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Social Identity Equilibrium: Structure and Implications

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Abstract

This thesis analyzes the existence and characteristics of a macroeconomic equilibrium of social identity with heterogeneous individuals and endogenous groups. For this purpose, the economic literature on social identity is first reviewed. Subsequently, a corresponding model is developed, and the resulting equilibrium is analyzed using two different distributions of heterogeneous individuals. The thesis concludes that the main determinant for the emergence of social groups in equilibrium is the relationship between identity-related utility parameters and the maximum value of an attribute in the population. By explaining the determinants of the existence of social groups in the societal equilibrium, the model also provides insights into the determinants of social cohesion.

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1 Introduction

An essential part of every person's life is social connections. We all strive to surround ourselves with people who bring us some subjective benefit. How and with whom an individual forms social connections is therefore a significant question in human life. The choice of such connections and their associated effects on our well-being are also understood as social identity. Tajfel (1974, p. 69) defines social identity as the "[. . .] part of an individual's self-concept which derives from his knowledge of his membership of a social group (or groups) together with the emotional significance attached to that membership.". The choice of individual social identity thus refers to choosing membership in social groups, given their impact on individual well-being.

Social identification is not only important for psychological well-being but also has major effects on human behavior and, consequently, on economic decisions. More specifically, it has been shown that mere membership in groups influences individual decisions (Charness et al., 2007; Goldstein et al., 2008). Explaining how and with which groups individuals identify is therefore a central component of the analysis of economic behavior. If the emergence and determinants of social identity can be explained both at the micro- and macro-level, this represents an important contribution to understanding behavior and how changes in external factors influence behavior through the social component.

In economic literature, the concept of identity, i.e., the perception of oneself, as part of individual utility was introduced about 25 years ago by Akerlof and Kranton (2000). Since then, the topic has gained increasing popularity, although it still constitutes only a small part of economic research. Another significant contribution to the economic analysis of social identity was made by Shayo (2009) through the development of a theoretical framework that models the choice of social identity specifically. This framework serves as the foundation for many subsequent analyses of social identity in economics. Another contribution worth mentioning here is the study of Grossman and Helpman (2021). While in the model framework introduced by Shayo (2009) an individual can only identify with one group, Grossman and Helpman (2021) extend the model by allowing for membership in multiple groups. This significantly expands the scope for analyzing social identity.

As I will argue in the course of this thesis, however, a part of the analysis of social identity does not seem to have been considered in the economic literature so far. Specifically, there appears to be no consideration of the macroeconomic equilibrium with endogenous social groups. This means that existing models assume a specific number of social groups with which individuals can identify. However, this is a significant limitation, as it is far more realistic that social groups emerge from the identification decisions of individuals themselves. Therefore, the goal of this thesis is to develop such a model with endogenous social groups at the macroeconomic level. Specifically, the model aims to describe how individuals choose social identity, i.e., their membership in social groups, how such an optimal choice is characterized, and which groups exist when all individuals behave optimally. In this regard, the model will build on the framework introduced by Shayo (2009) and further developed by Grossman and Helpman (2021), but extends this by allowing for endogenous social groups. Overall, this thesis seeks to answer the question of how an equilibrium of social identity with endogenous groups and heterogeneous individuals can be described and under what conditions such an equilibrium exists.

The remainder of this thesis is organized as follows. In the second section, I will discuss the economic literature on social identity, with a particular focus on the macroeconomic analysis of the choice of social identity. The third section then introduces the model framework for analyzing the equilibrium of social identity with endogenous groups. The subsequent fourth section analyzes the conditions under which the model actually leads to the existence of social groups in equilibrium, thus examining whether such an equilibrium exists. In the fifth section, the obtained results are discussed. The sixth and final section provides a conclusion.

2 Related literature

This section of the paper aims to provide an overview of how social identity is discussed within the field of economics. While I will briefly touch on key insights from social psychology and sociology, the primary focus will be on the role of social identity in economic research rather than an extensive analysis of studies from these fields. Thus, the objective is to explore the extent to which economic literature incorporates the concept of social identity. Furthermore, in order to offer the reader a better overview of the state of research in this area, this section is divided into two main categories. The first part examines the formation and choice of social identity. More specifically, it discusses the studies that analyze how individuals choose their social identity. The second part then shifts the focus to the effects of social identification, investigating how identifying with a particular group influences individual behavior and decision-making.

2.1 Choosing social identity

As mentioned above, this section explores the question of how individuals choose their social identity. It begins with a discussion of some fundamental concepts and an introduction to the early applications of social identity in economics. This is followed by an overview of empirical analyses examining the determinants of the social identity choice. Subsequently, studies focusing on the development of models of social identity at the micro level are discussed. The final subsection then addresses the most relevant part for this thesis, namely the discussion of studies that focus on the development of social identity models at the macro level.

2.1.1 General background

The theoretical foundations of social identity were largely developed by Tajfel (1974) and Tajfel and Turner (1979, 1986). They explain that a crucial aspect of human psychology is how individuals perceive their social connections with others. The basis of social identification lies in the fact that individuals categorize their social environment based on actions, intentions, attitudes, and beliefs (Tajfel, 1974, p. 69). In this sense, an individual divides their social surroundings into distinct groups and assigns people to these groups based on personal perception and subjective decision criteria. Given this categorization of the social environment, the individual then identifies with specific categories or groups that emerge from this classification. More precisely, social identity can be defined as "[...] part of an individual's self-concept which derives from his knowledge of his membership of a social group (or groups) together with the emotional significance attached to that membership." (Tajfel, 1974, p. 69). If we assume that individuals can choose their social identity - meaning they can decide with which categories or groups they identify - then it follows from the above definition that an individual will identify with groups that hold positive emotional significance for them. In economic terms, this implies that individuals identify with groups that increase their utility. To understand which factors influence the choice of social identity, it is useful to first examine the reasons behind the categorization of the social environment and the subsequent social identification in general. Hogg et al. (2008) identify three fundamental motivational patterns in this context. The first pattern relates to the positive effect of group membership on an individual's self-esteem. According to this, individuals generally feel better when they perceive themselves as part of a group compared to when they do not belong to any group. The second motivational channel of social identification stems from individuals' existential anxiety and their inherent desire for validation of their perceptions and behaviors in order to reduce these concerns. They seek this validation in social groups. The third motivational channel is the need to reduce uncertainty about oneself. Since social groups typically imply certain stereotypes through their members, identifying with a group provides an individual with a reference point for their own behavior. By aligning their behavior with this reference, the individual can reduce uncertainty about themselves.

While the first channel broadly explains why individuals join groups, the second and third channels also allow broad conclusions to be drawn about which groups an individual identifies with. More precisely, both the second and third channels imply that individuals tend to identify with groups whose members are, in some way, similar to themselves. The reduction of existential concerns through group membership is stronger when group members are more similar to the individual. Likewise, the uncertainty about oneself decreases the more closely the group's implied stereotype aligns with one's own characteristics. Since a group's stereotype is shaped by its members, this means that self-uncertainty diminishes more significantly when individuals identify with groups whose members are similar to them. This preference for similarity is in line with the aversion to inequality known from behavioral economics, as shown, for example, by the influential studies of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000).

Another factor that determines which groups an individual joins and which ones they do not join is the fundamental value a given group holds for the individual (Tajfel & Turner, 1979). The value of a group is subjectively determined by comparing the value-related attributes within the group to those of other groups. Consequently, every group holds a certain value for an individual, and the choice to identify with specific groups is influenced by this perceived value. This value can also be understood as status. An individual, therefore, prefers to identify with groups that have a high status over those with a lower status[1].

In summary, it can be concluded that the distance between an individual and the members of a group, as well as the relative status of a group, are the fundamental criteria in choosing one's social identity. Individuals tend to prefer groups that have a high relative status and whose members are, in some way, similar to themselves.

With an understanding of what social identity is, how it emerges, and the main determinants of the social identity choice from social psychology, we can now examine the applications of the social identity concept in economics. Akerlof and Kranton (2000) were the first to introduce the concept of identity into economic research. They focused on identity as an individual's self-perception and its influence on economic outcomes. By incorporating identity as a determinant of the utility function, they laid the foundation for the economic analysis of the significance of identity. Although they argue that self-perception is largely shaped by the social environment - since individuals compare themselves to the behavioral patterns implied by certain social categories - Akerlof and Kranton (2000) do not specifically analyze social identity itself, meaning the question of which categories or groups an individual chooses to identify with. Nevertheless, the introduction of identity as a utility-influencing factor has opened the door to a new field of research: the analysis of the role of identity and social identity in the economic context.

2.1.2 Empirical analyses

A part of the analysis of social identity involves the empirical examination of the determinants of social identity choice. From insights in social psychology, we know that the fundamental criteria for group affiliation are the distance to individuals within the group and the status of the group. Hett et al. (2017) empirically demonstrate, through a laboratory experiment based on revealed preferences, that individuals tend to identify with groups that have high status and to which they have low social distance. Furthermore, Atkin et al. (2021) extend this perspective by showing that, in addition to the

[1] Roccas (2003) demonstrate empirically the importance of status for the identity choice. However, since this study is not within the field of economics, it will not be included in the following section but rather serves as additional support for the explained significance of group status.

fundamental factors of status and distance, economic costs also play a role in social identity choice. Using a revealed preference approach based on food consumption in India, they demonstrate that the costs associated with group identification are a relevant factor when choosing which group to identify with. In another empirical analysis, Mayer and Puller (2008) use student data to determine that race is the primary factor in the process of developing social networks. In a similar study, Marmaros and Sacerdote (2006) find that, in addition to race, geographical proximity is also a key determinant in the formation of social networks.

2.1.3 Micro-models

In addition to the empirical examination of the determinants of social identity choice, theoretical models that describe this process constitute another important part of the economic literature on social identity. These models analyze the decision of which groups an individual joins or identifies with using microeconomic frameworks. This section provides a brief overview of several key studies that have developed relevant models.

For instance, Fryer and Jackson (2008) developed a model of categorization in which individuals sort their experiences into specific categories. This model can be linked to the step preceding social identification—social categorization—as described by Tajfel (1974). More precisely, the model explains how individuals categorize their experiences and perceptions, which then serve as the basis for later behavior. A key aspect of the approach of Fryer and Jackson (2008) is the assumption that the number of possible categories is limited. As a result, individuals must store heterogeneous experiences and perceptions within a single category. The optimal categorization, according to their model, is achieved by minimizing variation within each category across all available categories. When applied to the question of how individuals choose their social identity, the model developed by Fryer and Jackson (2008) does not directly answer this question but rather provides information on the groups that are salient to an individual. In other words, the model helps to understand which groups an individual considers when making their social identity choice. Furthermore, taking into account the definition of optimal categorization chosen in Fryer and Jackson (2008), that variation within a category is minimized, and applying this to the social context, it follows that individuals aim to sort similar people into the same category. This implies that the perceived groups resulting from this process consist of members who are relatively similar, aligning with the previously discussed principle of social distance within groups.

Graham (2017) also builds on the concept of social distance within groups and specifically models the individual choice of social identification from a microeconomic perspective. More precisely, the study presents a network formation model in which the connections between individuals are based on the concept of homophily — the preference for similarity. Additionally, Graham (2017) incorporates the concept of degree heterogeneity, which suggests that different individuals have

varying numbers of social connections. The model thus uses the preference for similarity to explain how individuals form social groups with others.
Another comprehensive model of social group formation and individual social identity choice was developed by Milchtaich and Winter (2002). It models both statically and dynamically how individuals group together based on their preference for similarity. Additionally, Milchtaich and Winter (2002) examine the stability of group segregation. A group structure is considered stable if no individual prefers to switch to another group. Thus, they also analyze a possible equilibrium of social identification. While their analysis closely aligns with the objectives of this thesis, their approach remains within the scope of microeconomics. This thesis, however, aims to describe such an equilibrium from a macroeconomic perspective.

2.1.4 Macro-models

After examining economic studies that focus on empirical and microeconomic analyses of the social identity choice, we can now turn to the most relevant area of economic literature for this thesis, namely, studies that analyze social identity from a macroeconomic perspective.
One of the earliest contributions in this context is the theoretical model of social identity choice developed in Shayo (2009). This model emphasizes the endogeneity of both social identity and individual behavior. Based on the behavioral patterns analyzed, the study draws conclusions about the influence of social identity on preferences for redistribution. Although the latter aspect is of less interest for this thesis, the model developed in Shayo (2009) is highly relevant for deriving and understanding the analyses that follow in this thesis. The crucial element here is the fundamental model structure related to the choice of social identity. Specifically, the model incorporates the fundamental decision criteria for social identification from social psychology, status and distance, as determinants of individual utility. In this context, the utility of an individual i is described as

$$U_i(t) = \pi_i(t) - \beta d_{iJ}^2 + \gamma S_J(t),$$

where t represents the tax rate and $\pi_i(t)$ is the materialistic payoff of an individual i that depends on the tax rate (Shayo, 2009, p. 151). The second part of the utility function then describes the individual's utility derived from being a member of a group $J \in G$, with G denoting the total number of groups. This utility is determined by both the individual's distance from the group, d_{iJ}^2, and the status of the group, $S_J(t)$. Shayo (2009, p. 150) further defines the distance as

$$d_{iJ} = \left(\sum_{h=1}^{H} w_h (q_i^h - q_J^h)^2 \right)^{1/2}$$

where, for the combination of an individual i and a group J, the Euclidean distance between the attributes of the individual and those of the group members represents the degree of similarity between the individual and the group. More precisely, q_i^h represents the level of attribute $h \in H$ for the individual, while q_J^h represents the average level of this attribute within the group. Additionally, weight factors w_h are included in the summation of attribute-specific distances to account for the fact that individuals consider some attributes more important than others. Furthermore, Shayo (2009, p. 151) specifies that the status of a group is given by

$$S_J(t) = \sigma_0^J + \sigma_1^J(\tilde{\pi}_J(t) - \tilde{\pi}_{r(J)}(t))$$

where σ_0^J includes all exogenous factors influencing status, σ_1^J is a positive constant, and the status of a group J increases with the difference between the material payoff of the group itself, $\tilde{\pi}_J(t)$, and the material payoff of a reference group, $\tilde{\pi}_{r(J)}(t)$.

The model assumes that individuals can be either poor or rich (Shayo, 2009, p. 149). Additionally, it is assumed that there are three possible groups with which individuals can identify: the two social classes, poor and rich, and the nation as a whole (Shayo, 2009, p. 150). As a result, the social identity choice of a poor or rich individual is reduced to deciding whether to identify with their own social class or with the nation. In general, the study describes the equilibrium of social identification in the way that individuals identify with those groups that provide them with the highest utility (Shayo, 2009, p. 151). Accordingly, an individual compares the utility profiles of the different groups and subsequently identifies with the utility-maximizing group. According to the above utility function, both the status of the group and the distance between the individual and the members of the group are included in the identification decision.

