment
  - Tefft (2011) finds e.g. a positive association between the unemployment rate and Google's depression search index
  - Sullivan and von Wachter (2009) find that job displacement increases the mortality rate of workers permanently
- Using a theoretical framework as above can help to understand the channels through which e.g. unemployment or short-term labour market contracts (understood as a stressor)
  - affect mental health (the level of stress) and
  - can lead to negative externalities (via verbal and even physical aggression)
- A theoretical framework also makes novel predictions (e.g. the emphasis on personality, defined in a theoretically consistent way)

- The ultimate goal would be to include mental costs of unemployment, non-standard work or poverty in macroeconomic models studying equality and inequality in a society
- Hypothesis
  - more flexible labour markets are good for wages, output and growth
  - more flexible labour markets are bad for well-being of workers
  - question: which effect is stronger?
  - potential conclusion: strong government intervention imposing contracts of at least 5 years length
- We look forward to seeing you in our seminars and/ or for writing a Master thesis :-)

## 14.6 What have we learned?

Background

- Stress is a feeling that
  - everybody experiences (at least) every now and then
  - is widely acknowledged to be of relevance for society as a whole
- Stress induces various coping styles
- The model presented here looked at smooth coping and emotional outbursts
  - Smooth coping stands for controlled and cognitive approach to emotion regulation
  - Emotional outbursts stand for more impulsive, costless and fast approach
  - Emotional outbursts tend to be socially harmful (in contrast to constructive smooth coping)

Dynamics of stress and coping

- Determinants of smooth coping
  - cost and benefits of smooth coping
  - cost and benefits of outbursts
- Prevalence of outbursts (outburst theorem)
  - personality: stress-prone vs. stress-resistant individuals and on
  - appraisal type $\phi$, situation $p$, ability $a$ and autonomous stress-reduction potential $\delta_0$
- Do temporary shocks have permanent effects?
  - Personality matters a lot
  - Reducing stressors temporarily removes symptoms (high stress, frequent outbursts) ...
  - ... and can permanently reduce stress for stress-prone individual
  - Shocks can permanently push (stress-prone) individual to outburst cycles

What is the economic implication

- Framework needed that incorporates mental health in macroeconomic and labour market analyses
- Labour markets should not only help to increase efficiency of production
- Labour markets should also help employees to live a healthy life
- A lot to be studied and researched ...

# 15 Exercises

## 15.1 Unemployment

1. Using the setup from the lecture, compute the change in the unemployment rate over time (differential equation in $u(t)$) as a function of the separation rate $\lambda$, and the job finding rate $p(\theta)$.

2. Find the optimal wage following Nash bargaining between the firm and the worker, express the wage in terms of the unemployment benefit $b$, the revenue of the firm $y$, and the expected cost of maintaining a vacancy $\theta(t)k$.

   Use the following results:

$$\dot{U}(t) = \rho U(t) - b - p(\theta(t))(W(t) - U(t)) \tag{15.1}$$

$$\dot{W}(t) = \rho W(t) - w(t) - \lambda(U(t) - W(t)) \tag{15.2}$$

$$\dot{V}(t) = \rho V(t) + k - q(\theta(t))(J(t) - V(t)) \tag{15.3}$$

$$\dot{J}(t) = \rho J(t) - (y - w(t)) - \lambda(V(t) - J(t)) \tag{15.4}$$

3. Derive the second differential equation of the classical matching approach to unemployment describing the evolution of labour market tightness.

4. In order to draw the phase diagram in $u(t)$ and $\theta(t)$, we need to understand the zero-motion lines for the system. Using the zero-motion line for $\dot{\theta}(t)$ derived in (3) above, verify that the value of $\theta(t)$ that solves it is indeed positive.

5. Draw the phase diagram for the equilibrium path of market tightness as a function of the unemployment rate, i.e. when $\dot{\theta} = 0$ and $\dot{u} = 0$. Give an interpretation. (Hint: Use your answer from (2) above, as well as the differential equation of $J$ given in (2)).

6. What is the effect of a change in output $y$ on labour market tightness $\theta(t)$ in equilibrium? (Hint: use implicit differentiation).

