Johannes Gutenberg University Mainz
Master in International Economics and Public Policy

Labour market theory: Wage inequality, redistribution and international trade

Summer Term

Klaus Wälde (lecture) and Hoang Van Khieu (tutorial)
www.macro.economics.uni-mainz.de

February 5, 2026

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- Organizational preliminaries
  - Website for course can be found at www.macro.economics.uni-mainz.de → Teaching → Master → Summer Term
  - Website provides
    - table of contents
    - slides with problem sets
    - links to literature
- Teaching & learning style
  - What do you want to understand? → collect questions
  - Term paper at end → active individual learning
  - (see schedule)

1 Motivation and some facts on German, European and OECD labour markets

1.1 Motivation and contents

- Motivation
  - Many individuals are concerned about large and potentially rising inequalities in modern societies
  - Inequalities can be found in labour income, overall income, wealth, consumption and happiness
  - When there is inequality, the question is: is there *too much* inequality?
  - If this is the view, the subsequent question would ask: how can inequality be reduced?
  - [The question after this one would inquire into implementing changes in a society]

- Contents
  - This lecture looks at labour income inequality (as a starting point)
  - The first chapter looks at facts about inequality in OECD countries
  - The first part of the lecture asks and offers explanations from the modern labour market literature why there is wage inequality
  - The second part studies principles of taxation and shows how tax systems affect and help to reduce inequality
  - Third part applies these insights to international trade
    - There are gains from trade and winners and losers
    - Which tax system would imply Pareto-improving gains from trade?
  - Part IV summarizes and provides an outlook to redistribution beyond the tax system

Labour market theory:
Wage inequality, redistribution and international trade

1. Motivation and some facts on German, European and OECD labour markets

Part I The origins of wage inequality

2. Luck – The example of the pure search model
3. Luck and endogenous wage distributions
4. Education and experience – Human capital

Part II Redistribution

5. The basics of positive tax theory
6. Taxation and inequality

Part III International trade, wage inequality and redistribution

7. The literature on trade and labour markets
8. A simple (autarky) model with heterogeneous agents
9. Opening up to trade
10. Redistribution and Pareto-improving gains from trade

Part IV Summary

11. Conclusion and long-term redistribution

1.2 Learning objectives

1.2.1 ... on unemployment

- How high are the unemployment rates in Europe, Germany and other OECD countries?
- What are typical patterns of unemployment rates over time?
- What are typical differences and commonalities across countries?
- What do we know about flows into and out of unemployment?

1.2.2 ... on wages

- How does the real wage in Germany evolve over time?
- What about the evolution of the wage distribution in Germany?
- What do we know about wage inequality in the US and other countries?
- Why should we care about unemployment and wages and their distributions?

1.2.3 ... on poverty

- How is poverty defined (for OECD countries)?
- How is poverty measured?
- What are poverty rates in OECD countries and how do they change over time?
- What is the effect of policy on poverty?

1.3 The case of Germany

1.3.1 ... on unemployment

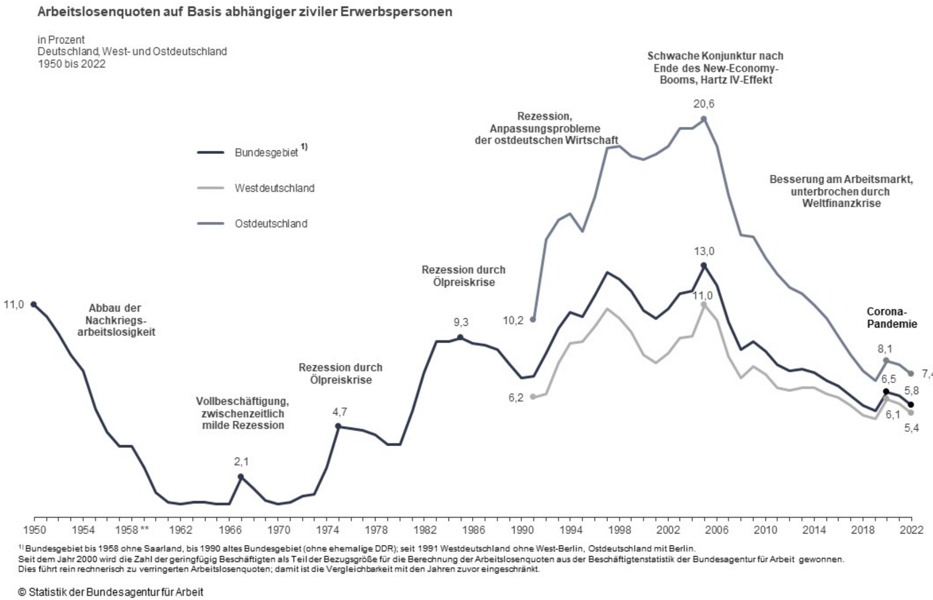


Figure 1 *Unemployment rate in Germany in percent from Bundesagentur für Arbeit*

- Decline after WW II up to 1960
- Basically steady increase over time as of early 1970s
  - major events: oil price shocks, reunification
  - up to 2005 high persistence (weak decreases once negative shocks are over)
  - “sick man of Europe”
- After 2005, big, fast and persistent drop
  - ... despite “big recession” due to financial crisis
  - ... importance of Hartz reforms implemented between 2003 and 2005 (to be discussed later)

1.3.2 ... on unemployment flows

population	
labour force	out of labour force
employees	children
civil servants	educational system
self-employed	homemaker/ mother/ father
unemployed	retirement
labour market programmes	

Table 1 *Classifying a population by economic status*

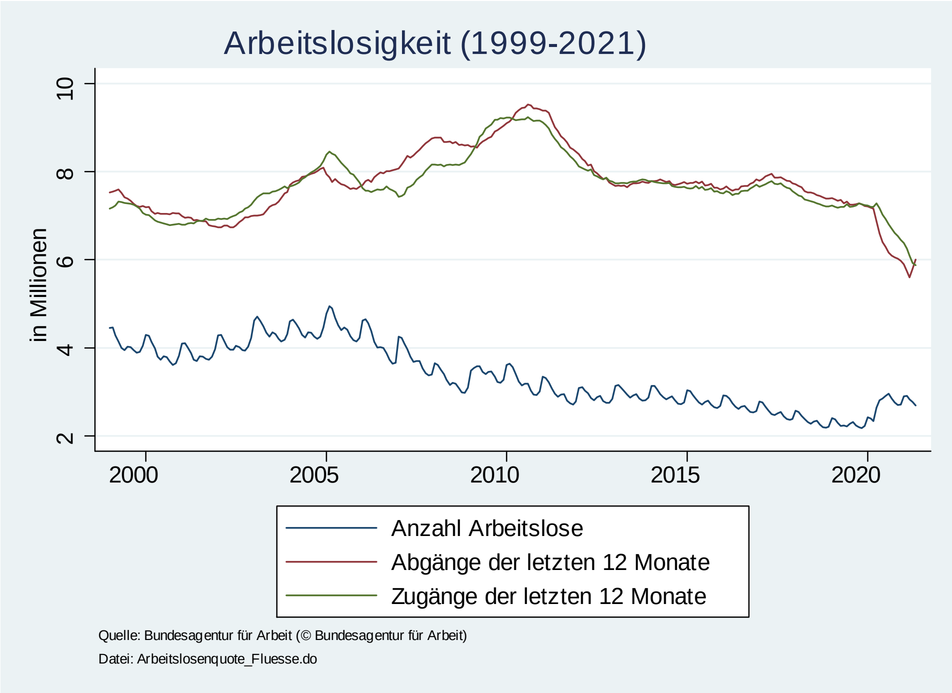


Figure 2 *Stocks ("Anzahl Arbeitslose") and flows ("Abgänge" und "Zugänge") on the German labour market*

1.3.3 … on wages

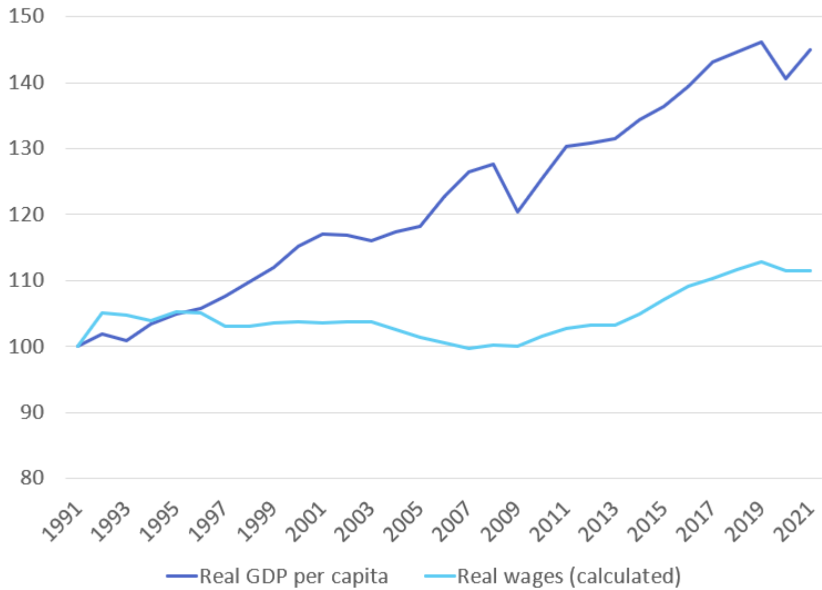


Figure 3 *Real wage and real GDP per capita (1991 =100), Germany 1991–2021. Sources: extended version of Krebs and Scheffel, 2013, Fig. 4, Statistisches Bundesamt (see real_wages_and_GDP_Germany_1979_2021_os.xlsx)*