In summary, the model developed in Shayo (2009) has made a significant contribution to incorporating social identity into economic analysis. This is particularly due to its use of distance and status as determinants of the utility derived from social identification, which aligns with the main findings from social psychology. Moreover, the developed model framework has opened the door for theoretical analyses of social identification decisions for different individuals and groups. However, within the study itself, such an analysis is only conducted to a limited extent, as both individual and group heterogeneity are restricted. Since individuals are assumed to be either poor or rich and the only social groups considered are the two social classes and the nation, the analytical framework is limited in this regard. Additionally, the endogeneity of the existence of social groups is not considered.

Even in the further development and discussion of the model in Shayo (2020), these aspects are not addressed. Although the investigation of social identity from an economic perspective is significantly more extensive in Shayo (2020) - covering discussions of micro evidence as well as applications to redistribution, nationalism, immigration, trade, and populism — it still does not introduce a greater differentiation of individuals, nor does it relax the restrictions on the number of social groups and their endogeneity.

Another study dedicated to developing a theoretical model of the social identity choice is Grossman and Helpman (2021). The main goal of this study is to develop a model of social identification to analyze preferences for trade policy within an economy. The fundamental idea is that heterogeneous individuals, who identify with certain groups, interact with political parties that offer specific trade policies in the form of tariffs. The model then analyzes which individuals identify with which groups and how this social identification affects voting behavior and the resulting tariffs.
Again, my focus is less on the part of Grossman and Helpman (2021) that applies identity choice specifically to trade policy. Instead, I aim to discuss the developed model in relation to the choice of social identity itself. In general, the model structure closely resembles the one introduced in Shayo (2009, 2020). More precisely, Grossman and Helpman (2021, p. 1106) also assume that individual utility consists of both a materialistic component and a component derived from social identification. Specifically, the utility of an individual in the developed model has the structure

$$u_i = c_{Xi} + v(c_{Zi}) + \sum_{g \in G} I_i^g [A_i^g + \alpha_i^g \overline{v}^g - \beta_i^g (v_i - \overline{v}^g)^2]$$

(Grossman & Helpman, 2021, p. 1107). Here, c_{Xi} represents the consumption of the export good by individual i, and $v(c_{Zi})$ represents the consumption of the import good, which depends on the trade tariff. Thus, the materialistic utility is described by the consumption of import and export goods. The second term in the utility function explicitly refers to the utility derived from identification. The expression within the brackets represents the utility of identifying with a group $g \in G$. As in Shayo (2009, 2020), this utility is positively influenced by the status of the group and negatively influenced by the distance between the individual and the group. The former is described by a certain level of pride in group membership, A_i^g, which is both individual- and group-specific, and by the average material utility in the group, $\overline{v}^g$, weighted by the utility parameter α_i^g. The distance between an individual and a group is determined by the squared distance between the individual's material utility, v_i, and the group's average material utility. The distance is also weighted by the utility parameter β_i^g. The identification utilities for all groups are then summed, and the binary variable $I_i^g \in \{0, 1\}$ is introduced to indicate with which groups an individual identifies, i.e., which groups ultimately influence individual utility. Here, $I_i^g = 0$ if the individual does not identify with the group and $I_i^g = 1$ if the individual does.
This highlights the most significant difference in preferences between Grossman and Helpman (2021) and the structure in Shayo (2009, 2020): an individual can identify with more than one

group.
Like in Shayo (2009, 2020), Grossman and Helpman (2021, p. 1105) assume that individuals are divided into two categories. In this case, there are low-skilled and high-skilled workers. Additionally, there are three groups: the working class, the elite, and the nation as a whole (Grossman & Helpman, 2021, p. 1107). As in Shayo (2009, 2020), it is assumed that individuals of a given class can only identify with their own socioeconomic group due to sufficiently large distance parameters β_i^g, meaning that cross-identification does not occur (Grossman & Helpman, 2021, p. 1107f.). The remaining question is then whether, and if so, which individuals additionally identify with the nation as a whole. This occurs if such identification has a positive effect on the individual's overall utility. In that case, the optimal behavior for individuals is to choose the binary variable I_i^g in a way that maximizes utility. In equilibrium, each individual is a member of those groups that maximize their subjective utility and has no incentive to change their group affiliation. This is in line with the concept of social identity equilibrium introduced in Shayo (2009).
Summarizing the model structure presented in Grossman and Helpman (2021) for analyzing the individual choice of social identity, we can say that the model—especially in comparison to Shayo (2009, 2020)—has several strengths. On the one hand, it also models the individual choice of social identity using the fundamental decision criteria from social psychology, group status and distance. On the other hand, and this is a novel aspect compared to Shayo (2009, 2020), it allows for individuals to identify with more than one group. This makes the model presented in Grossman and Helpman (2021) somewhat more realistic.
However, the model's assumptions are still restrictive. The assumption that individuals are either low-skilled or high-skilled significantly limits individual heterogeneity. Similarly, restricting the number of groups to three—the working class, the elite, and the nation—is a relatively broad representation of reality. As in Shayo (2009, 2020), Grossman and Helpman (2021) also do not allow for the endogeneity of group existence. Furthermore, the equilibrium analysis is relatively trivial due to the simple model structure, as it relies on a case-wise comparison of utility profiles under specific identification regimes. While this is beneficial within the specific model, such an analysis would likely become too complex for describing a societal equilibrium of social identities if more groups and endogenous group formation were considered.

Another study that focuses on the analysis of social identity in a macroeconomic context is La Ferrara (2002). The primary goal of this study is to examine the impact of income inequality on group membership. Furthermore, a distinction is made between two types of groups: open-access groups and restricted-access groups. While individuals can freely join open-access groups, entry into restricted-access groups requires approval from existing group members. Given these two types of groups and the presence of income inequality, the study investigates which individuals join

or do not join specific groups.
The fundamental idea of the model developed in La Ferrara (2002) is that groups provide certain excludable goods. The benefit of being a member of a group lies in gaining access to these excludable goods. Thus, this model does not rely on the fundamental decision criteria form social psychology, group status and distance, but instead applies a new approach. This represents a clear distinction between La Ferrara (2002) and the previously discussed studies by Shayo (2009, 2020) and Grossman and Helpman (2021). Regarding the excludable good, it is also assumed that the benefit derived from it decreases as the group size increases, due to congestion effects (La Ferrara, 2002, p. 240). Consequently, the benefit of group membership declines as the number of group members grows.
The equilibrium analysis in La Ferrara (2002) is also relatively simple. Specifically, one group, which is either an open-access or a restricted-access group, is considered, and the study examines which individuals derive greater utility from joining the group compared to remaining outside of it. Whether individuals decide to join a group then depends on the type of group and the wealth of the individuals. More precisely, La Ferrara (2002, p. 239) finds that in open-access groups, membership consists of individuals from the lower end of the wealth distribution up to a certain threshold. In restricted-access groups, members are individuals from the upper end of the wealth distribution. This is largely due to the fact that group membership in the restricted-access case is subject to voting by existing members. Since the composition of these two types of groups consists of individuals from the lower and upper parts of the wealth distribution, respectively, the effect of inequality depends on the specific shape of the wealth distribution and how increasing inequality affects these two segments of the distribution La Ferrara (2002, p. 239).
Following the theoretical analysis, La Ferrara (2002) empirically examines these patterns using survey data from rural Tanzania and finds results that align with the theoretical predictions.
With regard to the main objective of this thesis, examining an equilibrium of social identity, La Ferrara (2002) is particularly interesting because it allows for unrestricted heterogeneity of individuals, specifically in terms of wealth. This represents a novel approach compared to the previously discussed studies. On the other hand, the equilibrium analysis in La Ferrara (2002) has certain weaknesses when considering a societal equilibrium. Firstly, the equilibrium is only analyzed theoretically in relation to a single group (La Ferrara, 2002, p. 250). The existence and endogeneity of multiple groups in equilibrium are not examined. Another downside is that La Ferrara (2002) does not consider group status and distance as key determinants of social identity or group membership, thereby not reflecting relevant findings from social psychology.

Lindqvist and Östling (2013) also analyze social identities at the macroeconomic level. More specifically, they model the interaction between social identity and redistribution. For the modeling of individuals' choice of social identity, Lindqvist and Östling (2013) build on the model structure developed in Shayo (2009), meaning they also use group status and the distance between the individual and the group as determinants of social identity. It is assumed that individuals can be classified both by ethnicity and by social class. Each individual is therefore assigned both an ethnicity and a specific social class. The social groups are thus, by assumption, the ethnicities and social classes, and it is assumed that there is no cross-identification, meaning, for example, that an individual belonging to a certain social class cannot identify with a different social class. The same applies to identification with an ethnicity (Lindqvist & Östling, 2013, p. 473). The choice of social identification is therefore limited to whether the individual identifies with their ethnicity or their social class.

Since the model explicitly focuses on the interaction between social identity and redistribution, the latter is incorporated into the modeling of the social identity choice. More precisely, individuals make their identification decision given a certain prevailing tax rate. After these identification decisions are made, individuals then vote for a particular redistribution policy based on their social identity. Since this thesis focuses on the analysis of the equilibrium of social identities, I do not intend to delve deeper into the results on redistribution analyzed in Lindqvist and Östling (2013) but will instead focus on the study's findings regarding social identity.

Since Lindqvist and Östling (2013) specifically build on the model framework from Shayo (2009), the discussion of social identity equilibrium is very similar. Lindqvist and Östling (2013) also apply the social identity equilibrium concept from Shayo (2009), which states that the economy is in equilibrium when all individuals identify with the utility-maximizing groups and have no incentive to deviate from this identification pattern. Likewise, the number of groups and the possible identification alternatives remain limited, in this case, to ethnicities and social classes. Although there are more groups than in Shayo (2009), where only the social classes of the poor and rich as well as the entire nation exist, the number of social groups in Lindqvist and Östling (2013) remains constrained and is not endogenous. On the other hand, since Lindqvist and Östling (2013) also use group status and distance as determinants of social identity, their analyses are in line with the fundamental principles of social identity in social psychology.

Sambanis and Shayo (2013) too use the model framework developed in Shayo (2009) to analyze social identity. Specifically, the goal in Sambanis and Shayo (2013) is to analyze social conflict by using social identity. It is assumed that there are two ethnic groups as well as the nation as a whole, meaning that individuals can identify with one of three social groups (Sambanis & Shayo, 2013, p. 301). The individuals are further distinguished by their level of some resource endowment and their individual effort to fight for their social group's benefit. The latter depends on the resource endowment, meaning that individuals with higher endowments can contribute more than those with lower resource endowments (Sambanis & Shayo, 2013, p. 301). Individuals' social identification with either their ethnic group or the nation as a whole then depends, as in Shayo (2009), on the status of the respective group and the perceived distance between the individual and the group. The definition of the equilibrium of social identification also follows the concept described in Shayo (2009, p. 151). With regard to social conflict, Sambanis and Shayo (2013) analyze how the equilibrium social identification influences the respective fighting efforts. However, since this is of lesser relevance to this thesis, the focus remains on the analysis of the equilibrium of social identities. Since this equilibrium in Sambanis and Shayo (2013) is nearly the same as in Shayo (2009), the conclusions are also the same. Here, too, there are shortcomings regarding the variety of social groups in the economy, as well as their existence and endogeneity in equilibrium.

Another - very technical - study that examines endogenous social identity is Gennaioli and Tabellini (2019). The central idea of this paper is that individuals socially identify with a group, which then influences their beliefs, ultimately affecting their political preferences and voting behavior. More specifically, Gennaioli and Tabellini (2019) assume that there are four social groups in total, consisting of two cultural groups, the socially conservative and the socially progressive, and two economic groups, the upper and lower class (Gennaioli & Tabellini, 2019, p. 2386). Individuals then can identify either with their cultural or their economic group (Gennaioli & Tabellini, 2019, p. 2387).
The determinants of social identity assumed in Gennaioli and Tabellini (2019) include, first, the distance between the individual and the group and, second, the maximization of conflict between the in-group and the out-group (Gennaioli & Tabellini, 2019, p. 2387). In relation to the findings from Oakes (1987), the latter should incorporate the fact that individuals prefer the greatest possible distinction between their own group and other groups. The concept used in Gennaioli and Tabellini (2019) for the choice of social identification in equilibrium also differs from the concepts of the previously presented studies. Specifically, it does not focus on individual utility maximization but rather on choosing the group that maximizes the difference towards the out-group (Gennaioli & Tabellini, 2019, p. 2388f.). Gennaioli and Tabellini (2019) identify the relative importance of cultural versus economic policy for an individual as the decisive factor in the equilibrium choice of social identity (Gennaioli & Tabellini, 2019, p. 2389).
Given the endogenously determined social identities, individuals then form their beliefs about their income expectations and cultural views (Gennaioli & Tabellini, 2019, p. 2391). This, in turn, influences their political preferences and voting behavior. Since this part of Gennaioli and Tabellini (2019) is not the primary focus of this thesis, I will proceed directly to the discussion of the analysis of social identification.
As in most of the previously discussed studies on the macroeconomic analysis of social identity,

Gennaioli and Tabellini (2019) also use distance as a key determinant of social identity. However, the introduction of conflict maximization between in-group and out-group is a novel aspect. Since Gennaioli and Tabellini (2019) draw on findings from social psychology in this regard, their modeling of social identity is consistent with corresponding research in that field. However, an interesting distinction in Gennaioli and Tabellini (2019) is that group status does not play a role. Overall, Gennaioli and Tabellini (2019) provide a new contribution to the macroeconomic analysis of social identity. However, in this model too, the number of groups is limited due to their superficial character. Furthermore, the endogeneity of social groups is also not considered in Gennaioli and Tabellini (2019)).

In general, it can be observed that none of the presented studies and models analyzing the choice of social identities at the macroeconomic level account for the endogeneity of groups. As mentioned in the introductory section of this thesis, however, a comprehensive investigation of the equilibrium of social identity requires that groups emerge according to individuals' social preferences and are therefore not exogenously given. The assumptions made in the reviewed studies regarding the existence of social groups thus impose limitations on equilibrium analysis.
Additionally, the fact that some of the discussed studies pre-assign individuals to specific categories, from which their social identification then follows, also restricts the analysis. For example, the assumption that individuals with an income below the mean income are automatically categorized as poor and subsequently identify with the group of the poor is quite restrictive. It is uncertain whether such a group of "the poor" even exists in this context and whether all poor individuals indeed identify with it.
Overall, it can be concluded that existing analyses of the equilibrium of social identities rely on several restrictive assumptions. An equilibrium analysis with heterogeneous individuals and endogenous social groups does not appear to exist yet.