## 15.2 Labour supply (reminder)

1. Derive the optimal amount of leisure in (14.4).

2. How does optimal labour supply look like?

3. Does labour supply increase or decrease in the real wage?

Johannes-Gutenberg Universität Mainz

Master in International Economics and Public Policy 1st Semester

# Advanced Macroeconomics

2023/2024 winter term

Klaus Wälde (lecture) and Motoaki Takahashi (tutorial)
www.macro.economics.uni-mainz.de

# Part IV
# Conclusion

## 17 What did we learn from the individual fields?

### 17.1 Economic growth

- Central empirical questions
  - Why are some countries rich, why are some others poor?
  - Do countries converge to the same long-run level of income?
  - Is there a reduction of the poverty rate and of inequality as measured by the Gini coefficient?
- Current view of convergence debate
  - Poverty persists but the absolute number declines
  - Inequality as measured by Gini declines as well but very slowly

- Theory of economic growth
  - From exogenous factors of growth to endogenous, economically determined drivers of growth
  - Policy and politics play a crucial role in shaping the growth path of a society
- Contribution of psychological views
  - Impulse control and savings (Fudenberg and Levine, 2006)
  - Dual-self model provides new determinants (costs of self-control) of saving rate
  - Behavioural growth extends existing views on the growth process and allows for novel predictions
  - Empirical relevance to be investigated

## 17.2 Business cycles

- Central empirical questions
  - Why do growth rates of countries fluctuate over time?
  - How do we measure these fluctuations?
  - Can we date the beginning and end of a business cycle?
- The current approach to measurement
  - Various type of filters that produce trend vs cyclical component of a cycle
  - Dates of peak and trough of a cycle differ across methods but are sufficiently close
- Theories of business cycles
  - Real-business cycle (vs natural volatility vs sunspot cycles – see appendix)
  - Exogenous shocks (vs endogenous shocks vs mood)
  - Efficient factor allocation vs potential inefficiency vs inefficiency

- Contribution of psychological views
  - Time-consistency and time-inconsistent behaviour is widely observed
  - This might play a role for theories of business-cycles as well
  - Aggregate events that trigger reoptimisation would lead to changes in consumption and saving behaviour
  - Could be used to understand how moods affect production and growth
  - (Sunspot models can be carried one step further)

## 17.3 Unemployment

- Central empirical questions
  - How can unemployment meaningfully be defined?
  - How high are unemployment rates in Germany and how do they change over time?
  - How do unemployment rates differ across countries?
  - Who is most affected by unemployment? Skill, age, region ...
  - How often do individuals become unemployed and how many of them (stocks vs flows)?
- The central theoretical questions
  - Why is there unemployment?
  - How can one reduce unemployment?
  - Can unemployment be reduced without creating poverty?

- Theories of unemployment
  - Traditional theories of labour supply (voluntary unemployment)
  - Traditional theories of real wage rigidities (involuntary unemployment)
  - Pure search views – stresses worker's behaviour
  - Matching models with vacancy creation – stresses the job creation by firms
- Contribution of psychological views
  - Unemployment creates stress
  - Additional channel through which unemployment reduces well-being
  - Economic policy should not orient itself only at material aspects like income and consumption but also at feelings
  - There is a trade-off for policy as well between material growth and well-being

# 18 Overall conclusion

- A lot can be learned from economic analysis about growth, business cycles, unemployment and other macroeconomic questions
- There are many macroeconomic questions that need further investigation
- Generalizing the "model of man" in economics to allow for more psychological thinking is useful per se
- More psychological thinking also promises to yield a better understanding of macroeconomic questions

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Johannes-Gutenberg Universität Mainz

Master in International Economics and Public Policy 1st Semester

# Advanced Macroeconomics

2023/2024 winter term

Klaus Wälde (lecture) and Motoaki Takahashi (tutorial)
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# A Appendix

These are answers to earlier Q&A sessions. They might help to understand the material covered in the course.

## A.1 Steady states, balanced growth paths and saddle paths

- A simple definition for a steady state that works for most real (i.e. non-monetary) economic models without growth is the following: A steady state is a solution to a dynamic system where all variables are constant. More formally: A steady state $x^*$ of a dynamic system $\dot{x}(t) = f(x(t))$ satisfies $f(x^*) = 0$.