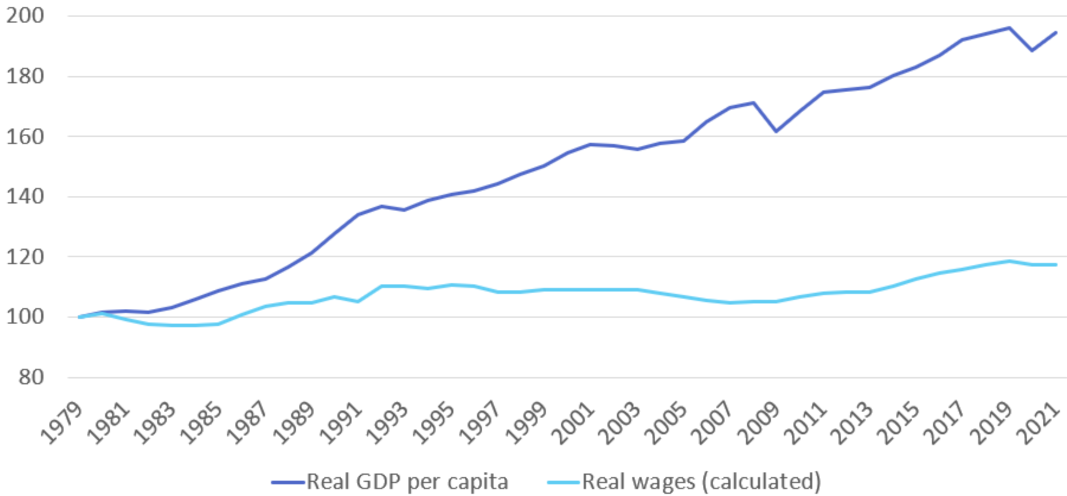


Figure 4 *Real wage and real GDP per capita (1979 =100), Germany 1979–2021 (source and comments, see previous figure)*

- Background for previous figures
  - real GDP data as published by the German Federal Statistical Office
  - real wage index calculated by using nominal wage index and the consumer price index (nominal wage/CPI *100)
  - nominal wage index shows how the average gross monthly earnings including special payments of all full-time, part-time and marginally employed workers in the manufacturing and service sectors would have developed if the structure of the workforce had been the same in the respective comparison period as in the previous year (Email Corina Neuerer, German Federal Statistical Office, Jan 2024)
  - aim of the nominal wage index is to reflect the pure development of earnings: should not be influenced by structural effects. Changes in the number and composition of employees (e.g. the ratio of full-time, part-time and marginal employees) should therefore have no influence on the development of earnings (Corina Neuerer)
  - there are minor deviations between calculated real wage index and officially reported index as published since 2007:
    - data on part-time and marginal part-time employees as well as special payments have only been recorded in the official surveys since 2007
    - earlier time series on nominal wage development were linked and back-calculated with the help of data from the National Accounts (Corina Neuerer)

- Wage dynamics
  - Real wages basically constant if not falling since 1992
  - GDP per capita rising (suggesting a rise in total factor productivity)
  - What are the mechanisms behind this (almost) constant real wage?
  - Unemployment rate rising until 2005 and falling thereafter
  - Shift in distributional shares between capital and labour - see Jaumotte and Tytell (2007, fig. 5.7) and next page
  - Big question waiting for a detailed answer

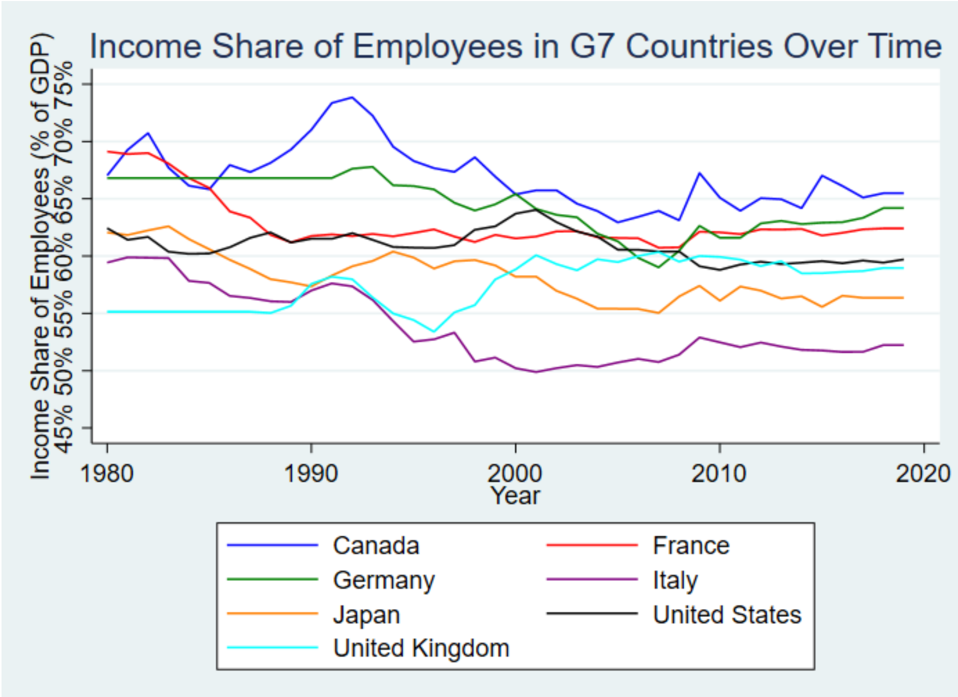


Figure 5 *Share of labour income in GDP in G7 countries 1980 - 2019 (Source: Penn World Table, https://www.rug.nl/ggdc/productivity/pwt/related-research, share_labour_in_GDP.do)*

1.3.4 ... on wage distributions

- How do wage distributions look like?

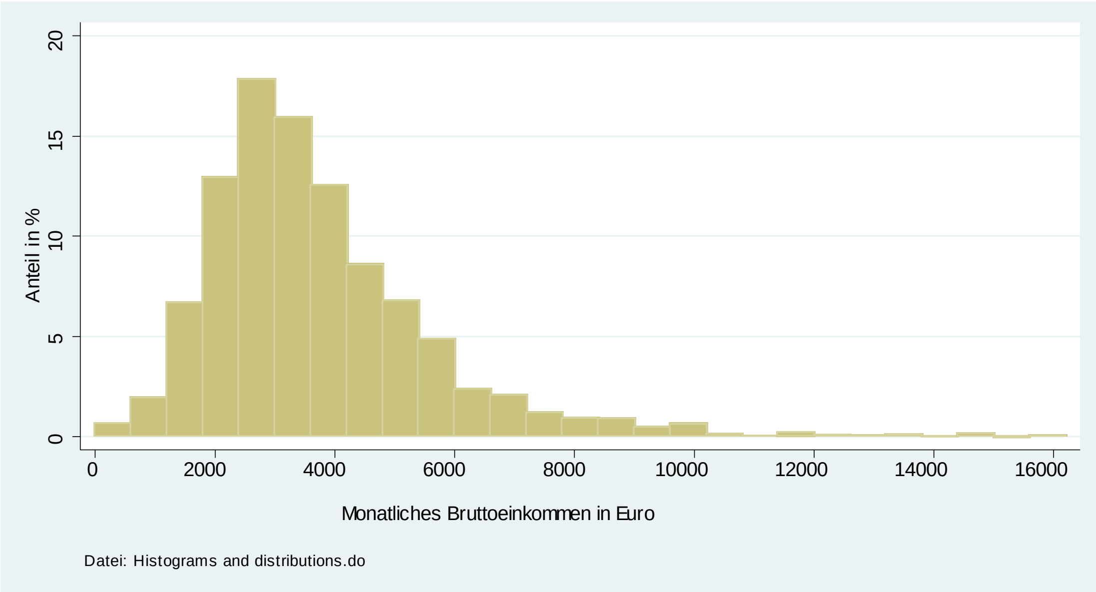


Figure 6 *Gross monthly labour income in 2020 of full time employed individuals (30 hours per week or more, GSOEP, in prices of 2020)*