2.2 The influence of groups on individuals

This part of the literature review now focuses on the implications of social identification for individual behavior. While this is not the main focus of this thesis, understanding the influence of social identity on behavior helps in interpreting results and their implications.

In a qualitative study, Granovetter (2018) analyzes the effect of social structures on economic outcomes. The study argues that the influence of such structures can primarily be explained through the flow of information, the rewarding and punishing of behavior, and trust within social networks. Using these channels, Granovetter (2018) explains the significance of social connections for labor markets, prices, productivity, and other economic factors. Similarly, Charness and Chen (2020) provide a comprehensive review of relevant literature discussing the impact of social identity on economic behavior.

In a more specific application, Holm (2016) examines the influence of social identity on preferences for redistribution under federal and regional identification. Using the modeling framework from Shayo (2009), Holm (2016) finds that preferences for redistribution under federal identification can shift against individuals' economic interests compared to the case of regional identification. Additionally, Bonomi et al. (2021) also develop a theoretical model demonstrating the influence of social identification on behavior. Through this model, Bonomi et al. (2021) show that identification with certain groups affects individuals' information and beliefs, which in turn influences their voting behavior.

The majority of studies examining the effect of social identity on individual behavior in an economic context, however, are empirical. It is important to note that the studies discussed in the following originate in the field of economics. Empirical analyses from sociology, psychology, or related disciplines are not covered here.

One channel through which social identity influences individual behavior is its effect on preferences and perception of the environment. Charness et al. (2007) find that the salience of groups and corresponding membership generally shape preferences and individual perception. Similarly, Goldstein et al. (2008) provide empirical evidence that mere group salience affects individual behavior, with the effect being stronger when the group is more similar to the individual in the decision-making context.

Another channel of influence is the effect of social identification on social behavior. Hett et al. (2017) analyze that identity preferences and corresponding social identities shape social preferences, thereby affecting behavior toward other groups. Similarly, Goette et al. (2006) provide empirical evidence that group membership leads to higher cooperation within a group than between groups. Chen and Li (2009) also demonstrate that social identity influences social preferences, as individuals exhibit greater altruism toward members of their own group than toward members of other groups. Regarding income inequality, Gangadharan et al. (2019) find antisocial behavior among low-income individuals toward high-income individuals. Moreover, Bernhard et al. (2006) present evidence of a general preference for one's own group in enforcing altruistic norms. In an experimental study, Hargreaves Heap and Zizzo (2009) analyze how group membership increases discrimination against other groups, which in turn reduces overall individual trust. However, they also identify a psychological benefit of group affiliation. In another study, Leider et al. (2009) present evidence suggesting that individuals exhibit directed altruism in favor of members of their social networks. In a similar way, McLeish and Oxoby (2011) find that cooperative behavior increases when individuals share a social identity.

Another channel through which social identity affects individual behavior is its impact on economic behavior. Benjamin et al. (2010) show that social identity influences both discount rates and risk aversion. Afridi et al. (2015) analyze the effect of social identity on performance using a specific real-world application. Additionally, Benjamin et al. (2016) provide empirical evidence that social identity affects economic choices and preferences. Bursztyn and Jensen (2015) further highlight the effects of social identity on individual investment in education and Hoff and Pandey (2006) demonstrate that social identity and social conflict influence self-confidence and willingness to learn. In relation to the models discussed earlier by Shayo (2009, 2020) and Sambanis and Shayo (2013), Klor and Shayo (2010) provide empirical evidence that social identity affects preferences for redistribution, as individuals are willing to forgo personal payoffs to increase the payoff of their group. Furthermore, in another study, Halberstam and Knight (2016) show that group membership influences individuals' access to and processing of information, relating their findings to the context discussed by Bonomi et al. (2021).

3 A model of social identity

This part of the thesis is dedicated to the development of a theoretical model of social identity with heterogeneous individuals and endogenous social groups. Specifically, the general structure of the economy assumed here is first explained. This is followed by an analysis of the decision problem of individuals regarding their social identity and the resulting optimal behavior. Finally, this section concludes with the general definition of social identity equilibrium within the framework of the model presented here.

3.1 Structure of the economy

This part describes the general structure of the economy in the model. In particular, the goal is to introduce the members of the economy, in our case the individuals, the way they differ, and the groups they can join.

3.1.1 Preferences

We have N individuals who have the utility function

$$u_i = u(v_i) + \sum_{g \in G} I_i^g * u_g(v_i), \tag{1}$$

which is closely related to the structure of Shayo (2009, 2020) and Grossman and Helpman (2021). In particular, the individual i derives utility from two sources. First, $u(v_i)$ represents the utility that the individual derives from having some attribute v_i. One can think of the attribute as, for example, the wage, some attitude towards a political topic, or the degree of support for some football club. Therefore, the first part of the utility function represents the part of utility that the individual derives directly from the attribute. The second source of utility comes from the utility of identification. Each individual can be a member of a group, and being part of a group yields utility. More specifically, $u_g(v_i)$ yields the utility from identifying with the group $g \in G$, with G denoting the total number of groups in the economy. Such utility can take different forms depending on the context. Some examples are visualized in Figure (1). The utility of identifying with a group can be an increasing function in v_i. Thus, the higher the value of the attribute of the individual, the greater the benefit of belonging to a group. This could, for example, apply to elite clubs, where the richest members of society meet, or fan clubs of football teams, where the more you support a team, the greater the benefit of identifying as a fan. Another possible option for the form of utility of identification could be a decreasing function in v_i. In this case, the utility from identifying with a group is always larger, the lower the individual's value of the attribute. An example of this could include minimalists, for whom the less material possessions they have, the greater the benefit of identifying with the group of minimalists. In addition to these specific forms, the utility of identification can take on any functional form. However, following the literature, the most realistic functional form of the utility of identification is a downward opening parabola around some value of v_i. This represents that individuals derive more utility the more similar they are to a certain value. The utility from identification then decreases the further away the individual is from this value. It is not important in which direction the individual deviates. Such a form of the function for the utility from identification ensures that the preference of individuals for similarity within a social group is taken into account. Although this is also the case in the event of a monotonically increasing or monotonically decreasing utility function, these relate exclusively to scenarios in which high or low values of an attribute are strictly preferred. A downward opening parabolic utility function for group identification, on the other hand, has a more general purpose. This means that the value of the attribute is of secondary importance and the distance between the attribute value of the individual and the predominant value of the attribute in the group is paramount. So if there exists a group that forms around any given attribute value, the closer the individual's attribute value is to the predominant attribute value in the group, the greater the benefit that the individual derives from identifying with this group. On the other hand, the utility from identification decreases the further away the individual is from the predominant attribute value. Because of the more general scope of a utility function with such functional form and because it best reflects the processes described in the literature on social identity, this thesis will focus on such a function for utility from identifica-

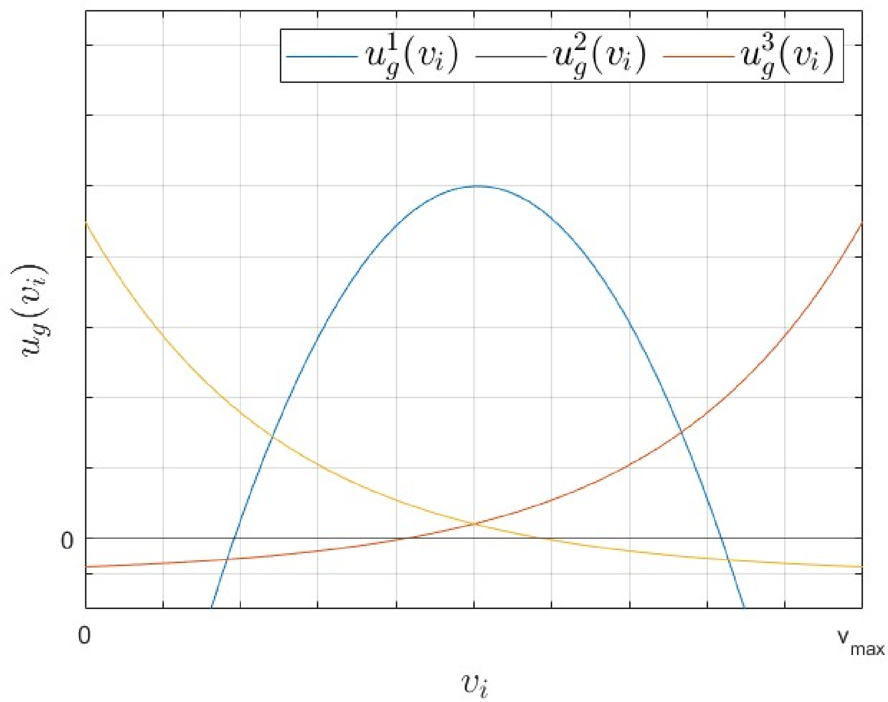

Figure 1: Possible functional forms of utility from identification

tion. Thus, the functional form chosen here is consistent with the form of utility from identification chosen in relevant studies on model-based analyses of social identity, such as Shayo (2009, 2020) and Grossman and Helpman (2021).

Given the utility that an individual derives from identifying with a social group, the final component of the individual utility function is the summation of the group-specific utilities from identification $u_g(v_i)$ across all groups G with which the individual identifies. In this context, the binary variable $I_i^g \in \{0, 1\}$ marks the identification decision by taking the value one if the individual identifies with the group g, and zero if the individual does not identify with the respective group. As a result, all groups with which the individual does not identify are excluded from the utility function. Therefore, these patterns specifically relate to the structure introduced in Grossman and Helpman (2021).

3.1.2 Population

Having described the utility of individuals, the question now arises as to how individuals differ in our economy. It is important to differentiate between individual characteristics, as otherwise all decisions would be the same. This would lead to the same behavior in society as a whole, which in turn implies the same social identification of all individuals. Therefore, we need some form

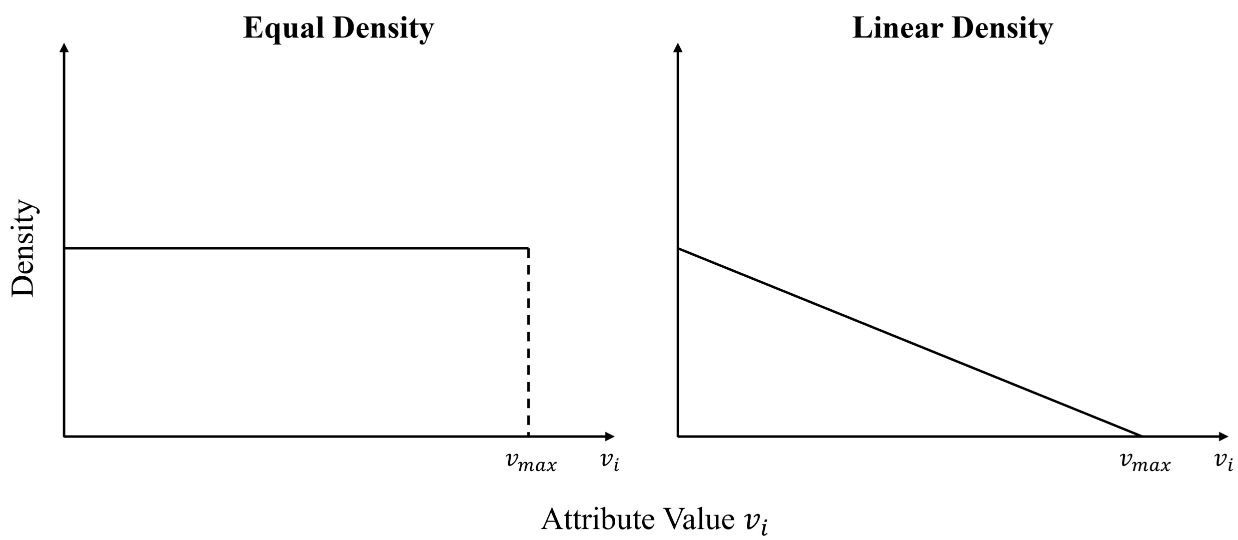

Figure 2: Illustration of density functions

of diversity between individuals in order to be able to investigate what effects these differences have on individual decisions regarding social identification. In this thesis, I assume that individuals differ only in the level of the attribute v_i. Thus, the functional form of overall utility (1), utility from the attribute itself, $u(v_i)$, and utility from identifying with a group, $u_g(v_i)$, are the same for all individuals our economy. Assuming that individuals differ in their level of the attribute implies that there is a distribution of the levels of the attribute, hereinafter referred to as attribute values, across individuals. Furthermore, I assume at this point that, on the one hand, the attribute value is identically and independently distributed (i.i.d.) between the individuals and, on the other hand, the number of individuals in the economy, N, is large. This allows the law of large numbers to be applied. Accordingly, the probability that an individual has a certain attribute value v_i corresponds to the proportion of this attribute value in the population. A density function of the attribute value thus corresponds to the representation of the various attribute values in the population (Wälde, 2011, p. 172). Moreover, I will investigate two scenarios with different distributions of attribute values in the following analysis. In the first scenario, v_i follows an equal density. Thus, all values of v_i have the same density, or, in other words, each value of v_i has the same representation in society. The second scenario assumes a linear density of v_i where the likelihood of some value of v_i falls linearly the higher the value. Figure 2 visualizes the different scenarios.