- A corresponding definition for a balanced growth path would read: On a balanced growth path, all variables grow with *one* constant (positive) rate. More formally: A balanced growth path satisfies $\dot{x}(t)/x(t) = g$ for all variables $x(t)$.
  From this definition we understand immediately that it does not apply to all growth models. In the Solow model with exogenous technological progress and population growth, we find that some variables grow with $g$, while others grow with $n$ or even $n + g$. In the growth model by Grossman and Helpman, the number $n(t)$ of varieties increases by $g$ while the value of a firm falls by this rate.
  A refined definition of a balanced growth model would therefore read: On a balanced growth path, all endogenous variables in the model grow at constant *rates*. Formally:

$\dot{x}_i(t)/x(t) = g_i$. The growth path would be balanced in the sense that growth rates do not change. But growth rates are not necessarily the same for all variables. Equivalently, one could defined a balanced growth path as a solution to a dynamic system where growth rates of endogenous variables do not change.

- A saddle path is one possible path of a dynamic system/ of an economy. A saddle path can
  (i) lead to a steady state (as in Cass-Koopmans-Ramsey model)
  (ii) lead to a balanced growth path (as in Solow model with technological progress and population growth)
  (iii) be a balanced growth path (as in Grossman Helpman model)

## A.2 Utility functions

- CRRA and CARA utility functions

(taken from 'Applied Intertemporal Optimization', ch. 8.1.7 – see www.waelde.com/aio)

Before concluding this first analysis of optimal behaviour in an uncertain world, it is useful to explicitly introduce CRRA (constant relative risk aversion) and CARA (constant absolute risk aversion) utility functions. Both are widely used in various applications.

The Arrow-Pratt measure of absolute risk aversion is

$$\text{APM\_ARA} \equiv -\frac{u''(c)}{u'(c)}$$

and the measure of relative risk aversion is

$$\text{APM\_RRA} \equiv -\frac{cu''(c)}{u'(c)}.$$

An individual with uncertain consumption at a small risk would be willing to give up a certain absolute amount of consumption which is proportional to $-u''(c)/u'(c)$ to obtain certain consumption. The relative amount she would be willing to give up is proportional to the measure of relative risk aversion.

The CRRA utility function is the same function as the CES utility function which we know from deterministic setups. Inserting a CES utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad \sigma \geq 0$$

into these two measures of risk aversion gives a measure of absolute risk aversion of

$$-\frac{u''(c)}{u'(c)} = -\frac{-\sigma c^{-\sigma-1}}{c^{-\sigma}} = \frac{\sigma}{c}$$

and a measure of relative risk aversion of

$$-\frac{cu''(c)}{u'(c)} = \sigma,$$

which is minus the inverse of the intertemporal elasticity of substitution. This is why the CES utility function is also called CRRA utility function. Even though this is not consistently done in the literature, it seems more appropriate to use the term CRRA (or CARA) in setups with uncertainty only. In a certain world without risk, risk-aversion plays no role.

Note that $\sigma$ needs to be non-negative. For negative $\sigma$, utility functions would be convex, i.e. marginal utility from consumption would rise in consumption. This is not considered to be empirically plausible. For $\sigma$

The fact that the same parameter $\sigma$ captures risk aversion and intertemporal elasticity of substitution is not always desirable as two different concepts should be captured by different parameters. The latter can be achieved by using a recursive utility function of the Epstein-Zin type.

The typical example for a utility function with constant absolute risk aversion is the exponential utility function $u(c(\tau)) = -e^{-\sigma c}$ where $\sigma$ is the measure of absolute risk aversion. Given relatively constant risk-premia over time, the CRRA utility function seems to be preferable for applications.

See "further reading" on references to a more in-depth analysis of these issues.

- The role of changing relativ risk aversion (changes in $\sigma$)