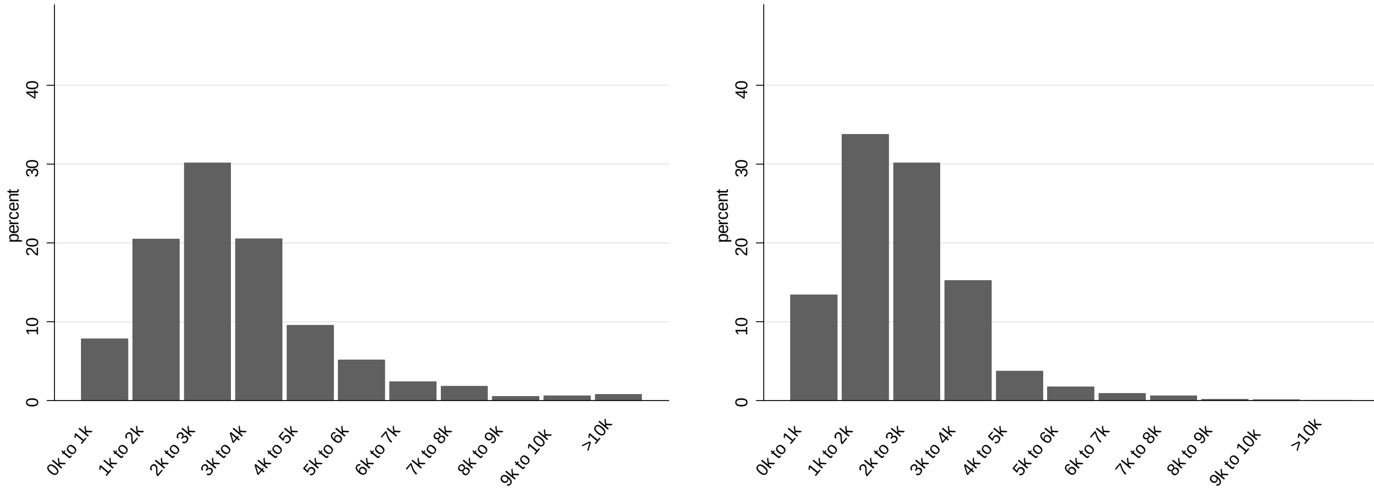


Figure 7 *Gross monthly labour income in 2013 of full time employed men (left) and women (right) (30 hours per week or more, GSOEP, in prices of 2010)*

- Wage distributions and residuals of wage distributions
  - What is the residual of a wage distribution?
  - Imagine a regression of the wage levels w for a given year like

$$w = \beta_0 + \beta_1 \text{skill} + \beta_2 \text{age} + \beta_3 \text{age}^2 + \beta_4 \text{gender} + \varepsilon \tag{1.1}$$

    where skill is a dummy variable for university degree and age is biological age
  - The independent variables in the wage regression, i.e. $\beta_0 + \beta_1 \text{skill} + \beta_2 \text{age} + \beta_3 \text{age}^2 + \beta_4 \text{gender}$, explain a part of the variation in the dependent variable w
  - The rest of the variation ends up in the error term ε
  - Sometimes ε is referred to as unobserved heterogeneity
  - The distribution of the residual ε (in almost all cases) displays less variation than the distribution of wages

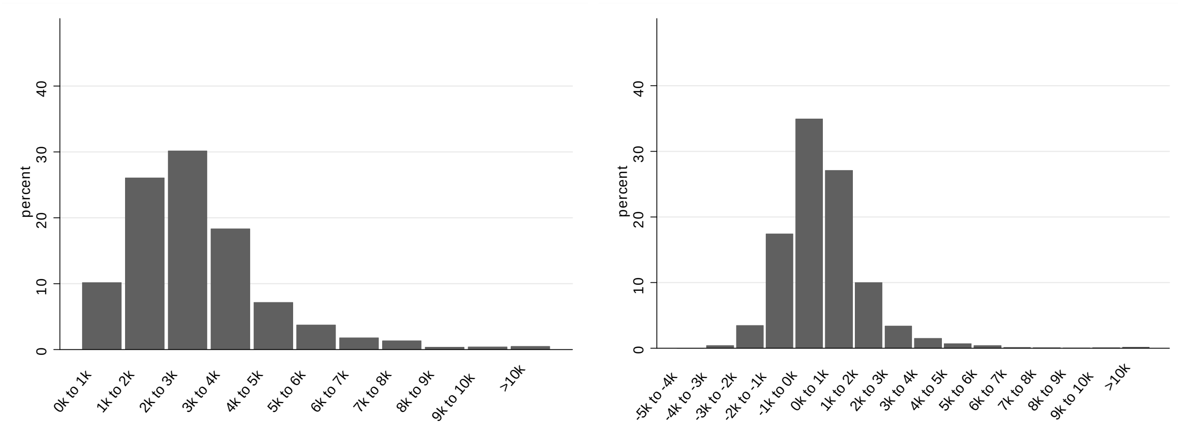


Figure 8 *Gross monthly labour income in 2013 of full time employed men&women (left) and residual labour income men&women (right) (30 hours per week or more, GSOEP, in prices of 2010)*

- Where does change in inequality come from?
- from the composition of the labour force? → composition effect, can be explained
- from unobserved factors? → change in residual, can not be explained

- Prasad (2004) IMF papers, “The Unbearable Stability of the German Wage Structure: Evidence and Interpretation”
  - wage structure is very stable in Germany
  - GSOEP data (see www.soep.de) from DIW Berlin, 1984 - 1997
  - see his figure 1 (on next slide), according to standard deviation, wage distribution did not become more unequal
  - explanation: institutional factors (labour market regulations, trade unions)
  - downside: strong increase of the unemployment rate of low-skilled workers

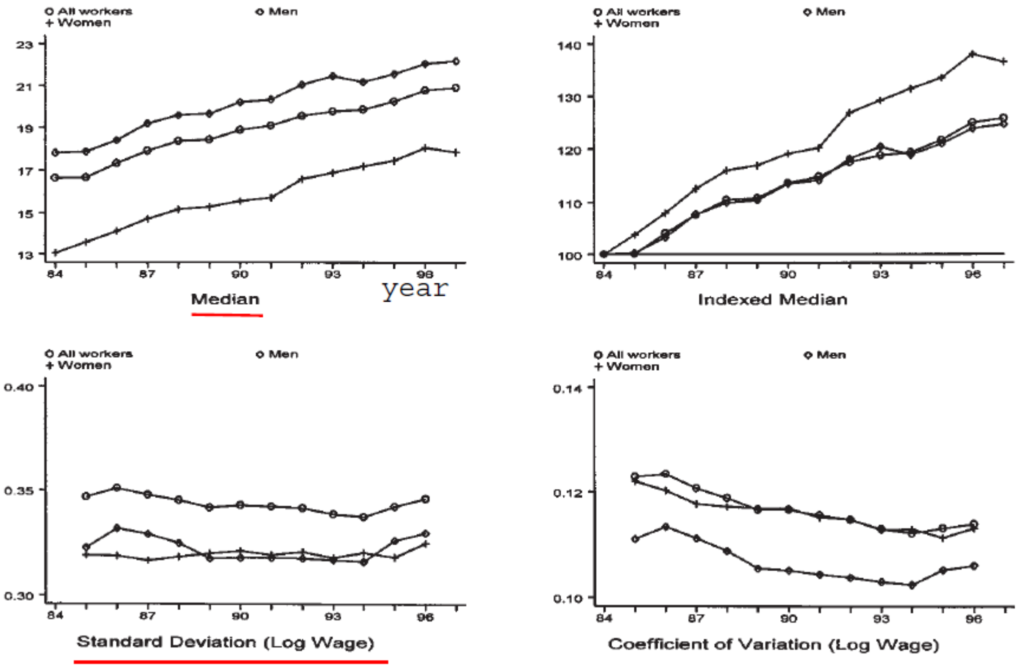


Figure 9 *Median and Dispersion Measures for Hourly Wage (Prasad, 2004, fig. 1)*

- Dustmann Ludsteck and Schönberg (2009)
  - Central finding: wage structure in Germany is not as stable as thought so far
  - The approach
    - use a different data set (LIAB from iab.de, administrative data set)
    - look at a longer time horizon
  - Comparing to the US
    - similar changes at the upper end (compare evolution of e.g. 85th percentile)
    - lower end inequality rose in US in 1980s and in Germany in 1990s
  - Explanation for the lower end
    - Decline in unionization rate
    - Changes in work-force composition

- What are percentiles again?