3.1.3 Groups

The next important component of our economy, besides the individuals themselves, is the groups with which individuals identify. As already briefly mentioned in the explanation of the individual utility function, I assume that there is an arbitrary number of G groups in the economy. Since this model explicitly considers the endogeneity of social groups, a group can initially be understood as a specific set of group-utility parameters or identification parameters, respectively. More precisely, a social group is initially described only by an abstract offer of group-related utility parameters. These parameters refer to the fundamental decision parameters for social identity introduced in Section 2.1.1: group status and the effect of distance between the individual and the group on individual utility. A social group is therefore initially only a specific offer of the status of the group, as well as how strongly distance is valued in this specific group. These parameters are exogenous for individuals, and they make their identification decisions according to the patterns explained in the following section. Consequently, the offer of social groups is initially exogenous, but the existence of such groups is endogenous because it depends on whether individuals actually identify with a group with a specific set of group-utility parameters or not. This ensures that social groups in this model emerge from the identification behavior of individuals and are therefore endogenous. The next question that arises is around which attribute values the groups are formed, given the group-utility parameters mentioned above. In general, this can be anywhere on the distribution of v. This follows from the assumption about the functional form of individual utility from identification. Since a downward-opening parabolic utility function is assumed here, it is not only very high or very low attribute values that yield high utility from identification. Instead, what matters is the distance between an individual's attribute value and the prevailing attribute value within the group. The prevailing attribute value in a group can also be defined as the average attribute value within the group. For the following analyses I use the expression

$$\overline{v}^g = \int_{v_g^{min}}^{v_g^{max}} v f(v) dv, \tag{2}$$

as a general definition of the group mean. The mean attribute value in a group is therefore defined by the product of the individual attribute value v and its density in the population, which follows from the density function $f(v)$, integrated over the attribute values over which the group extends. Since the members of a group can have different attribute values, it follows that there is a minimum attribute value in a group, v_g^{min}, and a maximum attribute value v_g^{max}. Hence, these determine the limits of the integral. At this point, it is worth noting that, since individuals decide on their group membership, the mean value is theoretically endogenous to the individual. In this setup, however, I assume that the groups are large enough so that the individual influence on the group mean is marginal. This assumption allows us to ignore the influence of an individual on the group mean, which effectively makes the group mean an exogenous variable for the individual identification decision, simplifying the analysis considerably.

Where exactly the group is located on the distribution of v_i depends on the individuals and their distribution across the attribute values. This is described in more detail in the following sections 3.2 and 4 . However, the general idea here is that a group can only be located at a certain value of v if a sufficient number of individuals have an attribute value that is close to this location. This also follows from the assumption regarding utility from identification that individuals prefer similarity. In this context, the aim of the following analysis in this thesis is to theoretically determine the specific location of a group as measured by the attribute value prevailing within the group, or, in other words, the average attribute value in the group. It should be noted that I will not be looking at the process of formation of groups, but will only examine whether and around which attribute value groups exist and which individuals identify with such groups.

3.2 Decision problem of the individual

A central component of the analysis of the equilibrium of social identity is the decision problem of individuals. In this case, this refers to the choice of which groups the individual wants to identify with and which not. Given the general individual utility function (1) from the previous section, this decision refers to the choice of the value of the binary variable I_i^g for each group g which is consistent with the approach chosen by Grossman and Helpman (2021). More specifically, this means that the individual chooses $I_i^g = 1$ if he or she wants to identify with the group g and $I_i^g = 0$ if not. Furthermore, I_i^g is the only endogenous variable in this setup and thus also the control variable of the decision problem.

3.2.1 The maximization problem

Given the general individual utility function (1) and that individuals maximize their utility by identifying with certain groups, which is represented by the choice of I_i^g, the maximization problem can be defined as

$$\max_{I_i^g} u_i = u(v_i) + \sum_{g \in G} I_i^g * u_g(v_i). \tag{3}$$

3.2.2 Optimal behavior

With the maximization problem from above and knowing that the decision problem of individuals is choosing the right I_i^g for each group $g \in G$, we can now turn to the analysis of the optimal behavior of individuals. From the maximization problem we can deduce that the choice of the optimal I_i^g depends solely on the utility from identification that a specific group yields. This is because I_i^g is a binary variable. If the individual derives a positive utility from identifying with a group, i.e. $u_g(v_i) > 0$, then the only rational decision is to choose $I_i^g = 1$. The individual thus identifies with the group and the utility from identification is included in the general utility function. Since the former is positive, the overall utility of the individual increases. If, on the other hand, the utility from identification with a group is negative, i.e. $u_g(v_i) < 0$, the only rational decision for the individual in this case is to choose $I_i^g = 0$. Thus, the individual does not identify with the group and the corresponding utility from identification is not added to the general utility, as it would reduce it.

In order to better understand the cases in which an individual identifies with a group, I will now introduce specific functions for the two sources of utility, i.e. the utility from the attribute itself, $u(v_i)$, and the utility from identification, $u_g(v_i)$. Accordingly, the two are defined as

$$u(v_i) = v_i \tag{4}$$

$$u_g(v_i) = A^g - \beta^g(v_i - \overline{v}^g)^2 \tag{5}$$

which is closely related to the functional forms chosen in Grossman and Helpman (2021). Thus, the utility from the attribute itself, (4), is described by the value of the attribute that an individual has. Therefore, the higher the value of the attribute, the higher the utility from the attribute, and vice versa. The utility from identifying with a group, i.e. being a member of a group, (5),is defined by a constant parameter $A^g > 0$, which describes the general feeling of satisfaction that the individual derives from being a member of a certain group g, which relates to the concept of group status introduced in part 2.1.1 and the distance distance term. The latter consists of the squared difference between the attribute value of the individual, v_i, and the average attribute value in the group, $\overline{v}^g$, as well as the utility parameter $\beta^g \geq 0$, which describes how strong the negative effect of the distance between the attribute value of an individual and the average attribute value in the group is on individual utility in a particular group g. Moreover, subtracting the distance term from the satisfaction of identification indicates that distance reduces the utility from identification, and since the distance is squared, it does not matter in which direction the individual deviates from the group average, but only the scale of the distance is relevant. Thereby such a functional form of utility from identification relates to the downward opening parabola case described in section 3.1. From the above assumption that I_i^g is the only endogenous variable, it follows that the utility parameters A^g and β^g, as well as the attribute value v_i are exogenous. It is therefore assumed that the individual has no influence on these three parameters. Another assumption I make at this point is that there is no variation in the utility parameters A^g and β^g across individuals. This means that for all individuals, both the general feeling of satisfaction that results from group membership, A^g,

and the aversion to distance, described by β^g, are the same for a given group. The aim of all these assumptions is to simplify the following analyses.
From the individual maximization problem follows that the individual maximizes utility by joining all groups that yield some positive utility from identification. Thus, the individual chooses $I_i^g = 1$ for such groups and $I_i^g = 0$ for all other groups. Utility from identifying with a group is positive if the condition

$$A^g - \beta^g (v_i - \overline{v}^g)^2 > 0 \tag{6}$$

is true. Put differently, the individual joins all groups for which satisfaction of membership is greater than the disutility of distance between the individual and the mean of the group in terms of the attribute value. Solving this condition yields

$$\overline{v}^g - \sqrt{\frac{A^g}{\beta^g}} < v_i < \overline{v}^g + \sqrt{\frac{A^g}{\beta^g}}, \tag{7}$$

which represents the range of values of v_i around the group mean, for which the individual decides to join the group[2]. Having a range around the group mean as membership condition is straightforward because group membership not only yields positive utility but also disutility from difference between the individual and the group average. Since distance refers to deviations from the mean in both directions, the membership condition must incorporate the limits above and below the group mean, for which utility of identification is still positive. From the membership condition (7) follows that all individuals that have a value of v_i that is in the defined range decide to join the group whereas all other individuals will not join, since the disutility of difference outweighs the satisfaction of membership for values of v_i that deviate much from the group mean. The idea behind the membership condition (7) is visualized by Figure 3. Individuals with values of v_i that fall in the range around the mean value of the group $\overline{v}^g$ where the the satisfaction of membership outweighs the disutility from difference decide to join the group, and all other individuals decide not to join. The limits of this range are denoted by

$$v^{g,low} = \overline{v}^g - \sqrt{C^g}, \quad with \quad C^g = \frac{A^g}{\beta^g}, \tag{8}$$

and

$$v^{g,up} = \overline{v}^g + \sqrt{C^g}, \tag{9}$$

and mark the threshold values of v_i for which an individual decides to join the group given the relationship of group-utility parameters C^g, refering to the range described in the membership condition (7).

[2]See Appendix A.1 for a formal derivation of this result.

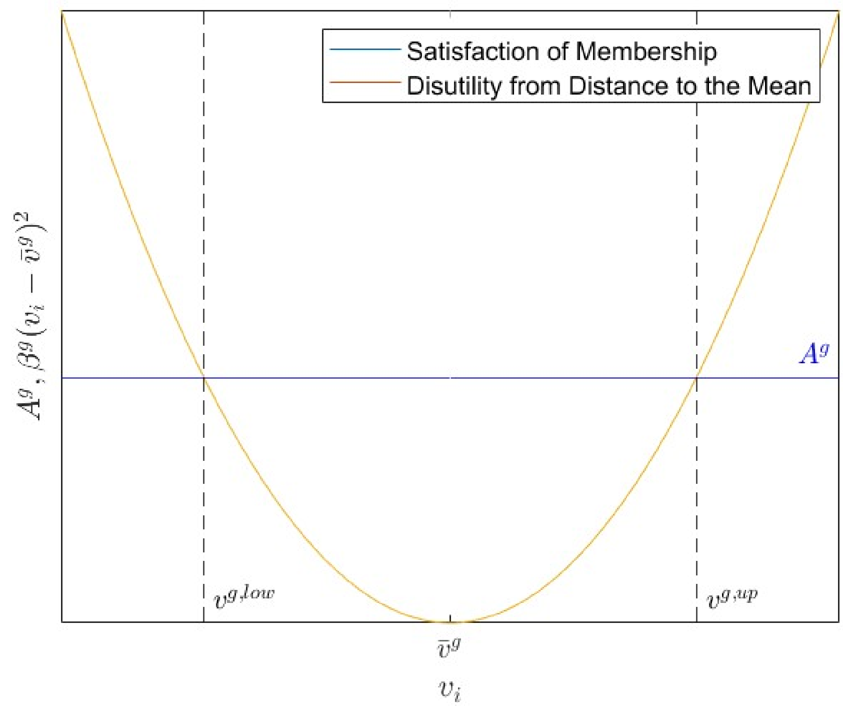

Figure 3: Group membership condition

3.3 Definition of equilibrium

Now that the optimal behavior of the individuals is identified, the question arises as to how the equilibrium of the membership decision is described. In general, the equilibrium is reached when all individuals join those groups that give them a positive utility from identification, do not join all other groups, and there is no more incentive to deviate from the decision made. As we have seen in the previous section, the optimal decision of identification can only be made if the mean attribute value in a particular group is known. We therefore need an expression of the group mean (2) in equilibrium. At the same time, the equilibrium group mean must take into account the decision pattern for group identification of the individuals explained in section 3.2.2. In this way, the groups are described, given that the individuals behave optimally, which leads to the endogeneity of social groups in equilibrium. From the optimal behavior described in section 3.2.2, we know that there are threshold values of the attribute value for which an individual barely enters a group. The lower limit is described by (8) and the upper limit by (9). Since it is assumed that individuals only differ in their attribute values v, these thresholds apply to all individuals. Given these threshold values, it can be concluded that the limits of a group in equilibrium can also be described by the threshold values from the optimal individual behavior. If the group were to extend over a larger range, then there would still be individuals in the group for whom the utility from identification with this group is not positive. This would mean that the equilibrium would not be reached, as it would be optimal for these individuals to leave the group. Conversely, if the group were to span a range that is smaller than the range defined by the threshold values of optimal individual behavior, then there would be individuals for whom identification with this group would increase their utility, but who are not yet members of this group. Here, too, the equilibrium would not be reached, as it would be optimal for these individuals to join the group.

We can therefore substitute the thresholds for group membership from the optimal behavior for the limits of the integral in (2) and obtain the general expression of the group mean in equilibrium

$$\overline{v}^g = \int_{\overline{v}^g - \sqrt{C^g}}^{\overline{v}^g + \sqrt{C^g}} v f(v) dv, \tag{10}$$

which depends on both the utility parameters and the population characteristics represented by the density function $f(v)$. Given the optimal behavior of the individuals from section 3.2.2 and the above group mean in equilibrium, it can be determined which individuals identify with the group and which do not. In this case, there would no longer be any incentives for deviations in the identification decisions and the equilibrium is described.

4 Does an equilibrium exist?

The next question that arises is whether, given the definition of equilibrium described above, such an equilibrium actually exists. This part of the thesis is devoted to the investigation of such an existence. It first describes how the existence of an equilibrium can be determined. This is followed by an analysis of the equilibrium using the density functions of the attribute value introduced in section 3.1, starting with the equal density and followed by the linear density. The idea behind this methodology is to stepwise approach the equilibrium characteristics by increasing the complexity of the density function. As we will see, this directly influences the complexity of the group mean $\overline{v}^g$, the existence of groups and thus our equilibrium.

4.1 General properties

The central question of the analysis of the equilibrium is whether there is a value for $\overline{v}^g$ that solves the equation (10). In other words, is there a value for $\overline{v}^g$ such that the integral on the right-hand side of the equation is equal to this value. If so, then there is a group at this attribute value in equilibrium, and individuals make their identification decision regarding this group based on their distance to this value, as described in section 3.2.2. However, if no value for $\overline{v}^g$ can be found that solves equation (10), then no group exists in equilibrium. In order to find a specific value for $\overline{v}^g$, that is to calculate the integral in (10), the specific density function of the attribute values is required. The following sections therefore apply the density functions introduced in section 3.1.

4.2 Equal distribution of the attribute

I will start the analysis of the equilibrium by using an equal probability density function (PDF).

4.2.1 Properties of the equal density function

With an equal density function, all values of v have the same density or, in other words, each value of v has the same representation in the population of individuals. Given the equal distribution, the probability density function (PDF) takes the form

$$f_{EQ}(v) = \overline{f},$$

with $\overline{f}$ denoting the density of some value of v. By definition $\overline{f}$ is the same for all values of v in the case of the equal PDF. Moreover, it is possible to further specify the equal density by taking into account that the area under the density function must always equal one. This requirement ensures that all individual densities together result in one. Using this condition and given that $v \in [0, v_{max}]$, with v_{max} denoting the maximum value of the attribute in the population, it follows

$$\overline{f} = \frac{1}{v_{max}}, \tag{11}$$

which relates the density of each v to the maximum value of the attribute in the population[3].

4.2.2 Equilibrium properties with equal density

Inserting the specific form of the density function in our definition of the group mean and also using (8) and (9) as the limits of the integral yields the equilibrium expression for the group mean given the equal density of attribute values

$$\overline{v}^g = \int_{\overline{v}^g - \sqrt{C^g}}^{\overline{v}^g + \sqrt{C^g}} v\overline{f}dv. \tag{12}$$

[3]See Appendix A.2 for a formal derivation of the specific parameter expression in the case of an equal density.