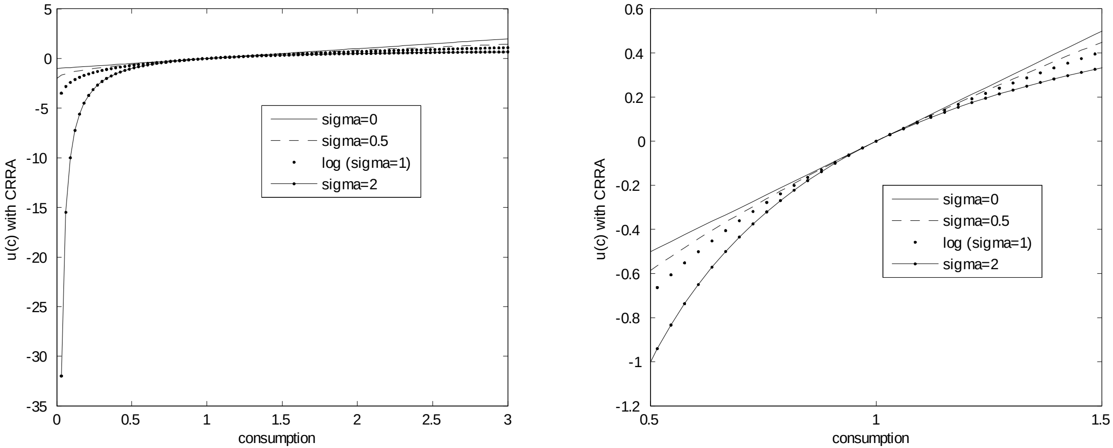


**Figure 55** $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ *for different values of relativ risk aversion* $\sigma$

- Curvature and risk

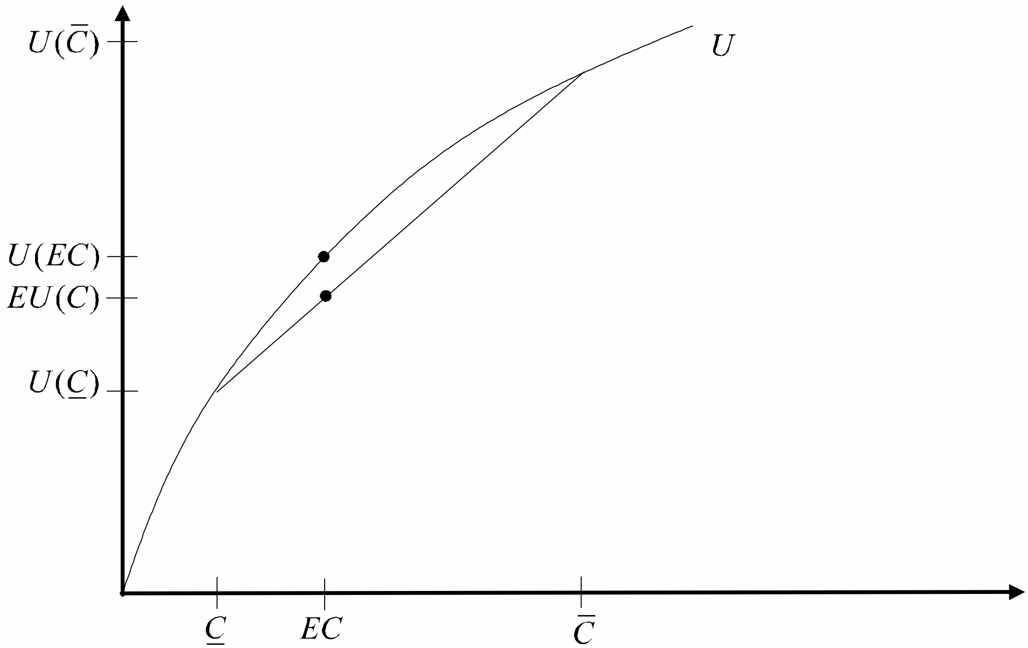


**Figure 56** *The effect of uncertainty on expected utility*

Imagine consumption fluctuates between a lower and an upper level, $\underline{C}$ and $\bar{C}$, respectively. Let expected consumption be given by

$$EC = p\underline{C} + (1-p)\,\bar{C}$$

where $p$ is the probability that consumption is low. Then expected utility $EU(C)$ is the same as utility from expected consumption $U(EC)$ if the individual is risk neutral, i.e. if the utility function is linear $(u(C) = a + bC)$,

$$\begin{aligned} EU(C) &= pU(\underline{C}) + (1-p)\,U\left(\bar{C}\right) = p\left[a + b\underline{C}\right] + (1-p)\left[a + b\bar{C}\right] \\ &= a + b\left[p\underline{C} + (1-p)\,\bar{C}\right] = U(EC)\,. \end{aligned}$$

The expected utility level is given by the dot on the straight line above.