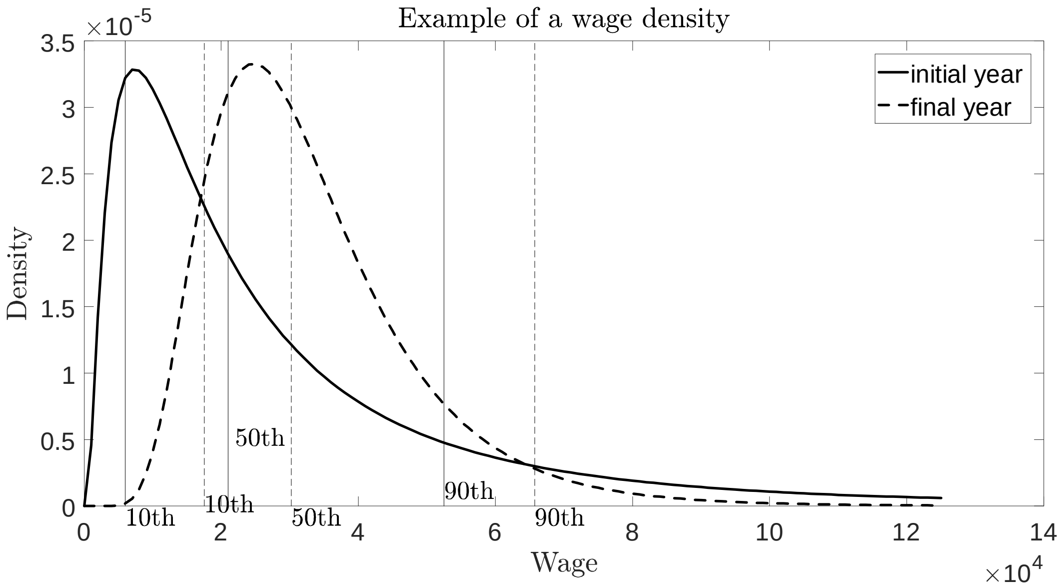


Figure 10 *10th, 50th and 90th percentiles in two lognormal densities*

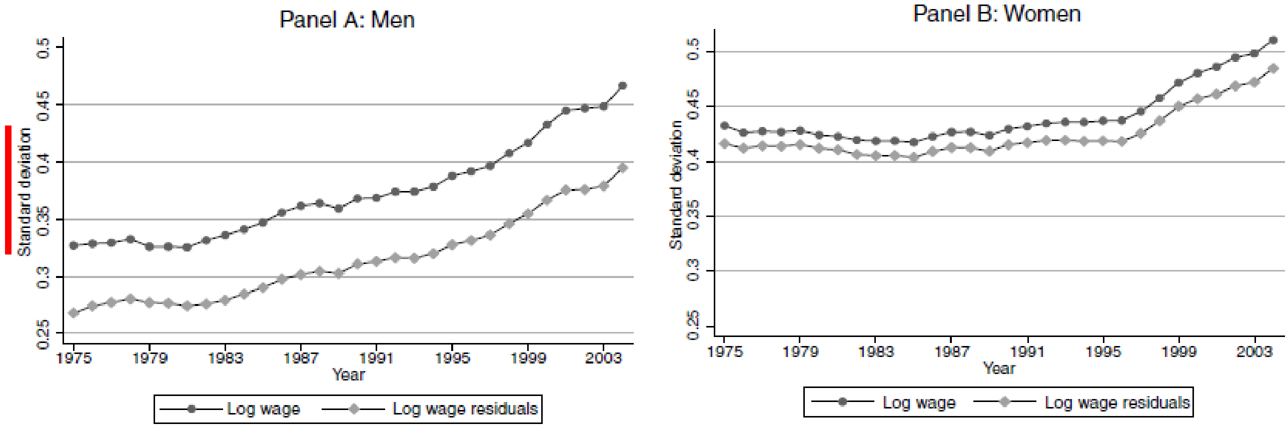


Figure 11 *Evolution of the Standard Deviation of Log Wages and Log Wage Residuals. Source: Dustmann et al. (2009)*

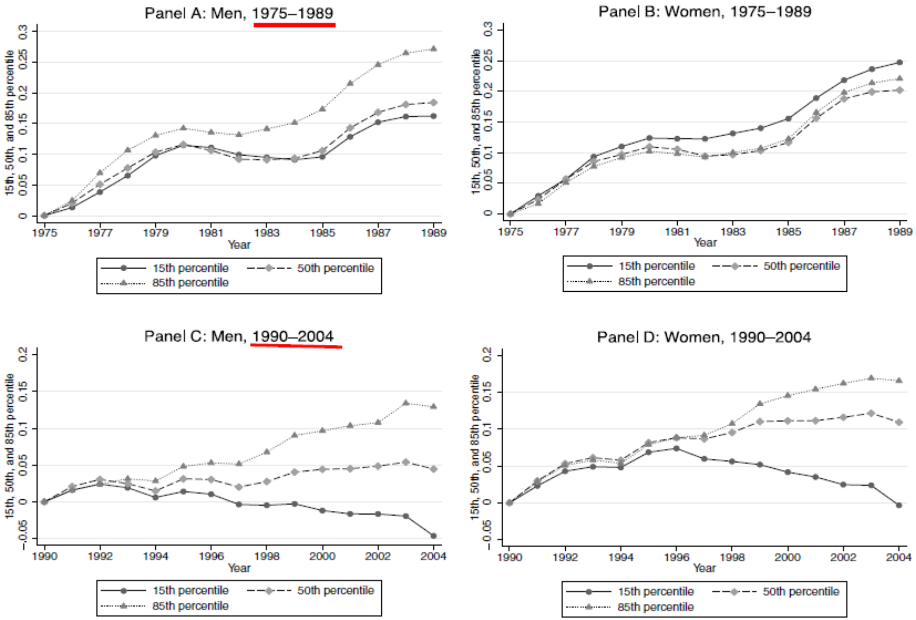


Figure 12 *Indexed Wage Growth of the 15th, 50th, and 85th Percentiles: The Pre- versus the Postunification Period. Source: Dustmann et al. (2009)*

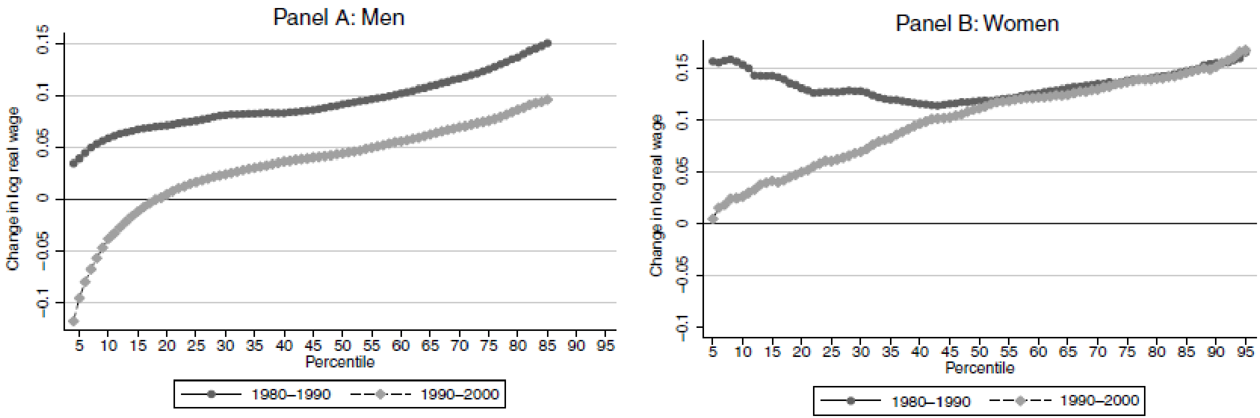


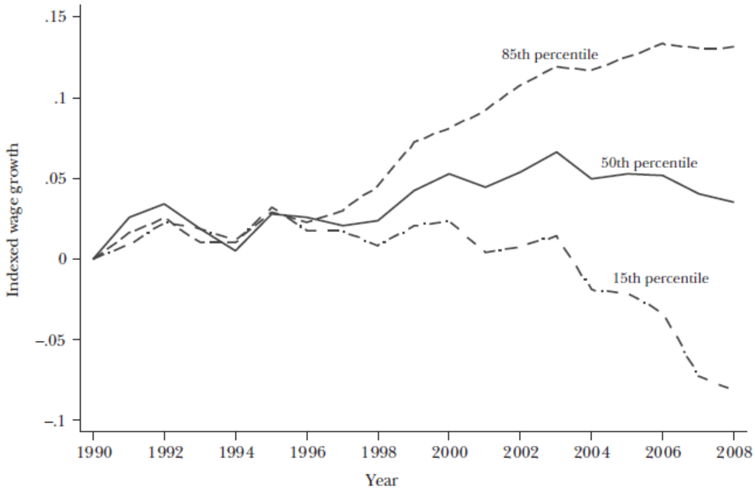
Figure 13 *Wage Growth by Percentile: The 1980s versus the 1990s. Source: Dustmann et al. (2009)*

- Conclusion: unsatisfactory contradiction between Dustmann et al. and Prasad – precise reasons to be found

- More recent work
  - (General) trend to take firm-characteristics into account when understanding wage distributions (see e.g. Card et al., 2013, for evidence and (4.1) below for theory)
  - Interesting sources are also domestic outsourcing (contractors, temp agencies or franchises). They account for 9% of increase in wage inequality since the 1980s (Goldschmidt and Schmieder, 2017)
  - These studies tend to support the Dustmann et al. (2009) finding of rising wage inequality in Germany
  - See next figure for an illustration from Dustmann et al. (2014)
  - Subsequent figure updates to 2016 (employing Drechsel-Grau et al. 2022)

Figure 2

Indexed Wage Growth of the 15th, 50th, 85th Percentiles, West Germany, 1990–2008



Notes: Calculations based on SIAB Sample for West German Full-Time Workers between 20 and 60 years of age. The figure shows the indexed (log) real wage growth of the 15th, 50th, and 85th percentiles of the wage distribution, with 1990 as the base year. Nominal wages are deflated using the consumer price index (1995 = 100) provided by the German Federal Statistical Office.