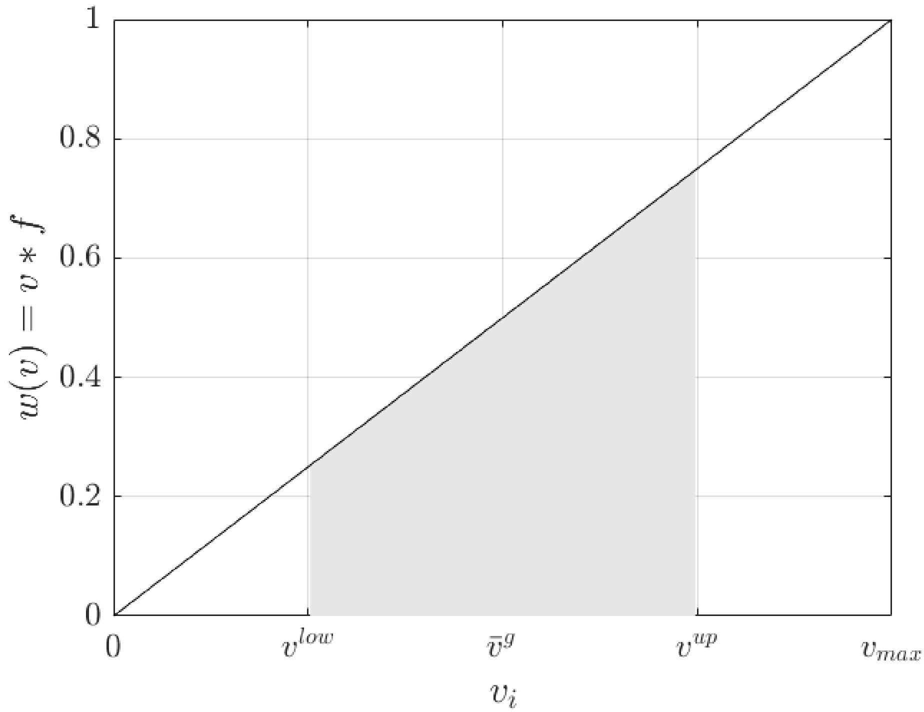

Figure 4: Equilibrium concept for an equal density

Thus, the equilibrium group mean is the area underneath the weighing function $w(v) = v\overline{f}$ between the threshold values of the individual group membership decision. Figure (4) visualizes the weighing function of an equal PDF. Furthermore, the gray area represents the theoretical concept of the equilibrium group mean described in section 4.1, which is the integral underneath the weighing function from the lowest value of the attribute in the group, $v^{g,low}$, to the highest value $v^{g,up}$. It is very important to note that we only have a valid result, if this area actually equals the respective $\overline{v}^g$ on the abscissa. Thus, finding a solution for the equilibrium group average implies finding those values of $\overline{v}^g$, where both the area and the value on the abscissa equalize given the preference parameters A^g and β^g from the optimal behavior of individuals.

4.2.3 Analytical solution

Having described the theoretical idea of how to find the average value of the attribute in a group in equilibrium given an equal PDF, I will now turn to the analytical calculation of such value. More specifically, the equilibrium value of $\overline{v}^g$ can be found by solving (12) for $\overline{v}^g$. Performing the calculation yields[4]

$$0 = (2\frac{1}{v_{max}}\sqrt{C} - 1)\overline{v}^g, \qquad with \quad C^g = \frac{A^g}{\beta^g}, \tag{13}$$

[4]See Appendix A.3 for a formal calculation of the equilibrium solution with equal density.

which suggests two cases for our solution which can be denoted by

$$\overline{v}^g = \begin{cases} 0, & \text{if } \quad 2\frac{1}{v_{max}}\sqrt{C^g} \neq 1 \\ [0; v_{max}], & \text{if } \quad 2\frac{1}{v_{max}}\sqrt{C^g} = 1. \end{cases} \tag{14}$$

For the first case, we can divide (13) by the term in the brackets and receive zero as solution. For the second case, the term in the brackets equals zero and every value of $\overline{v}^g$ solves the equation. Moreover, the second case is only true if

$$2\frac{1}{v_{max}}\sqrt{C^g} = 1 \Leftrightarrow \sqrt{C^g} = \frac{v_{max}}{2}$$

is fulfilled. Thus, the second case of the solution applies only for a very specific relationship between the utility parameters which are represented by C^g and the maximum value of the attribute v_{max}.

4.2.4 Numerical solution

Besides solving the equilibrium group mean analytically, we can also investigate the property of the equilibrium by a numerical approach. In that case we set certain values for the constant parameters and evaluate whether there exists a group with some mean value of the attribute at each v_i. Formally, the numerical solution calculates the difference

$$g(\overline{v}^g) = \overline{v}^g - \int_{v^{g,low}}^{v^{g,up}} v f(v) dv \tag{15}$$

at each value of $\overline{v}^g \in [0; v_{max}]$. Thus, the function $g(\overline{v}^g)$ calculates the difference between some value of $\overline{v}^g$ and the respective group mean that would emerge given that individuals act optimal, which is represented by the integral shown in (2). Then the next step is to determine for which values of $\overline{v}^g$ the integral equals the value itself. This is the case for all values for which $g(\overline{v}^g)$ is zero.

In order to numerically find the equilibrium values of $\overline{v}^g$ for an environment where the distribution of the attribute v follows an equal PDF, we have to insert our equal density (11) into the integral. Next, we define values for the constant parameters, which in our case is the relationship of utility parameters[5], $\sqrt{C}^g$, and the maximum value of the attribute, v_{max}. Let us start by analyzing the solution where $\sqrt{C^g} \neq \frac{v_{max}}{2}$. Figure (5) visualizes the function (15) as well as the value of the integral (2) for each $\overline{v}^g$, given that the attribute follows an equal density. Moreover, I define the

[5]Of course we could also define values for the satisfaction from group membership, A^g, and the disutility of difference, β^g, separately. However, to save time and reduce the code, I have abbreviated it here and defined $\sqrt{C^g}$ directly.

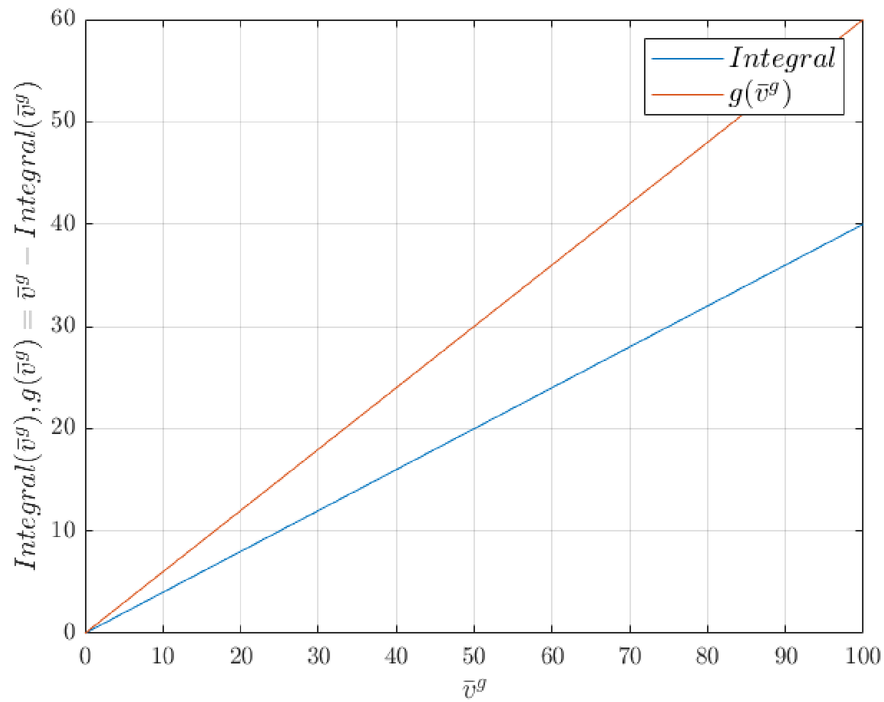

Figure 5: Numerical solution for the equilibrium group average with $\sqrt{C^g} = 20$

maximum value of the attribute to be one hundred and the square root of relative utility parameters, $\sqrt{C}^g$, to be twenty. This is of course arbitrary, but is in line with the above condition, so we are still in the first case solution of (14).

We can observe that for all values of $\overline{v}^g$ except $\overline{v}^g = 0$ the integral is smaller than the respective value of $\overline{v}^g$, resulting in (15) to be positive for all positive values of $\overline{v}^g$. However, I find exactly the same solution of the equilibrium group average as in the analytical approach, namely at $\overline{v}^g = 0$.
Let us now turn to the second case of the solution for the equilibrium group average with equal density of the attribute. This is a special case and, as already described, only applies for a certain relationship between utility parameters and the maximum value of the attribute. More precisely, we are in the second solution case if $\sqrt{C^g} = \frac{v_{max}}{2}$. If this equality is fulfilled, then the analytical solution suggests that each value of $\overline{v}^g$ is an equilibrium group mean of the attribute. If we now apply the numerical approach explained above by defining $\sqrt{C}^g$ accordingly and calculating (15) again, we see that for each value of $\overline{v}^g$ the integral corresponds to the respective value and therefore (15) is always zero (see Figure 6). This suggests that in the special case every value of $\overline{v}^g$ is an equilibrium solution for the group average (2). As in the first case, this is consistent with the analytical solution.

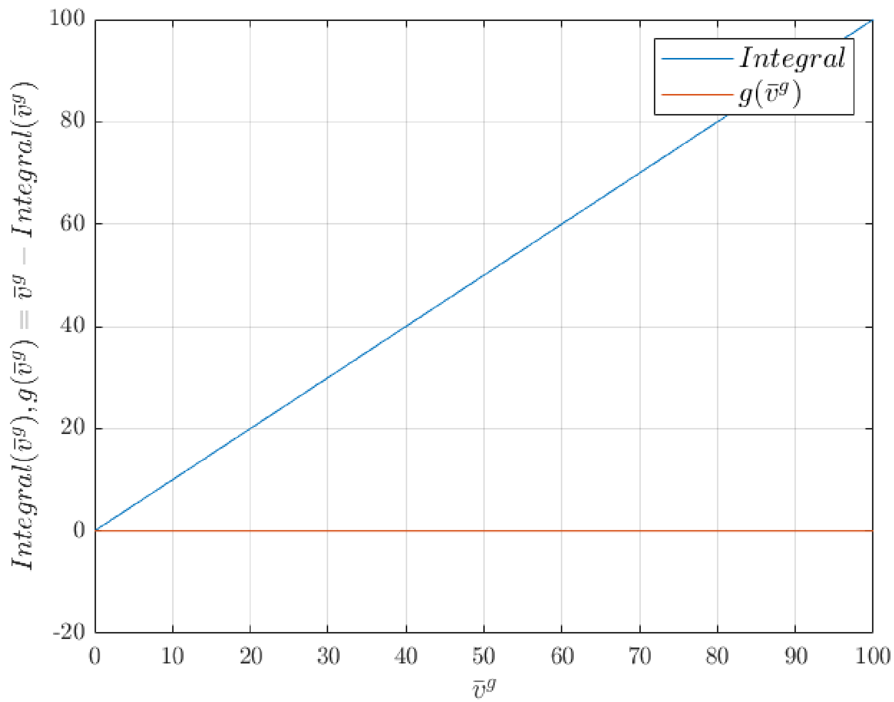

Figure 6: Numerical solution for the equilibrium group average with the special case that $\sqrt{C} = \frac{v_{max}}{2}$

4.2.5 Technical interpretation of results

Now that the results of the mean value of the attribute in a group in equilibrium have been calculated both analytically and numerically, and both approaches come to the same result, we can now turn to an initial interpretation. I will start this from a purely technical perspective for the time being. The aim is to discuss the results obtained for the case of an equal density of attribute values in the population against the background of the specified framework of the model and to evaluate their validity.

Starting with the first case of the solution in equilibrium, where $\overline{v}^g = 0$, we can see that such a solution is impossible for the group mean of the attribute value. The reason for this lies in the definition of the group mean in equilibrium and in the range of possible values of the attribute. By definition, the group average of the attribute in equilibrium is the mean of the attribute values of the individuals in the group, given that all individuals behave optimally. As explained in section 3.2, the optimal behavior of individuals leads to the threshold values of the attribute for group membership. In this context, (8) defines the lower limit of the attribute at which individuals just decide to be part of the group, and (9) the upper limit. It follows that the group extends over a range of attribute values, whereby the concrete definition of the threshold values (8) and (9) implies that the group average in equilibrium lies exactly in the middle of the threshold values.

Having a group average of zero then implies that the group is made up of individuals with attribute values ranging from $-\sqrt{C^g}$ at the lower limit to $\sqrt{C^g}$ at the upper limit. However, such a range cannot be possible by construction, as v cannot take on negative values but is only defined for values greater than or equal to zero. Therefore, a group with a corresponding range would only contain individuals with attribute values between zero and $\sqrt{C^g}$. In this case, the group average in equilibrium according to the definition (2) would not be zero, but instead would be a positive number. Since this is not consistent with the analytical and numerical result, it can be concluded that a group average of zero is not possible due to the model construction and the restriction of the attribute values to exclusively positive values. This also indicates that there exists no group for an equal density of attribute values in equilibrium, provided we are not in the special case.