When the utility function is concave $(\sigma > 0)$, expected utility from consumption is smaller than utility from expected consumption. Utility from expected consumption, $U(EC)$, is the dot on the concave utility function above.

## A.3 Why marginal utility equals the shadow price of wealth

Why does the planer equate marginal utility from consumption $u'(C(t))$ in $t$ with the shadow price of wealth $\lambda(t)$ in $t$? Why does $u'(C(t)) = \lambda(t)$ from (3.8) need to hold from an intuitive perspective?

- The costs from more consumption

Let us start from the following figure. There is an optimal consumption path $C(\tau)$ and the individual/ planer decides at $t$, for whatever reason (e.g. change in interest rate or income) to increase or decrease utility by one unit.

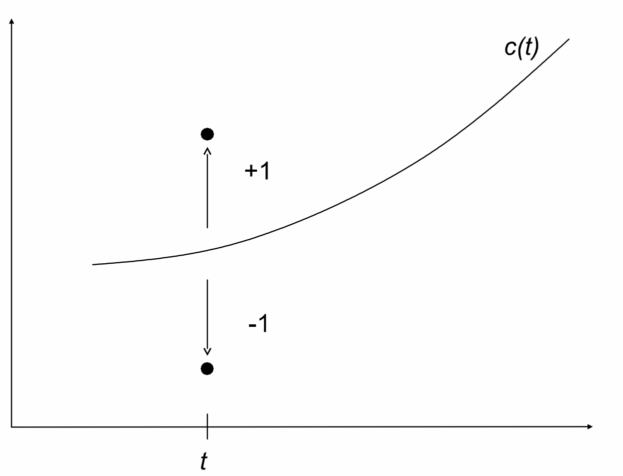


**Figure 57** *The consumption path and a "marginal increase or decrease by one unit"*

To make this discrete change consistent with marginal changes in theory, we should think of this one unit as being small. How does this change affect wealth or the capital stock? The constraint reads

$$\dot{K}(t) = Y(K(t), L) - \delta K(t) - C(t).$$

Hence, the marginal increase of the consumption level ("by one unit") reduces the capital stock marginally ("by one unit"). This is the loss from more consumption today: the capital stock falls.

- The gains from more consumption

Where do we see the gains from more consumption today? Lets look at the next figure.

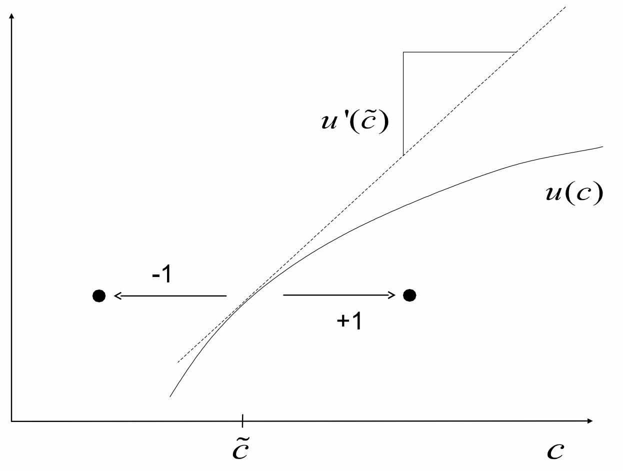


**Figure 58** *The effect on utility of a marginal increase or decrease*

Let us imagine the consumption level before the change is at $\tilde{C}$. The "marginal" effect by increasing consumption today by one unit is then described by marginal utility at this consumption level $\tilde{C}$, i.e. by $u'\left(\tilde{C}\right)$. This marginal utility gives us the gain from more consumption today.

- The costs from more consumption today expressed via the shadow price $\lambda(t)$ of wealth

What is then $\lambda(t)$? To understand this, we first need to understand what the value function is. The value function gives us the highest possible utility level when the capital stock is $K(t)$ and the planer behaves optimally, i.e.

$$V(K(t)) = \max_{\{C(t)\}} U(t)$$

where $U(t)$ is our objective function from (3.5), $U(t) = \int_t^\infty e^{-\rho[\tau - t]} u(C(\tau))\, d\tau$.