Figure 14 *Increase in wage inequality in Germany (source: Dustmann et al., 2014)*

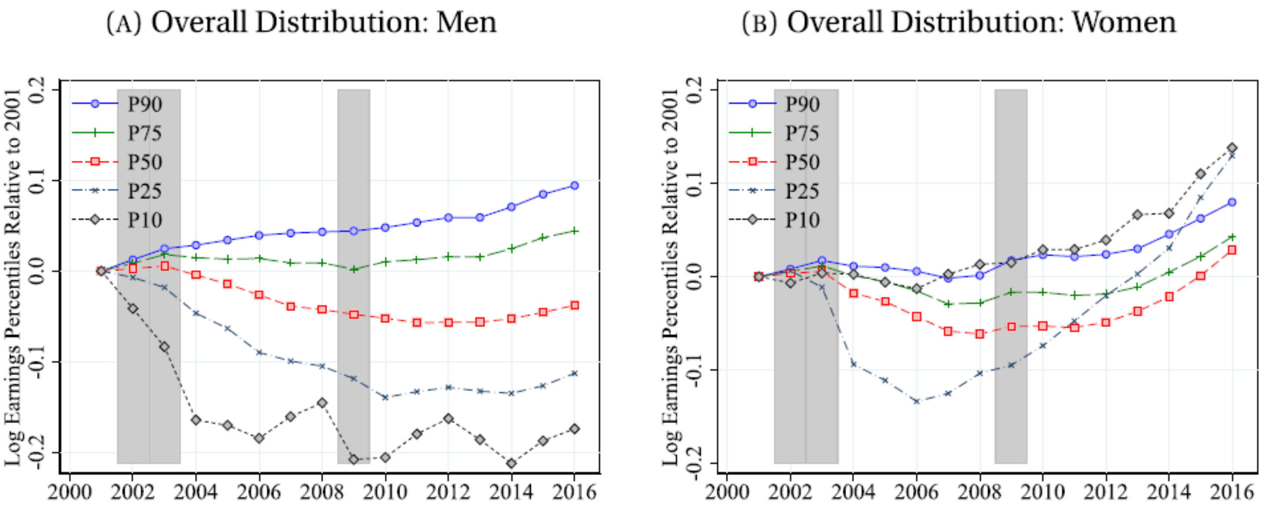


Figure 15 *Continuing increase in inequality up to 2009 and more stability (men) to reversal (women) thereafter (source: Drechsel-Grau et al. 2022)*

1.4 Poverty as a summary measure of inequality

- We studied unemployment and wage inequality up to now
- Why care about inequality?
- Poverty within OECD countries is an important concern
  - for humans with empathy
  - for policy makers (with empathy)
  - for efficiency reasons (via political extremism and crime – Master thesis)
- This concluding section suggests poverty as a summary measure for inequality
  - Inequality is a problem for the poor
  - Some facts about poverty
  - Some ethics considerations

1.4.1 ... on poverty in OECD countries

- OECD (2008) Growing Unequal? Income distribution and poverty in OECD countries
  - Poverty is of course first and foremost an international problem (low-/ middle- /high-income countries)
  - Poverty is also an "intranational" problem, i.e. a problem within countries
- Defining poverty: the share of individuals with equivalised disposable income less than 40, 50 and 60% of the median income
- Among OECD countries, relative poverty rate of Mexico is highest being followed by Turkey, the US and Japan (OECD, 2008)

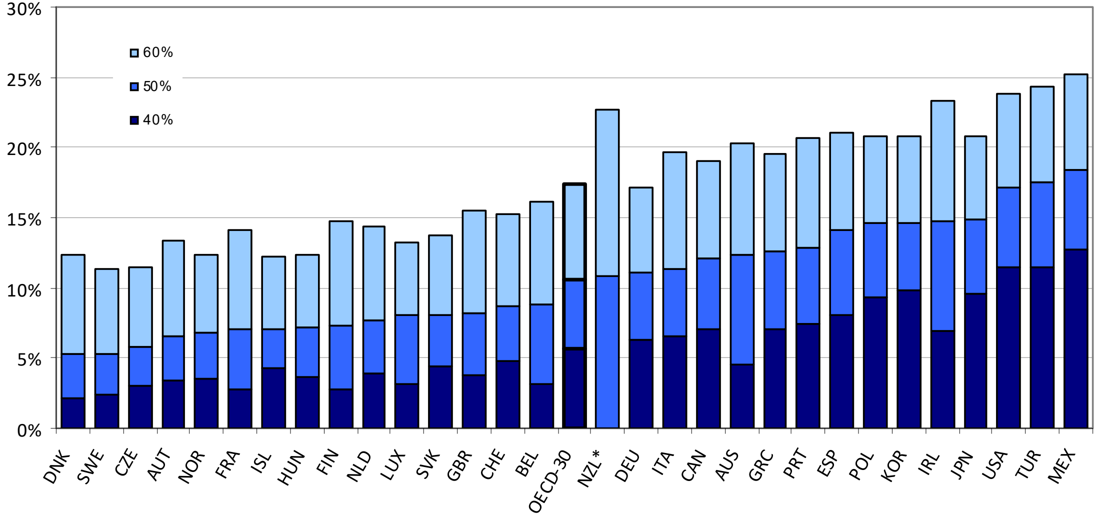


Figure 16 *Poverty rates in OECD countries*

- Some numbers for G7 countries (relative poverty in %, mid 2000, 60% of median income, OECD 2008)

country	US	Japan	Italy	Canada	Germany	UK	France
poverty rate	23.9	20.8	19.7	19.0	17.2	15.5	14.1

- Trends in poverty (OECD 2008)

→ Increase in relative poverty (headcount at 50% of median income, mid 1980s to mid 2000s)
→ decrease in absolute poverty (apart from Germany, experiencing increase from mid 1990s to 2005)

- Current values (OECD, current years)

→ https://data.oecd.org/inequality/poverty-rate.htm
→ 50% of median income
→ at around the same level as mid 2000
→ same order of magnitude as unemployment rates or even higher (US)

- The effect of policy

→ Valletta (2006, Table 6): Analysis of determinants of (inter alia) entry rates into and exit rates out of poverty (see next page)

→ strong impact of policy

DETERMINANTS OF POVERTY EXITS (WORKING-AGE HOUSEHOLDS; DISPOSABLE INCOME POVERTY)

(annual) Exit Rate →	Canada (1994–98)		Germany (1992–96)		Great Britain (1992–96)	
	32.4		44.5		51.6	
Explanatory Variables	Probability Effect	Mean	Probability Effect	Mean	Probability Effect	Mean
Head age (omitted = age 30–50)						
Head < 30	3.6	28.8	–7.7**	27.5	–9.9**	24.3
Head 51–65	–1.5	14.0	–6.4*	28.1	2.0	11.2
Head education (omitted = HS grad)						
Low education	–1.3	35.3	–2.4	25.0	N/A	N/A
High education	5.0**	30.8	–3.2	9.8	N/A	N/A
Individual age (omitted = adult)						
Child	–2.1**	30.8	–0.2	21.4	–3.2	38.9
Family type (omitted = 2 adults w/children)						
Single adult	–2.8*	16.8	–2.1	29.6	–18.9**	11.3
Adults no children	5.1**	6.5	1.6	11.3	–20.4**	4.2
Single with children	–2.3*	29.5	5.3	27.7	–0.7	31.4
Other family type	0.4	5.0	5.7	3.5	20.2	2.7
Number of full-time workers[a] (omitted = 1)						
No workers	–13.2**	75.1	–7.4*	76.1	–6.0*	76.1
Two workers	6.3**	3.0	–20.9**	2.6	–15.1	1.1
Increase in full-time work[a] (omitted = no change)						
More work—head	32.6**	11.6	43.5**	6.7	17.5**	12.5
More work—spouse	22.6**	5.2	40.2**	2.5	23.0**	2.6
Increase in months worked[b] (omitted = no change)						
More hours—head	12.7**	19.1	12.3**	14.1	9.4**	16.3
More hours—spouse	10.0**	10.7	18.3**	3.7	17.5**	7.4
Change in family type (omitted = no change)						
Marriage	37.8**	5.4	16.6**	2.8	47.6**	5.2
Other family change	14.1**	9.9	–0.2	6.3	1.4	4.1
Gov. transfers up >20%	10.6**	27.6	15.8**	56.5	17.2**	42.3
Log-likelihood	–7,356.1		–1,462.0		–1,054.7	
Number of observations	13,795		2,357		1,693	

- Simple time series "evidence"

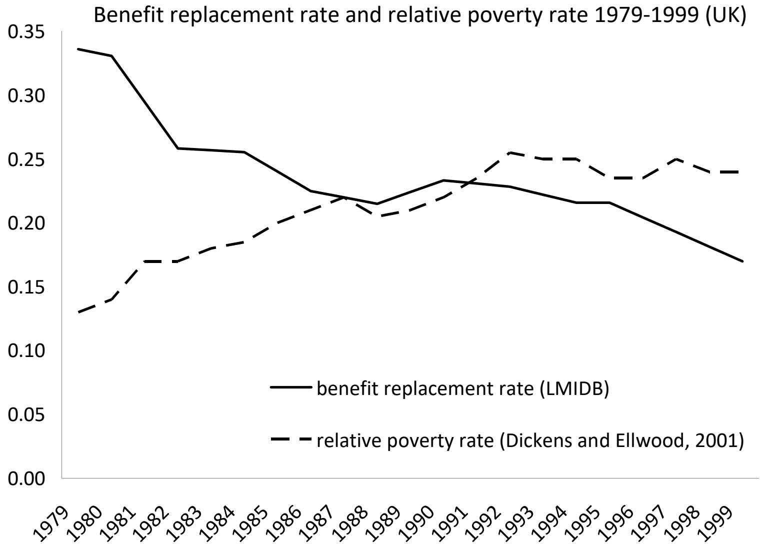


Figure 17 *Unemployment benefits and relative poverty (UK 1979 - 1999) Source: Dickens and Ellwood (2004) and LMIDB (labour market institutions database, see Nickell, 2006. Current version in AMECO database)*

1.4.2 The idea of this lecture

In which society do we want to live in?