Let us now look at the second solution case for the same density. This is a special case in which the utility parameters and the maximum attribute value are in a very specific relationship. More precisely, the special solution case only applies if $\sqrt{C^g} = \frac{v_{max}}{2}$ is fulfilled. If this is true, then both the analytical and the numerical approach state that all available attribute values $\overline{v}^g \in [0, v_{max}]$ are a solution for the group mean in equilibrium. Therefore, a group exists at every value of v. This is reasonable because, if we look at equation (13), we can use any possible value for $\overline{v}^g$, and since the term in the parenthesis is zero in the special case, we multiply by zero and the equation is therefore satisfied. However, we should bear in mind that the attribute values are limited to the range between zero and v_{max}. In this case, the result that there exists a group for each value of v cannot be correct. The reasoning here is the same as in the first solution case. Since a group always extends beyond a range around the average attribute value of the group, and this range is defined by $\sqrt{C^g}$, no groups can exist at the margins of the available attribute values. If a group is close to zero, or close to v_{max}, it would exceed the existing attribute values in the population. More precisely, this is the case for all groups whose mean attribute value is less than $\sqrt{C^g}$ away from zero or v_{max}. It follows that for the special case there is a minimum and a maximum value of the group mean in equilibrium and not every attribute value is a group mean. In the following, the minimum value of the group mean in equilibrium is denoted by $\overline{v}^g_{min}$ and the maximum value by $\overline{v}^g_{max}$. The solution of the special case is therefore that in the range $\overline{v}^g \in [\overline{v}^g_{min}, \overline{v}^g_{max}]$ each attribute value is a group mean in equilibrium, meaning that at each of these values there is a group in equilibrium. Figure 7 illustrates this result, showing both the equal density and the various groups marked by their utility of identification $u_g(v_i, \overline{v}^g)$. For the boundary solutions, the groups, measured by the range of attribute values over which they extend, end at the limit values of the attribute distribution. Thus, the attribute value of zero is the first attribute value at which individuals identify with the group around $\overline{v}^g_{min}$. On the other hand, the attribute value v_{max} is the last attribute value at which individuals still identify with the group around $\overline{v}^g_{max}$. Given that the distances of the attribute values

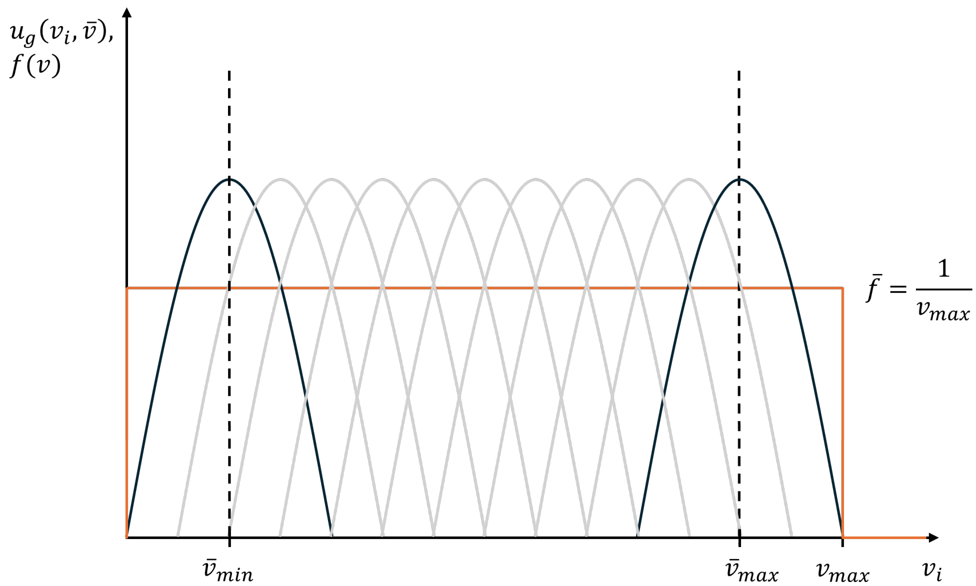

Figure 7: Visualization of the average attribute level in a group in equilibrium for the special case in an equal density environment

from the group mean, which define at which attribute values individuals still identify with a group, are defined by $\sqrt{C^g}$, it can be concluded that the lowest group mean in equilibrium is defined by

$$\overline{v}^g_{min} = \sqrt{C^g} \tag{16}$$

and the highest group mean is defined by

$$\overline{v}^g_{max} = v_{max} - \sqrt{C^g}. \tag{17}$$

Groups with attribute values outside this range would exceed zero or v_{max} and thus include non-existent attribute values. Such groups can therefore not be a solution in equilibrium.

4.3 Linear distribution of the attribute

In this part, I will now analyze the equilibrium characteristics with a linear density function. This is a step towards a relatively more realistic density function in terms of the representation of attributes in a society and compared to the equal density function, without increasing the complexity too drastically.

4.3.1 Properties of the density function

Given the linear distribution of the attribute, the density of some attribute value v_i decreases, the higher the value of the attribute. Thus, low levels of the attribute have a higher representation in society than high values of the attribute. Given these characteristics, the linear density function takes the form

$$f_L(v) = z - gv,$$

with z being the intercept and g the slope parameter of the function. Knowing that there exists some maximum value of the attribute, v_{max}, where the density reaches zero and that the area below the density function must equal one by definition, we have the two equations,

$$f_{LIN}(v_{max}) = 0$$

and

$$\int_0^{v_{max}} (z - gv)dv = 1,$$

with which we can derive specific values for the intercept and the slope, depending on v_{max}. Specifically, we can rewrite z and g as

$$z = \frac{2}{v_{max}}, \quad and \quad g = \frac{2}{v_{max}^2}$$

which allows us to adjust the density function such that it also includes the maximum value of the attribute[6]. The adjusted linear density function takes the form

$$f_{LIN}(v, v_{max}) = \frac{2}{v_{max}} - \frac{2}{v_{max}^2} * v$$

4.3.2 Equilibrium properties with linear density

The equilibrium properties with a linear density function follow the same structure as described in Section 4.2.2. Again, the equilibrium group average of the attribute level is described by individual preferences and the density function and therefore takes the form

$$\overline{v}^g = \int_{\overline{v}^g - \sqrt{C^g}}^{\overline{v}^g + \sqrt{C^g}} (\frac{2}{v_{max}} - \frac{2}{v_{max}^2} v) v dv, \tag{18}$$

now applying the linear density function. As in the case of an equal density function, the group mean in equilibrium is described by the area under the weighting function $w_{LIN}(v) = f_{LIN}(v)v$,

[6] See Appendix A.4 for a formal derivation of the specific parameter expression in the case of a linear density.

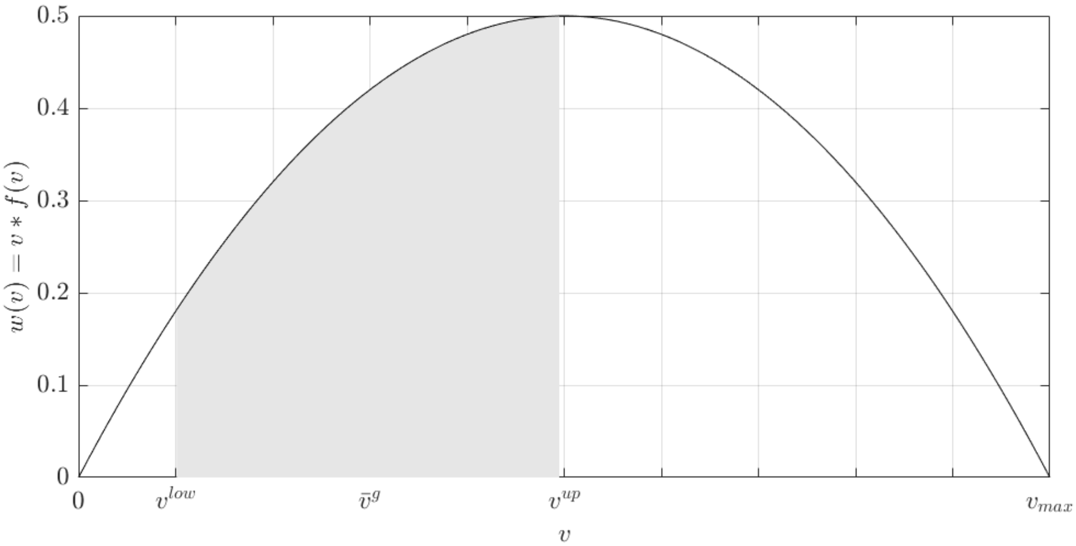

Figure 8: Equilibrium concept for a linear density

which is constrained by the threshold values for group membership derived from the optimal behavior, which also represent the limit values of the attribute in the group. Figure (8) visualizes the weighing function and the respective area. Again, there only exists a valid solution for the group average in equilibrium, if the area under the weighing function is the same as the specific $\overline{v}^g$ on the abscissa. Finding the values of $\overline{v}^g$ that satisfy the above equation is therefore the key objective when analyzing the equilibrium with linear density.

4.3.3 Analytical solution

Deriving the analytical solution for the linear density equilibrium follows the same idea as in Section 4.2.3. Specifically, we solve the equilibrium expression of the average attribute value in the group, equation (18), for $\overline{v}^g$ and analyze whether it is possible to find such equilibrium value or not[7]. Before we get to the final solution, it is worth pointing out an intermediate solution of (18), which is already informative. More precisely, we get the equation

$$(4\sqrt{C^g}v_{max} - v_{max}^2)\overline{v}^g = 4\sqrt{C^g}(\overline{v}^g)^2 + \frac{4}{3}(\sqrt{C^g})^3 \tag{19}$$

as an intermediate result. Here we can observe that we have a linear function in $\overline{v}^g$ on the left-hand side (LHS) and a quadratic function in $\overline{v}^g$ on the right-hand side (RHS). We therefore obtain a value of the equilibrium attribute value of a group if such value results in these two functions either

[7]See Appendix A.5 for the formal derivation of the equilibrium solution of the equilibrium group mean with linear density

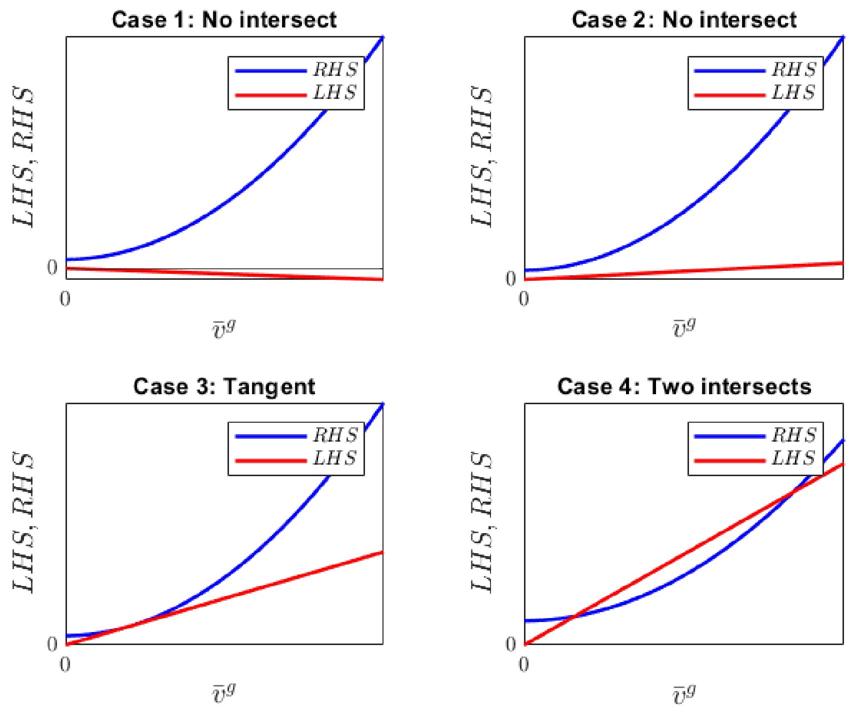

Figure 9: Qualitative illustration of possible solution cases of the equilibrium group average with linear density

touching or intersecting. Figure (9) illustrates the qualitative idea behind this approach[8,9]. It can be seen that, depending on the functional form of the two functions, we can obtain different solution cases. If the slope of the linear function on the left-hand side of equation (19) is negative, we will not get a solution. The reason for this is that this function starts at the origin and falls, while the quadratic function starts at a positive value and rises quadratically. Similarly, if the slope of the linear function is not sufficiently high, we will also not get a solution. However, if the slope of the linear function is high enough, then it is tangent to or intersects the quadratic function. In the first case there is a solution at the tangent point, in the second case there are two solutions, both at the intersection points.

[8]It is important to note that the specific functions of the intermediate result are not yet applied here. Rather, it is a purely qualitative representation of the general idea using general linear and quadratic functions. A particular application of the specific functions is given in Section 4.3.4, the numerical analysis of the equilibrium. This presentation is intended solely as a support for understanding the solution concept of the mean attribute value of a group in equilibrium with linear density. Whether the illustrated cases actually exist in the specific framework is not yet clear at this point.

[9]This graph was built with the assistance of ChatGPT (2025)

If we now solve equation (18) completely for $\overline{v}^g$, we get

$$\overline{v}^*_{1,2} = \frac{4\sqrt{C^g} - v_{max} \pm \sqrt{(v_{max} - 4\sqrt{C^g})^2 - \frac{64}{3}\frac{(C^g)^2}{v_{max}^2}}}{8\frac{\sqrt{C^g}}{v_{max}}}, \tag{20}$$

which is our closed-form solution for the group mean value of the attribute in equilibrium. As we can see, this solution also implies different scenarios for the equilibrium with linear density, depending on the parameters C^g and v_{max}.

First, if either the numerator or the denominator is negative, we do not have a result as we assume the attribute value to be non-negative. However, since we assume $C^g > 0$ and $v_{max} > 0$, the denominator is always positive. Obtaining a positive value of $\overline{v}^g$ therefore depends solely on the numerator being positive. We do also not get a result if the discriminant of the square root in the numerator is negative as this would yield complex numbers. Second, we get one solution if, in the case of subtracting the square root in the denominator, a negative value of $\overline{v}^g$ results, but adding the root results in a positive value. And finally we get two solutions if both the subtraction and the addition of the square root lead to a positive result. We can therefore note that the closed solution is consistent with the solution concept explained above using equation (18).

However, we still do not know which case actually applies. The general problem becomes apparent when we consider the conditions for the existence of a valid solution: first, the condition that the discriminant must be non-negative, and second, that the numerator must be positive overall. It can be observed that the influence of the parameter values C^g and v_{max} on these two conditions is opposing.

From the partial derivatives of the discriminant

$$D = (v_{max} - 4\sqrt{C^g})^2 - \frac{64}{3}\frac{(C^g)^2}{v_{max}^2}$$

with respect to the above parameters,

$$\frac{\partial D}{\partial v_{max}} \gtrless 0 \Leftrightarrow \frac{1}{4}\frac{v_{max}}{\sqrt{C^g}} + \frac{8}{3}(\sqrt{C^g})^3\frac{1}{v_{max}^3} \gtrless 1 \tag{21}$$

and

$$\frac{\partial D}{\partial C^g} \gtrless 0 \Leftrightarrow -\frac{4(v_{max} - 4\sqrt{C^g})}{\sqrt{C^g}} - \frac{128}{3}\frac{C^g}{v_{max}^2} \gtrless 0, \tag{22}$$

it can be determined that a higher v_{max} most likely has a positive effect on the discriminant, thereby increasing the probability of a non-negative discriminant[10]. In contrast, a higher C^g most likely has a negative effect on the discriminant, reducing the likelihood of a non-negative value for D. On the

[10]See Appendix A.6 for the derivation of the partial derivatives of the discriminant

other hand, based on the expression (20), it is easy to see that v_{max} has a negative effect on the first part of the numerator preceding the square root, while C^g has a positive effect.
Thus, a high value of v_{max} increases the probability of fulfilling the first condition for a valid solution, while decreasing the probability of fulfilling the second condition. This indicates an opposing influence of v_{max} on the existence of a solution. A similar observation applies to the effect of C^g. While a high C^g increases the numerator and thus the probability of fulfilling the second condition, it simultaneously leads to a lower value of D, making the non-fulfillment of the first condition more likely.
For these reasons, deriving the parameter combinations for which one or two solutions exist is highly complex. Unfortunately, corresponding results could not be obtained in the course of this thesis. This should be addressed in future research.