Once we behave optimally and we have chosen a time path $\{C(t)\}$ for consumption, we can then ask what the effect is of an increase of consumption on the value function. If we increase consumption marginally in $t$ and this reduces capital marginally in $t$, then the cost of more consumption today is given by the drop in the value function due to less capital. This is described by $\lambda(t)$. It is the shadow price of wealth and it equals (see Wälde, 2012, for a more formal background)

$$\lambda(t) = \frac{d}{dK(t)} V(K(t)),$$

i.e. by the derivative of the value function $V(.)$.

- Putting costs and gains together

When the planer behaves optimally, marginal gains and marginal costs are the same. Hence, (3.8) needs to hold.

## A.4 Early economic literature on Covid-19

- Brotherhood et al. (2020)
  - partial equilibrium with consumption-leisure choices + augmented SIR model
  - individuals can consume, work, work at home, enjoy leisure outside, or leisure at home
  - spending time outside (for work or leisure) increases infection risk and transmission rate
  - consumers can be either healthy, symptomatic (common cold or CoV-2), infected with CoV-2 (revealed upon testing), recovered, or dead
  - testing reveals whether it is CoV-2, leading to (targeted) quarantine
  - age differences matter for targeting and create heterogeneity in risk-taking
    - the young take more risk as they are more resistant
    - creating more risks for the old but also speeding up time to herd immunity
  - aggregate output: summing total labour income across health states and individuals

- Acemoglu et al. (2020)
  - multi-group age-based SIR model
  - focuses on targeted lockdown measures
  - quantifies effects on GDP, (excess) mortality rate in the absence of vaccine/cure, and infection across groups
  - economic loss is measured from each group: susceptible, infected, recovered, and dead

- Eichenbaum, Rebelo, and Trabandt (2020a)
  - general equilibrium with consumption-leisure choice + classical SIR framework
  - individuals optimally choose consumption and hours worked
  - and can be either susceptible, infected, recovered, or dead
  - infected individuals have lower labour productivity than susceptible and recovered individuals
  - consuming and working less reduce infection risk
  - government taxes consumption and redistributes via lump-sum transfers

- Eichenbaum, Rebelo, and Trabandt (2020b)
  - general equilibrium with consumption-leisure choice + classical SIR framework + testing
  - testing leads to lower infection rates, lower death rates, and a reduction in the size of output drop
  - infected individuals do not work and finance consumption via transfers from the government levied from taxing non-infected
- Eichenbaum, Rebelo, and Trabandt (2020c)
  - general equilibrium with New Keynesian framework + classical SIR framework
  - due to sticky prices, recession is larger than in Neoclassical framework
  - inflation rate is reduced compared to the steady-state in an epidemic
  - → as consumption drops in an epidemic, firms face lower demand
  - optimal choice of prices is then lowered as firms maximise profits in the face of low demand

- Fernández-Villaverde and Jones (2020)
  - SIR(D) framework with social distancing
  - individuals can be susceptible, infected/infectious, resolving (i.e. infected but no longer infectious), dead, or recovered
  - social distancing captures how infectious a contact with a susceptible individual is (for the latter)
  - simulating the model, the authors forecast disease spread and time to herd immunity
- Krueger, Uhlig, and Xie (2020):
  - follow Eichenbaum, Rebelo, Trabandt (2020a)
  - introduce different likelihoods of contagion across consumption sectors in addition to general infection via social interactions
  - with heterogeneity in infection across sectors, consumption shifts to the low infection sector (e.g. shopping in a supermarket vs. online)
  - effect mitigates drop in aggregate consumption

- General framework can be constructed to encompass major results
  - individuals maximise lifetime utility in consumption, hours worked (in office or at home), and leisure (outside or at home)
  - consumers can be in one of four states: susceptible, infected (whether infectious or not), recovered, or dead
  - testing works as a revealing mechanism to determine who is infected and refine targeting of containment policies
  - discovery of a vaccine/cure eliminates (or at least severely reduces) future infection rates (assuming widespread availability and adoption)
  - epidemic has multiple effects on the economy:
    * being infected can reduce productivity, thus reducing output the higher the share of the population with the disease
    * working from home or isolating can reduce transmission and infection rates but also lower utility and labour income
    * consumption can shift to low-risk sectors reducing negative impact on aggregate consumption and output
    * inflation can slow down as firms choose lower prices in a low-demand environment