- What is “the society”?
  - Is it: world – country – region – neighborhood?
  - Here: country
- What are acceptable values for a society?
  - Equality or liberty?
  - Some bias towards equality (often referred to as “justice”) in this lecture ...
- Equality of what (Sen 1979)? (Compare initial question on slide 2)
  - Chances – income – outcome (well-being)?
  - Introduction and background: Roemer (1996, “Theories of distributive justice”)

- Poverty violates all three criteria for equality
- Poverty needs to be taken into account when
  - thinking about how to reduce unemployment or inequality
  - designing labour or trade (or other) policies

In which society do we want to live in? (cont'd)

- Reduction of unemployment is simple
  - Define only short-term unemployed as 'unemployed', the rest is defined away (as long-term unemployed receive no benefits – see OECD - benefits and wages)
  - Drawback: This is not a solution to the (individual) problem of unemployment and low income
- In which society do we want to live in?
  - Where unemployment is reduced *without* the creation of poverty
  - Reduce unemployment but preserve high *net* income for *low* income groups
  - When there are gains from trade, think about Pareto-improving gains from trade
- How can this be achieved? Wait for the end of this lecture ...

1.5 Exercises

1.5.1 German Socioeconomic Panel (GSOEP)

As a preparatory step to the subsequent questions, load the data set *labor_theory_data_pub_2.dta* into STATA. The data set contains nine variables:

persnr: person identifier
year: observation year
cpi: consumer price index (2010=100)
gebjahr: birth year
sex: respondent's gender ([1] man, [2] woman)
isced: highest degree attained, ISCED-1997 ([0] in school,..., [6] higher education)
tatzeit: actual weekly working hours
labgro: current monthly gross labor income in Euros
phrf: sampling weights

Missings are denoted by -1, -2, or -3. The sample includes only full-time employed respondents from Western Germany.

Note: The data extract is a 50 percent sample of the original GSOEP data, and the person identifier is randomly reassigned each year.

1. **Generating the Variables of Interest** For the subsequent analysis, generate five additional variables:

(a) *labgro_10*: monthly gross labor income in 2010 prices

(b) *hwage_10*: hourly wage in 2010 prices (real monthly labor income divided by the 4.3-fold of actual weekly working hours)

(c) *hours_phrf*: product of sampling weights and the 4.3-fold of actual weekly working hours

(d) *age*: respondent's age in years

(e) *skill*: indicator variable for ISCED-categories five and six

2. **Editing Extreme Values** Few—potentially spurious—outliers may substantially affect some statistics. In order to limit the impact of extreme values, income variables are typically truncated or winsorized. Therefore, calculate for each year the 1st and 99th percentile of the variable *hwage_10*, and generate the new variables

   (a) *hwage_10_truncated* by setting the values below the 1st and above the 99th percentile to missing, and

   (b) *hwage_10_winsorized* by replacing the values below the 1st and above the 99th percentile by the 1st and 99th percentile, respectively.
   Use the weights reported in *hours_phrf*.

1.5.2 The Evolution of Real Wages

Plot the evolution of truncated real hourly wages over the sample period for full-time employed men, and separately for the four age groups: 25–35, 35–45, 45–55, and 55–65. Use the weights reported in *hours_phrf*.

1.5.3 Wage Inequality

1. **The Skill Premium** The skill premium is perhaps the most prominent labor market wage premium. Therefore,

   (a) generate the new variable *log_hwage_10_truncated* as the logarithm of the variable *hwage_10_truncated*, and

   (b) calculate the log-wage premium enjoyed by male high-skill workers ($skill = 1$) relative to male low-skill workers ($skill = 0$) in each year and plot its evolution over time.
   Use the weights reported in *hours_phrf*.

2. **The Standard Deviation of Log-Wages** A popular and scale-invariant measure of wage inequality is the standard deviation of log-wages (discuss why):

   (a) Compute the standard deviation of full-time employed men's truncated hourly log-wages for each year separately, and plot its evolution over time.

(b) Regress—for each year separately—full-time employed men's truncated hourly log-wages on the skill indicator variable, and a second order polynomial in age. Plot the evolution of the standard deviation of the residuals over time.
Use in all calculations the weights reported in *hours_phrf*.

3. **The coefficient of variation** An alternative scale-invariant measure of wage inequality is the coefficient of variation (CoV).

    (a) Compute the CoV for the entire sample of fully employed individuals for each year separately and

    (b) plot its evolution over time.

4. **The Wage Distribution** In order to gain insights into the evolution of the entire wage distribution, complete the following three exercises:

    (a) Plot full-time employed men's truncated real hourly log-wages in a histogram against the density of the normal distribution separately for the years 1984–89 and the years 2002–07.

    (b) Calculate the differences between the wages at the *(i)* 90th and 50th percentile, *(ii)* 50th and 10the percentile, and *(iii)* 90th and 10th percentile for each year separately. Plot the evolution of the three statistics over time.

(c) For each percentile of the wage distribution, plot the change in wages between the years 1984–89 and 2002–07.
Use the weights reported in *hours_phrf*.

Part I
The origins of wage inequality

Wage income differs for many reasons

- education
- experience
- firm characteristics
- discrimination
- luck
- compare Mincer wage regressions as e.g. in (1.1)

We study the most important ones here – luck and human capital

2 Luck – the example of the pure search model

- Modern theories of unemployment: search and matching models ("Diamond-Mortensen-Pissarides models")
- Central concept: search
- Simplest (and most traditional) form: pure (one-sided) search
- This model also provides a first idea why wages differ for identical workers

2.1 Learning objectives

- Understand the simplest explanation of wage inequality
- Understand why people search
- Understand why people do not accept everything they find
- Understand determinants of search behaviour
- Understand why wage inequality persists when firms fix wages ("Burdett-Mortensen model")

2.2 Pure search

→ see Cahuc Zylberberg (2004, ch. 3)

2.2.1 The basic idea

- reason for search: lack of information about job availability and the wage paid per job
- setup
  - look at one unemployed worker
  - receives unemployment benefits
  - intensity of search is *not* chosen
  - can *not* look for another job once employed
  - stationary environment
- questions we can ask
  - which wage is accepted once an offer is made?
  - why do wages differ across individuals?

- unemployed does not know which wage will be offered once a job is found
- knows that all wages are drawn from the same (continuous cumulative) distribution $H(w)$ with density $h(w)$

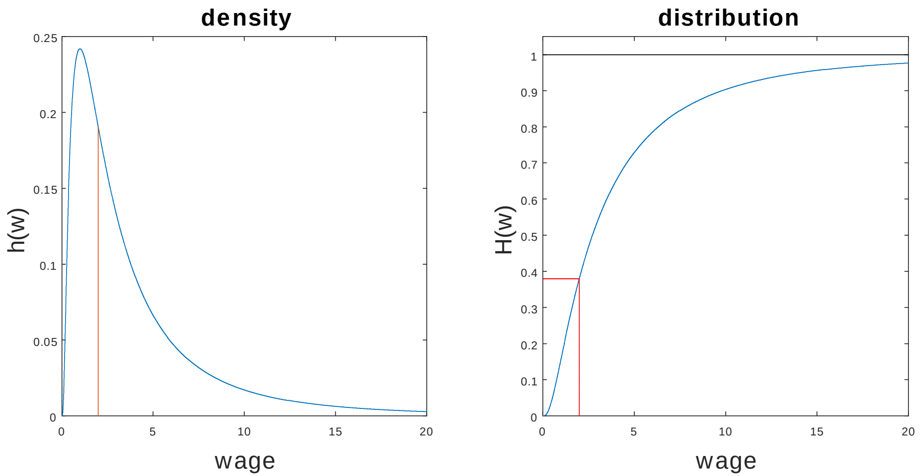
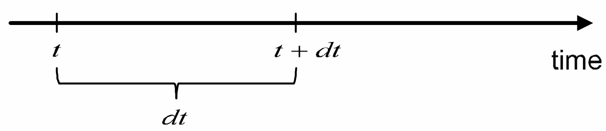


Figure 18 *Wage offer distribution: There is a random variable W with realization w (wage) and $Prob\,(W < w) = H(w) = \int_0^w h(w)\,dw$*

2.2.2 Expected utility once employed

- Worker is risk neutral
  - risk-neutrality means that the utility function is linear in income
  - here: utility function is given by real labour market income (wage or benefit)
- When employed the worker loses the job
  - at (separation) *rate* $s > 0$, meaning that
  - the *probability* to lose the job over period of time of length dt is given by sdt



  - (Poisson process in continuous time)
- real instantaneous interest rate r: invest a Euro in t and receive $1 + rdt$ Euro in $t + dt$
- discount factor of $\frac{1}{1+rdt}$ useful for computing present values