4.3.4 Numerical solution

Since the exact parameter combinations for which we can theoretically obtain a solution have not yet been found analytically, numerical solving is only possible through an iterative approximation. In this process, the parameter values are gradually adjusted, and it is observed whether a solution is approached or not. Due to this method, it makes sense to use a different approach than the one in Section 4.2.4. Specifically, we use the functions described in Section 4.3.3 from Equation (19) and check for which parameter values they converge. We define the left-hand side of Equation (19) as

$$h(\overline{v}^g) = (4\sqrt{C^g}v_{max} - v_{max}^2)\overline{v}^g \tag{23}$$

and the right-hand side as

$$k(\overline{v}^g) = 4\sqrt{C^g}(\overline{v}^g)^2 + \frac{4}{3}(\sqrt{C^g})^3 \tag{24}$$

A numerical solution in this case refers to finding the parameter combinations for which the two functions touch or intersect. We can now plot these two functions and obtain a visualization in Figure 10, which resembles the graphs from Figure 9. The first subplot shows a representation with two arbitrarily chosen parameter values for C^g and v_{max}. As can be seen, there is no point of tangency or intersection between the two functions in this case. One way to approach such a point is by adjusting the parameter values so that the slope of function $h(\overline{v}^g)$ is maximized. The slope-maximizing v_{max} is given by $v_{max} = 2\sqrt{C^g}$[11]. The second subplot presents the functions given the value of v_{max} that maximizes the slope of $h(\overline{v}^g)$, while keeping C^g constant. Although the two functions approach each other, no tangency or intersection is found here either. Another way to bring the functions closer can be to choose a lower value for C^g, thereby reducing the

[11] See Appendix A.7 for the calculation of the slope maximizing v_{max}.

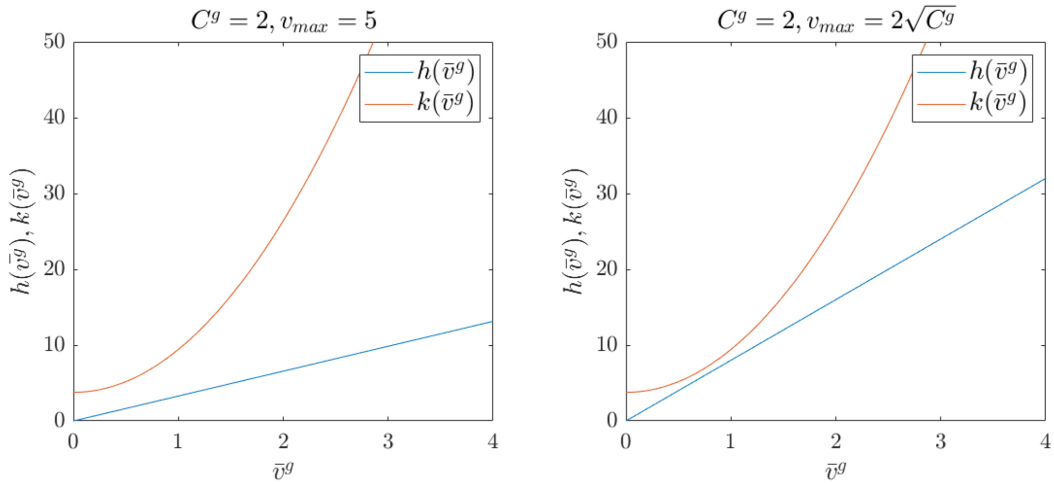

Figure 10: Numerical approach to the equilibrium group mean of the attribute with linear density

intercept of $k(\overline{v}^g)$. However, since this also negatively affects the slope of $h(\overline{v}^g)$, the influence of C^g on the probability of a solution is counteracting. Even after extensive testing, it has not been possible to numerically determine parameter values for C^g and v_{max} that yield one or two solutions. The fact that it has not yet been possible to make any analytical statements about the corresponding parameter values considerably complicates the numerical analysis. Overall, based on the numerical analysis, it remains unclear whether, in equilibrium with linear density, one or multiple groups exist.

4.3.5 Technical interpretation of results

As we have seen in the course of the analysis of the existence of groups in equilibrium with linear density, such existence primarily depends on two conditions. First, due to the constraint on attribute values to be within $v \in [0, v_{max}]$, a non-negative value must result. As we have seen, this is the case when the numerator in expression (20) is positive. Second, it must be ensured that the discriminant in (20) is non-negative; otherwise, we would not obtain real numbers. Additionally, the question arises whether, given that both conditions are met, one or two solutions exist. This depends on whether a solution exists both for the subtraction of the square root in the denominator of (20) and for the addition, or if this is only the case for one of the two. If two solutions can be obtained, this would imply that, for a specific set of group parameters A^g and β^g, which are summarized in C^g, two groups with different group averages of the attribute exist. However, whether and how many groups exist in equilibrium with linear density ultimately depends on the specific combinations of the parameter values C^g and v_{max} for which the conditions mentioned above are met. Since it has not yet been possible to determine these specifications, this remains, for now, the final conclusion regarding the existence of social groups in equilibrium with linear density.

5 Discussion

The goal of the model developed in this thesis was to create a framework for analyzing an equilibrium of social identity with endogenous groups. In such an equilibrium, the existence of groups should result from individuals' preferences and their resulting optimal behavior. Ultimately, such a model framework can then explain how individuals form groups based on their social preferences, which groups emerge, and which individuals identify with which groups. In addition to individual social preferences, the society itself plays a crucial role, as it is described in this model by the distribution of an attribute. As explained in Section 3.3, the equilibrium is then defined by the fact that all individuals identify with the groups that maximize their utility and that no group member has an incentive to leave their group.

Given the assumption that groups themselves are endogenous, meaning they result from the optimal behavior of individuals, the central question of the societal equilibrium of social identity is how many and which groups exist in equilibrium. To answer this, this thesis examines two scenarios that differ in their assumptions about the distribution of the attribute in society. Specifically, the first scenario assumes an equal density of all attribute values, while the second scenario assumes a linearly decreasing density.

As analyzed in Section 4.2, for a specific set of group-utility parameters, A^g and β^g, representing the general group-specific satisfaction from group membership and the utility parameter for distance, respectively, in the case of equal density, either no group exists in equilibrium or a group forms at every attribute value that is sufficiently far away from the minimum and maximum values. Which one of these two solutions applies depends on the combination of utility parameters and the maximum attribute value in the population. The normal case here is that no group exists in equilibrium given the equal density. The reason for this is that either too few or too many individuals would join the group, causing the group mean to shift in a way that no longer aligns with individuals' preferences. Only for a very specific combination of the above-mentioned parameter values does the special case apply, in which a group forms around all attribute values that are sufficiently far from the margins.

Figure 11 illustrates the interpretation of this solution. For a low density, which results from a high maximum attribute value in the population, too few individuals have an attribute value close to any hypothetical group mean. According to the preferences explained in Section3.2, too few individuals would join the group in this case. As a result, the group mean is always too low, regardless of where it is located on the distribution. This can also be observed in the right graph of Figure 11, which shows the concept of the equilibrium solution of the group mean for equal density for three different density levels, given a hypothetical group mean and constant identification parameters. In

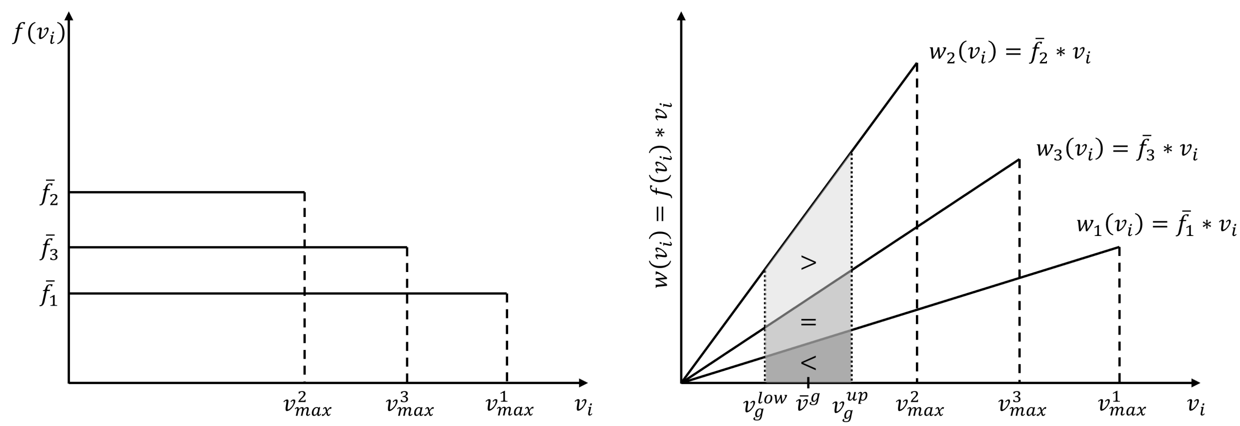

Figure 11: Visualization of the solution cases with equal density

the case of low density, the weighting function is relatively flat, and the area under the function is always smaller than the mean attribute value between the boundaries of the area. Therefore, no group forms at any point, as the condition for the existence of a group in equilibrium, as explained in Section 4.2.2, is not met.

The same applies when the maximum attribute value is very low, meaning that density is very high. Here, many individuals are located close together in terms of their attribute values, leading to too many individuals joining a hypothetical group. In this case, the resulting group mean would always be too high. This can also be observed in the right graph of Figure 11. If density is too high, then given the identification preferences, the area under the weighting function is too large at every point. Only for a very specific ratio of the maximum attribute value (which defines density) to the utility parameters is the condition described in Section 4.2.2 met. In this case, as described in Section 4.2.5, a group forms at every attribute value that is sufficiently far from the distribution's boundaries. However, since this only applies to a very specific ratio of utility and population parameters, this is considered a special case.

In the second scenario, under the assumption of a linearly decreasing density, either none, one, or two groups exist for a given set of group-utility parameters. Here, also, the combination of utility and population parameters, C^g and v_{max}, determines which of these three possible solutions applies.

At this point, the question arises as to how it should be understood that more than one group can exist for a given set of group-utility parameters. Essentially, this case considers how many groups form for the same combination of satisfaction from identification and disutility from distance. If more than one group exists in equilibrium, this means that individuals with significantly different attribute values are willing to join such a group. However, since individuals dislike distance and therefore do not want to be in a group with others who have highly varying attribute values, two or more groups form. To illustrate this, it helps to consider an example where the attribute represents income. In this case, it might be that, for the same satisfaction of identification and the same disutility from distance, both low-income individuals and high-income individuals could be willing to identify with such a group. However, as distance is avoided, not all of these individuals can join the group, as the income in the group would then vary too much. For this reason, multiple groups form, each consisting of individuals with similar income levels. The parameters of satisfaction from identification and distance of these groups, however, are identical.

This equilibrium analysis, at this point, does not yet describe the societal equilibrium, as it is initially limited to a specific set of group-utility parameters. To reach such equilibrium of social identity, one could incorporate distributions of possible values for the parameters of satisfaction from identification and distance and subsequently analyze for which values one or more groups exist. Then, there would be a distribution of possible groups, described by different combinations of group-utility parameters, allowing an analysis of which of these combinations lead to the formation of social groups according to the patterns described in Section 4. Therefore, this would describe the societal equilibrium of social identities with endogenous social groups.

Another general conclusion that can be drawn from the model presented here follows from the conditions for the existence of groups. In both the case of equal density and the scenario with linear density, the existence of social groups depends on the ratio between the relation of group-utility parameters, C^g, and the maximum value of the attribute in the population, v_{max}. For a group to exist, a specific ratio of utility and population parameters must be present. In other words, for given utility parameters, one or more groups only exist if the highest attribute value in the population takes on a certain value. This can be traced back to the preference for similarity among individuals, as introduced in Section 2.1.1. If, given the utility parameters, the maximum value of the attribute in society is too low, then no group forms, as society is so similar in terms of the attribute that it does not split into social groups but instead remains a single group[12]. Conversely, if the maximum attribute value in society is too high, then individuals are too different overall to be willing to form groups together.

Overall, it can therefore be concluded that the combination of utility parameters and the maximum attribute value determines the existence and number of social groups in equilibrium. This

[12]This is a purely logical conclusion. The results from section 4 suggest that there exists no group if the maximum attribute value is too low. Analysis of boundary solutions for $\beta \to 0$ should be part of future research. However, if we have preferences here, for example, where individuals accept a deviation of ± 10 for group membership, but the maximum attribute value in the population is only 2, then the group exceeds all individuals and there would be one large group. The fact that we do not get a solution in the analysis section here is most likely due to the technical requirements for the mathematical derivation of a solution.

aligns with another central conclusion of the model presented here. The number and existence of social groups fundamentally describe how interconnected individuals in a society are. If there are many different social groups in society, more individuals share the same social identity or feel closer to the other members of society than if there were no or only few social groups. This can also be understood as social cohesion. Thus, the results presented in this model are not only informative about the existence and number of social groups but also describe the determinants of social cohesion. If the utility parameters and the maximum attribute value approach the ratio that increases the number of social groups, social cohesion also increases, and vice versa.

6 Conclusion

As described in the review of economic literature on social identity conducted in this thesis, the body of research on the analysis of a social identity equilibrium remains highly limited. Studies that approach a macroeconomic analysis of such an equilibrium so far assume a given number of groups with which individuals can identify. An analysis of a social identity equilibrium with endogenous groups and heterogeneous individuals at the macro level appears to not yet exist in the economic literature. This means that existing macroeconomic models do not consider the formation of groups as an outcome of individual identity decisions. The aim of this thesis was to describe such an equilibrium. To achieve this, a model framework was developed that allows for the endogeneity of social groups and in which individuals differ according to an arbitrary attribute. Building on the framework introduced by Shayo (2009, 2020) and further developed by Grossman and Helpman (2021), the model developed in this thesis also assumes that individuals choose their social identity based on the two key determinants from social psychology: group status and distance. The resulting social identity equilibrium was then analyzed under the assumption of two different distributions of the attribute: an equal density and a linearly falling density. Moreover, in this thesis, the equilibrium is defined such that all individuals identify with the groups that maximize their utility and have no incentive to leave them. This part of the definition follows the concept of social identity equilibrium introduced by Shayo (2009). However, since this thesis assumes the endogeneity of social groups, the definition of equilibrium is extended by a second condition, namely that the groups in equilibrium must also be characterized by the optimal behavior of individuals. Therefore, the key question remains whether, and if so, how many social groups exist in such an equilibrium.

The analysis carried out in this thesis leads to the conclusion that, under both equal density and linearly falling density of the attribute, the number of social groups in equilibrium depends on the relationship between the group-utility parameters, i.e., the general satisfaction of membership and the distance to the group, and the maximum attribute value in the population. This means that social groups in equilibrium only exist if these parameters are within a specific ratio. Furthermore, the solution for the case of a linear density suggests that the number of social groups in equilibrium increases the closer the parameters align to a certain optimal ratio between group-utility parameters and the maximum attribute value in the population. Since the number of social groups also serves as an indicator of social connectivity within a society, the findings of this thesis also provide insights into the determinants of social cohesion.

While these results offer important insights into the analysis of a social identity equilibrium with endogenous groups and heterogeneous individuals, several open questions remain. A potential avenue for future research in this field could be the precise derivation of the parameter ratio for which, under a linear density, there exist either no groups, one group, or two groups for a given set of group-utility parameters. Another open question concerns the characterization of boundary solutions in cases where the utility parameter for distance approaches zero, that is, when the distance between an individual and a group no longer plays a role in the choice of social identity. Additionally, future research could explore the analysis of a social identity equilibrium under non-linear distributions of individuals.

Although much work remains to be done to fully understand the societal equilibrium of social identity, the model presented here appears to provide a solid foundation for analyzing the equilibrium of social identity with endogenous groups and, consequently, for understanding social cohesion.

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A Appendix

A.1 Derivation of the individual group membership condition

Utility from identification (5) must be positive

$$
\begin{aligned}
u_g(v_i) = A^g - \beta^g (v_i - \overline{v}^g)^2 > 0 \\
\Leftrightarrow A^g > \beta^g (v_i - \overline{v}^g)^2 \\
\Leftrightarrow \frac{A^g}{\beta^g} > (v_i - \overline{v}^g)^2 \\
\Leftrightarrow \overline{v}^g - \sqrt{\frac{A^g}{\beta^g}} < v_i < \overline{v}^g + \sqrt{\frac{A^g}{\beta^g}}
\end{aligned}
$$

A.2 Derivation of the specific form of the equal density function

For the equal density function, the area below the function and from the lowest to the highest value in the population must equal one

$$
\begin{aligned}
\int_0^{v_{max}} f_{EQ}(v) dv = 1 \\
\Leftrightarrow \int_0^{v_{max}} \overline{f} dv = [\overline{f} v]_0^{v_{max}} = \overline{f} v_{max} = 1 \\
\Leftrightarrow \overline{f} = \frac{1}{v_{max}}
\end{aligned}
$$

A.3 Calculation of the equilibrium solution with equal density

We can calculate the equilibrium solution of the average attribute value in a group by solving the expression of the group mean in equilibrium with equal density (12) for $\overline{v}^g$

$$
\begin{aligned}
\overline{v} &= \int_{\overline{v}^g - \sqrt{C^g}}^{\overline{v}^g + \sqrt{C^g}} v \overline{f} dv \\
\Leftrightarrow \quad &= \overline{f} \int_{\overline{v}^g - \sqrt{C^g}}^{\overline{v}^g + \sqrt{C^g}} v dv \\
\Leftrightarrow \quad &= \overline{f} [\frac{1}{2} v^2]_{\overline{v} - \sqrt{C^g}}^{\overline{v} + \sqrt{C^g}} = 2 \overline{f} \overline{v} \sqrt{C^g} \\
\Leftrightarrow \quad &= 2 \frac{1}{v_{max}} \overline{v} \sqrt{C^g} \\
\Leftrightarrow 0 &= (2 \frac{1}{v_{max}} \sqrt{C^g} - 1) \overline{v}, \qquad with \quad C^g = \frac{A^g}{\beta^g}
\end{aligned}
$$

A.4 Derivation of the specific functional form of the linear density function

Given the described conditions of the linear density function, that the area below the function must equal one and that the density at the maximum value equals zero, specific expressions for the intercept and the slope can be derived. The second condition implies

$$\begin{aligned} &\Leftrightarrow f(v_{max}) = 0 \\ &\Leftrightarrow z - gv_{max} = 0 \\ &\Leftrightarrow z = gv_{max} \end{aligned}$$

inserting the expression for z into the first condition yields

$$\begin{aligned} &\int_0^{v_{max}} (gv_{max} - gv)dv = 1 \\ &\Leftrightarrow [gv_{max}v - \frac{1}{2}gv^2]_0^{v_{max}} = 1 \\ &\Leftrightarrow gv_{max}^2 - \frac{1}{2}gv_{max}^2 = 1 \\ &\Leftrightarrow \frac{1}{2}gv_{max}^2 = 1 \\ &\Leftrightarrow g = \frac{2}{v_{max}^2} \end{aligned}$$

inserting the obtained expression for g back into the above expression for z yields

$$\begin{aligned} z &= \frac{2}{v_{max}^2}v_{max} \\ \Leftrightarrow z &= \frac{2}{v_{max}} \end{aligned}$$

A.5 Derivation of the analytical solution of the average attribute value in a group in equilibrium with linear density

As in the case of the equal density equilibrium, we can calculate the equilibrium solution of the average attribute value in a group by solving the expression of the group mean in equilibrium with equal density, (18), for $\bar{v}^g$

$$\bar{v}^g == \int_{\bar{v}^g-\sqrt{C^g}}^{\bar{v}^g+\sqrt{C^g}} v \left[\frac{2}{v_{max}} - \frac{2}{v_{max}^2} v\right] dv$$

$$\Leftrightarrow \bar{v}^g = \left[\frac{1}{2}\frac{2}{v_{max}}v^2 - \frac{1}{3}\frac{2}{v_{max}^2}v^3\right]_{\bar{v}^g-\sqrt{C^g}}^{\bar{v}^g+\sqrt{C^g}} = \left[\frac{1}{v_{max}}v^2 - \frac{2}{3}\frac{1}{v_{max}^2}v^3\right]_{\bar{v}^g-\sqrt{C^g}}^{\bar{v}^g+\sqrt{C^g}}$$

$$\Leftrightarrow \quad = \left\{\frac{1}{v_{max}}\left[\bar{v}^g + \sqrt{C^g}\right]^2 - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g + \sqrt{C^g}\right]^3\right\} - \left\{\frac{1}{v_{max}}\left[\bar{v}^g - \sqrt{C^g}\right]^2 - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g - \sqrt{C^g}\right]^3\right\}$$

$$\Leftrightarrow \quad = \frac{1}{v_{max}}\left[(\bar{v}^g)^2 + 2\bar{v}^g\sqrt{C^g} + C^g\right] - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g + \sqrt{C^g}\right]^3 - \frac{1}{v_{max}}\left[(\bar{v}^g)^2 - 2\bar{v}^g\sqrt{C^g} + C^g\right] + \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g - \sqrt{C^g}\right]^3$$

$$\Leftrightarrow \quad = \frac{1}{v_{max}}(\bar{v}^g)^2 + \frac{1}{v_{max}}2\bar{v}^g\sqrt{C^g} + \frac{1}{v_{max}}C^g - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g + \sqrt{C^g}\right]^3 - \frac{1}{v_{max}}(\bar{v}^g)^2 + \frac{1}{v_{max}}2\bar{v}^g\sqrt{C^g} - \frac{1}{v_{max}}C^g + \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g - \sqrt{C^g}\right]^3$$

$$\Leftrightarrow \quad = \frac{2\sqrt{C^g}}{v_{max}}\bar{v}^g - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g + \sqrt{C^g}\right]^3 + \frac{2\sqrt{C^g}}{v_{max}}\bar{v}^g + \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g - \sqrt{C^g}\right]^3$$

$$\Leftrightarrow 0 = \bar{v}^g - \frac{4\sqrt{C^g}}{v_{max}}\bar{v}^g + \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g + \sqrt{C^g}\right]^3 - \frac{2}{3}\frac{1}{v_{max}^2}\left[\bar{v}^g - \sqrt{C^g}\right]^3$$

$$\Leftrightarrow 0 = v_{max}\bar{v}^g - 4\sqrt{C^g}\bar{v}^g + \frac{2}{3}\frac{1}{v_{max}}\left[\bar{v}^g + \sqrt{C^g}\right]^3 - \frac{2}{3}\frac{1}{v_{max}}\left[\bar{v}^g - \sqrt{C^g}\right]^3$$

$$\Leftrightarrow 0 = v_{max}\bar{v}^g - 4\sqrt{C^g}\bar{v}^g + \frac{2}{3}\frac{1}{v_{max}}\left[(\bar{v}^g)^3 + 3(\bar{v}^g)^2\sqrt{C^g} + 3\bar{v}^gC^g + (\sqrt{C^g})^3\right] - \frac{2}{3}\frac{1}{v_{max}}\left[(\bar{v}^g)^3 - 3(\bar{v}^g)^2\sqrt{C^g} + 3\bar{v}^gC^g - (\sqrt{C^g})^3\right]$$

$$\Leftrightarrow 0 = v_{max}\bar{v}^g - 4\sqrt{C^g}\bar{v} + \frac{2}{3}\frac{1}{v_{max}}(\bar{v}^g)^3 + \frac{2}{v_{max}}(\bar{v}^g)^2\sqrt{C^g} + \frac{2}{v_{max}}\bar{v}^gC^g + \frac{2}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3 - \frac{2}{3}\frac{1}{v_{max}}(\bar{v}^g)^3 + \frac{2}{v_{max}}(\bar{v}^g)^2\sqrt{C^g} - \frac{2}{v_{max}}\bar{v}^gC^g + \frac{2}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3$$

$$\Leftrightarrow 0 = v_{max}\bar{v}^g - 4\sqrt{C^g}\bar{v}^g + \frac{4}{v_{max}}(\bar{v}^g)^2\sqrt{C^g} + \frac{4}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3$$

$$\Leftrightarrow 0 = \frac{4}{v_{max}}\sqrt{C^g}(\bar{v}^g)^2 + \left[v_{max} - 4\sqrt{C^g}\right]\bar{v}^g + \frac{4}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3$$

Now, we can apply the quadratic formula to solve for $\bar{v}$:

$$\bar{v}^g_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \;\; with \;\; a = \frac{4}{v_{max}}\sqrt{C^g}, \; b = \left[v_{max} - 4\sqrt{C^g}\right], \; c = \frac{4}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3$$

$$\Leftrightarrow \bar{v}^g_{1,2} = \frac{-\left[v_{max} - 4\sqrt{C^g}\right] \pm \sqrt{\left[v_{max} - 4\sqrt{C^g}\right]^2 - 4\left[\frac{4}{v_{max}}\sqrt{C^g}\right]\left[\frac{4}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3\right]}}{2\frac{4}{v_{max}}\sqrt{C^g}}$$

$$\Leftrightarrow \bar{v}^g_{1,2} = \frac{4\sqrt{C^g} - v_{max} \pm \sqrt{\left[v_{max} - 4\sqrt{C^g}\right]^2 - \frac{16}{v_{max}}\sqrt{C^g}\frac{4}{3}\frac{1}{v_{max}}(\sqrt{C^g})^3}}{\frac{8}{v_{max}}\sqrt{C^g}}$$

$$\Leftrightarrow \bar{v}^g_{1,2} = \frac{4\sqrt{C^g} - v_{max} \pm \sqrt{\left[v_{max} - 4\sqrt{C^g}\right]^2 - \frac{64}{3}\frac{1}{v_{max}^2}(C^g)^2}}{\frac{8}{v_{max}}\sqrt{C^g}}$$

A.6 Calculation of the partial derivatives of the discriminant in the linear density equilibrium solution

Taking the partial derivative of the discriminant in (20),

$$D = (v_{max} - 4\sqrt{C^g})^2 - \frac{64}{3}\frac{(C^g)^2}{v_{max}^2}$$

with respect to the maximum attribute level in the population and controlling for whether this is positive or negative yields

$$\begin{aligned} \frac{\partial D}{\partial v_{max}} &\gtrless 0 \\ \Leftrightarrow 2(v_{max} - 4\sqrt{C^g}) + \frac{64}{3}(C^g)^2 v_{max}^{-3} &\gtrless 0 \\ \Leftrightarrow 2v_{max} - 8\sqrt{C^g} + \frac{64}{3}(C^g)^2 v_{max}^{-3} &\gtrless 0 \\ \Leftrightarrow 2v_{max} + \frac{64}{3}(C^g)^2 v_{max}^{-3} &\gtrless 8\sqrt{C^g} \\ \Leftrightarrow \frac{1}{4}\frac{v_{max}}{\sqrt{C^g}} + \frac{8}{3}\frac{1}{\sqrt{C^g}}(\sqrt{C^g})^4\frac{1}{v_{max}^3} &\gtrless 1 \\ \Leftrightarrow \frac{1}{4}\frac{v_{max}}{\sqrt{C^g}} + \frac{8}{3}(\sqrt{C^g})^3\frac{1}{v_{max}^3} &\gtrless 1 \end{aligned}$$

and with respect to the utility parameters

$$\begin{aligned}
\frac{\partial D}{\partial C^g} &\gtreqless 0 \\
\Leftrightarrow 2(v_{max} - 4\sqrt{C^g})\frac{-2}{\sqrt{C^g}} - \frac{128}{3}\frac{C^g}{v_{max}^2} &\gtreqless 0 \\
\Leftrightarrow -\frac{4(v_{max} - 4\sqrt{C^g})}{\sqrt{C^g}} - -\frac{128}{3}\frac{C^g}{v_{max}^2} &\gtreqless 0
\end{aligned}$$

A.7 Maximal slope of $h(\overline{v}^g)$

Deriving the expression for v_{max} that maximizes the slope of (23) implies maximizing the slope $m = 4\sqrt{C^g}v_{max} - v_{max}^2$ with respect to v_{max}

$$\begin{aligned}
\frac{\partial m}{\partial v_{max}} &= 0 \\
\Leftrightarrow 4\sqrt{C^g} - 2v_{max} &= 0 \\
\Leftrightarrow 2v_{max} &= 4\sqrt{C^g} \\
\Leftrightarrow v_{max} &= 2\sqrt{C^g}
\end{aligned}$$