- This gives us value of being employed between t and $t + dt$

$$V_e = \frac{1}{1 + rdt} \left[wdt + (1 - sdt) V_e + sdt V_u \right]$$

where w is the instantaneous wage rate and V_u is the value of being unemployed and $(1 - sdt)$ is the probability to keep the current job

- (technically: this is heading towards a Bellman equation, V_e is the value function of an employed worker)

- rearrange this to make it simpler → exercise (and compare advanced macro)

$$rV_e = w + s \left[V_u - V_e \right]$$

(this is presentation in terms of classic Bellman equation)

- rewrite this for later purposes as

$$V_e (w) - V_u = \frac{w - rV_u}{r + s} \tag{2.1}$$

2.2.3 The optimal search strategy

- we assume job searcher only meets one employer at a time
- an offer consists of a fixed wage w
- choice between 'accept' or 'reject'
- optimality criterion: is V_e or V_u higher?
- accept $\Leftrightarrow V_e(w) > V_u$, which from (2.1) is the case if and only if $w > rV_u \equiv x$
- we have thereby defined the reservation wage x
- intuition why ever reject
  - Disadvantage from accepting a job consists in the inability to further look for jobs (as there is *no* on-the-job search)
  - Employee is stuck with wage w for a potentially long time
  - It might be better to reject and hope for better offer (with higher wage w)

2.2.4 Expected utility once unemployed

- Arrival rate of job offer: λ
- λ reflects labour market conditions, personal characteristics (age, educational background), effort (time and carefulness put into writing applications, not modeled here)
- Unemployment benefits b and opportunity costs of search c give instantaneous utility when unemployed, $z \equiv b - c$
- Value of receiving an offer

$$V_\lambda = \int_0^x V_u h(w)\,dw + \int_x^\infty V_e(w)\,h(w)\,dw$$

- Value of being unemployed over a period of length dt

$$V_u = \frac{1}{1 + rdt}\left(zdt + \lambda dt V_\lambda + (1 - \lambda dt)\,V_u\right)$$

- Rearranging (see exercise 2.4.3, question 2), we get the Bellman equation for unemployed worker

$$rV_u = z + \lambda \int_x^\infty \left[V_e(w) - V_u\right] h(w)\,dw$$

2.2.5 Reservation wage

- Last equation has intuitive interpretation, but hard to use for comparative statics
- But note that it is also an expression for the reservation wage $x = rV_u$
- After further steps (see exercise 2.4.3, question 4), we get final expression for the reservation wage x

$$x = z + \lambda \frac{\int_x^\infty (w - x)\, h(w)\, dw}{r + s} \tag{2.2}$$

- Interpretation as above for rV_u, apart from $r + s$ in denominator
  - $\frac{\pi}{r}$ is the present value (when discounting with r) of receiving income (profits) π forever (see tutorial)
  - $\frac{\pi}{r+s}$is the present value of receiving π as long as it randomly stops at rate s
  - hence $\frac{\int_x^\infty (w-x)h(w)dw}{r+s}$ is the present value of receiving a wage above x until exit rate s hits
  - z is received instantaneously as a flow and λ is the arrival of a job offer

2.2.6 Hazard rates and average duration in unemployment

- What is hazard rate (exit rate with which an individual leaves unemployment)?

$$\text{exit rate} = \lambda \left[1 - H\left(x\right)\right]$$

  where λ is the job offer rate and $1 - H(x)$ is the probability of accepting a job

- What is the average duration T_u in unemployment?

$$T_u = \frac{1}{\lambda \left[1 - H\left(x\right)\right]} \tag{2.3}$$

  (using a standard property of Poisson processes, duration is exponentially distributed)

- This allows for many policy analyses concerning determinants of unemployment rate (duration in unemployment to be more precise)

2.2.7 Equilibrium wage distribution

- Why is there a distribution of wages?
  - All individuals have the same reservation wage
  - Individuals accept all wages above the reservation wage
  - Some are more lucky than others and therefore earn higher wages
- Equilibrium wage distribution plotted on next slide

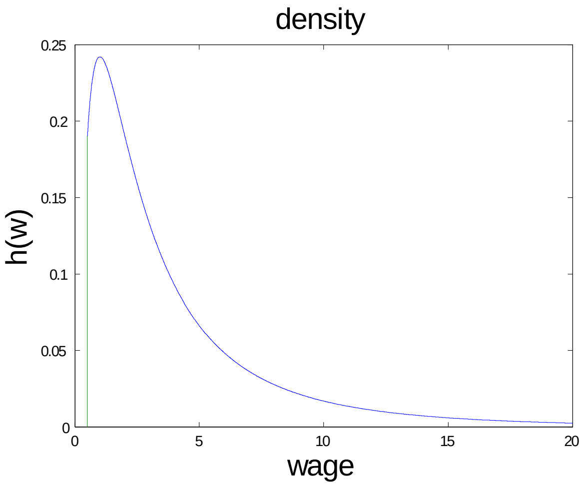


Figure 19 *Equilibrium wage distribution for a reservation wage of* $x = 0.5$

2.3 Conclusion

- How to reduce unemployment?
  - Question should be rephrased
    - this is not an equilibrium model
    - there is no unemployment rate
  - How to reduce average duration in unemployment?

$$T_u = \frac{1}{\lambda \left[1 - H\left(x\right)\right]}$$

- Simple answer
  - Reduce reservation wage x
  - Then: probability of accepting an offer $1 - H\left(x\right)$ rises and T_u falls
  - (but be aware of λ)

- How to reduce the reservation wage x?
  - Unemployment benefits z and job offer rate λ need to go down
  - Interest rate r and separation rate s need to go up
  - λ, r and s pretty exogenous to policy (but think of short-time work)
  - z goes down if lower unemployment benefits are paid
  - Unemployment would fall if reservation wage falls (besides for λ)
- The ambiguous effect of the job offer rate λ in reducing unemployment
  - A higher λ reduces duration (2.3) in unemployment directly
  - A higher λ increases the reservation wage and probability $1 - H(x)$ of accepting an offer goes down
- How to reduce wage inequality?
  - No answer from this model
  - [one could increase the reservation wage but this leads to unemployment]
  - Theory of origin of wage distribution needed

2.4 Exercises

2.4.1 The Leibniz Rule

The Leibniz rule for integrals (see, e.g., Wälde, 2012, pp. 93) reads

$$\frac{d}{dt}\left(\int_{a(t)}^{b(t)} f(x,t)dx\right) = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t}(x,t)dx - \frac{da}{dt}(t)f(a(t),t) + \frac{db}{dt}(t)f(b(t),t).$$

Apply the Leibniz Integral Rule to compute the following derivatives

1. $\frac{d}{dy}\int_a^y f(s)ds$,
2. $\frac{d}{dy}\int_a^y f(s,y)ds$,
3. $\frac{d}{dy}\int_a^b f(y)dy$,
4. $\frac{d}{dy}\int f(y)dy$.

Show that the integration by parts formula $\int_a^b \dot{x}ydt = [xy]_a^b - \int_a^b x\dot{y}dt$ holds.

2.4.2 Applications of the implicit function theorem

1. Prove that the expression $x^2 - xy^3 + y^5 = 17$ is an implicit function of y in terms of x in a neighborhood of $(x, y) = (5, 2)$.

2. Then estimate the value of y which corresponds to $x = 4.8$.

3. Consider $3x^2yz + xyz^2 = 30$ as defining x as an implicit function of y and z around the point $(x, y, z) = (1, 3, 2)$.

(a) If y increases to 3.2 and z remains at 2, use the Implicit Function Theorem to estimate the corresponding x.

(b) Use the quadratic formula to solve $3x^2yz + xyz^2 = 30$ for x as an explicit function of y and z. Use approximation by differentials on this explicit formula to estimate x when $y = 3.2$ and $z = 2$.

(c) Which way was easier?

2.4.3 Job Search and the Reservation Wage

(Cahuc and Zylberberg, 2004) Unemployed workers enjoy an net instantaneous income of $z > 0$. They receive job-offers according to a Poisson process at rate $\lambda > 0$. The wage, w, associated with a job-offer is randomly drawn from the continuously differentiable distribution $H(\cdot)$, which is assumed to have finite first moments. Jobs are destructed according to a Poisson process at rate $q > 0$. Let V_u denote the unemployed worker's discounted expected utility, and $V_e(w)$ that of an employed worker who receives the wage w. The parameter r denotes the exogenous real instantaneous rate of interest.

1. Derive the employed worker's discounted expected utility, V_e, as a function of the wage, w, the job destruction rate, q, the instantaneous interest rate, r, and the unemployed worker's discounted lifetime utility, V_u. *Hint:* As it is standard in continuous-time analysis, use the concept of the infinitesimal time period $dt > 0$. The eventual expression for the employed worker's discounted expected utility, V_e, will be that that emerges in the limit as $dt \to 0$.

2. Derive the unemployed worker's discounted expected utility, V_u, as a function of the net instantaneous income, z, the job-offer arrival rate, λ, the instantaneous interest rate, r, and the employed worker's discounted lifetime utility, V_e.

3. Show that the unemployed worker's optimal behavior exhibits the reservation wage property, i.e., there is a wage x, such that unemployed workers reject all job-offers associated with lower wages and accept all job-offers associated with higher wages.

4. Derive an expression for the reservation wage, x, that solely depends on the model's primitive parameters, i.e., you need to substitute for the employed and unemployed workers' discounted expected utility V_e and V_u, respectively.

5. How does the reservation wage, x, depend on the model's primitive parameters?

2.4.4 Properties of the reservation wage

1. Understand the effect of higher z

   (a) Consider the equation for the reservation wage (2.2) and compute dx/dz (using implicit function theorem - see exercise 2.4.2 and Leibniz' rule - see exercise 2.4.1)

   (b) Provide an intuition for this result

   (c) Consider the equation for duration in unemployment (2.3) and compute dT_u/dz

   (d) Provide an intuition for this result

2. Understand the effect of job offer rate λ (same steps as for z - but be careful for T_u (ambiguous effect))

3. Understand the effect of r (same steps as for z)

4. Understand the effect of separation rate s (same steps as for z)

2.4.5 Optimal consumption and labour

Assume that the household's preferences for consumption c and leisure l are represented by the following utility function

$$U(c, l) = \left[\gamma c^{\theta} + (1-\gamma) l^{\theta}\right]^{1/\theta}, \quad \theta < 1 \tag{2.4}$$

The household faces a nominal budget constraint described as

$$pc = (\bar{l} - l) w^{\text{nominal}} \tag{2.5}$$

where p, $\bar{l}$, and w^{nominal} are prices of consumption good, time endowment, and nominal wages, respectively.

1. Set up the household's utility maximization problem and derive the optimality conditions.
2. Derive the optimal consumption. (You may define and use $w = \frac{w^{\text{nominal}}}{p}$ for convenience.)
3. Derive the optimal leisure.
4. The elasticity of substitution is defined as $\varepsilon =: \frac{d\ln(c/l)}{d\ln(U_l/U_c)}$. Compute ε.

2.4.6 Wage elasticity of labour supply

Assume the firm's production function is of a Cobb-Douglas form

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{2.6}$$

A and K are assumed to be unchanged in the short run. Assume that the firm is a monopsony on the factor market.

1. Write down the firm's profit function and set up its profit maximization problem.

2. Derive the optimality condition and verify that

$$Y'(L(w)) = w\left[1 + \frac{1}{\eta(w)}\right]$$

   where $\eta(w) \equiv \frac{dL(w)}{dw}\frac{w}{L(w)}$.

3 Luck and endogenous wage distributions

3.1 Learning objectives

- The pure search model "explains" wage distributions by luck
- There is wage inequality as there is a wage offer distribution (see fig. 18)
  - Where does this wage offer distribution come from?
  - Pure search model does not provide an answer
  - There is wage inequality *because* [of an exogenous reason]
  - Not a full answer
- Could we think of an explanation why there is a wage offer distribution (as an equilibrium outcome)?
- Could it be that there is *unequal* pay for *equal* workers?
- Let us consider one of the most important analyses in modern labour economics
- The model by Burdett and Mortensen (1998) provides such an answer

3.2 The basic argument

- The workers
  - There is a continuum of *identical* workers
  - Workers can be unemployed or employed (like in pure search model)
  - Workers can search on-the-job (unlike in pure search model) and find a higher-paying job
  - There is a wage-offer distribution (like in the pure search model)
- The firm side
  - Crucial new aspect of Burdett-Mortensen model: it explains wage-offer distribution
  - There is a continuum of *identical* employers
  - Employers maximize expected profits (as the number of workers is uncertain)
  - Property of profit maximization: Firms do not offer identical wages!

- Why do firms offer different wages (and we obtain an endogenous wage offer distribution)?
  - Optimal behaviour of one firm depends on behaviour of other firms
    - A firm offering a wage w experiences a job-acceptance probability by a worker also as a function of the wages offered by other firms
    - There is a strategic interaction between firms
  - If all firms offered identical wages, it would pay to increase wage by tiny amount
    - A tiny increase increases the probability of acceptance of a wage offer by a worker to 1
    - This increases profits
  - Why do not all firms offer zero-profit wage?
    - A tiny reduction would also increases profits
    - It always pays to depart from any wage offered by a group of firms (no mass-points)
  - It can be shown that a continuous wage offer distribution results (see slides and references therein)

3.3 The model [towards end of lecture]

3.3.1 The setup

- There is a continuum of *identical* workers and employers
- The number of employers is normalized to 1 and the measure of workers is denoted by m
- A worker can be in two mutually independent states, i.e. unemployed (denoted 0) and employed (denoted 1)

- A worker switches between those two states following two Poisson processes
  - With an arrival rate of λ_0, the worker receives a job offer (i.e. meets a firm as in the pure search model)
  - With an arrival rate of δ, an employed worker loses her job
  - (The notation is from Burdett and Mortensen, 1998)
- Letting workers randomly search among employers, receiving an offer is modelled by a draw of a wage w from a "wage-offer" distribution $F(.)$
- There is on-the-job search
  - with an arrival rate of λ_1, the worker receives an offer while employed
  - big difference to labour market models seen so far
- When unemployed, the worker earns unemployment benefits b
- Workers are risk-neutral and discount future income at a rate r

3.3.2 Bellman equations

- The Bellman equation for the unemployed worker (see exercise 3.6.1)

$$rV_0 = b + \lambda_0 \left[\int_{-\infty}^{\infty} \max \left\{ V_1(x), V_0 \right\} dF(x) - V_0 \right] \tag{3.1}$$

- How can this equation be understood?
  - The value V_0 of being unemployed equals benefits b plus the expected gain for a successful match, i.e. job that pays a sufficiently high wage
  - The wage is high enough if the value $V_1(x)$ of accepting a job paying a wage x is higher than the value V_0 of being unemployed
  - A job offer therefore only implies a gain if $\max \left\{ V_1(x), V_0 \right\} = V_1(x)$
  - Why use the max-operator?
    * This is an alternative to computing the probability that x is above the reservation wage
    * Using such a probability, equation could be expressed as in the pure search or the matching model (see exercise 3.6.1, question 2)

- The Bellman equation for a worker earning a wage w

$$rV_1(w) = w + \delta\left[V_0 - V_1(w)\right] + \lambda_1 \left[\int_{-\infty}^{\infty} \max\left\{V_1(x), V_1(w)\right\} dF(x) - V_1(w)\right] \tag{3.2}$$

- Understanding the equation
  - The terms before λ_1 are familiar
    * the wage w and
    * the expected loss from losing the job at rate δ
  - The λ_1 term captures the expected gain from a job offer
  - This gain is positive
    * if $V_1(x) > V_1(w)$, i.e. only
    * if the wage x of the offer exceeds the wage w of the current job

3.3.3 The reservation wage and unemployment

- The reservation wage for unemployed workers
  - The reservation wage R is defined by $V_1(R) = V_0$
  - In words, accepting a job paying the reservation wage yields the same value as staying unemployed (see exercise 3.6.1, question 4 for a comparison with the pure-search reservation wage in sect. 2.2.3)
  - All wage offers at or above R are accepted by unemployed workers
- Using the definition $\overline{F}(x) \equiv 1 - F(x)$, the reservation wage becomes (see exercise 3.6.1, question 4)

$$R = b + (\lambda_0 - \lambda_1) \int_R^\infty \frac{\overline{F}(x)}{r + \delta + \mu_1 \overline{F}(x)} dx \tag{3.3}$$

- The reservation wage depends on
  - unemployment benefits b
  - the arrival rate λ_0 and λ_1 (actually only on their difference)
  - properties of the wage distribution as captured by $F(x)$ in integral
- Is there a reservation wage for employed workers? (It is the current wage w)

- The unemployment rate
  - The unemployment rate is given (again) by an equation fixing inflows and outflows

$$\delta\left[m-u\right]=\lambda_0\left[1-F\left(R\right)\right]u$$

  - Inflows result from the separation rate δ times the measure $m-u$ of employed workers
  - Outflows from unemployment are given by the job offer rate λ_0 times the probability $1-F\left(R\right)$ that an offer is accepted times the measure u of unemployed workers
  - Solving for the unemployment rate, we find

$$\frac{u}{m}=\frac{\delta}{\delta+\lambda_0\left[1-F\left(R\right)\right]} \tag{3.4}$$

3.4 Wage inequality [towards end of lecture]

3.4.1 Endogenous wage distributions – supply side

- So far, we have an exogenous wage-offer distribution $F(w)$
- How does the long-run stationary distribution of wages $G(w)$ in this economy look like?
  - One would expect this distribution $G(w)$ to differ from $F(w)$ as there is a reservation wage
  - Even in the long-run, however, this distribution $G(w)$ differs from truncated $F(w)$
- We start by looking at a surprisingly simple ordinary differential equation for $G(w,t)$
  - Remember that the number of workers receiving a wage equal or smaller than w is given by $G(w,t)[m-u(t)]$
  - The change in this number follows

$$\frac{d}{dt}G(w,t)[m-u(t)]$$
$$= \lambda_0 \max\{F(w)-F(R),0\} u(t) - [\delta + \lambda_1 [1-F(w)]] G(w,t)[m-u(t)]$$

  - This equation follows the usual inflow-outflow logic seen earlier in other matching models (see next figure)

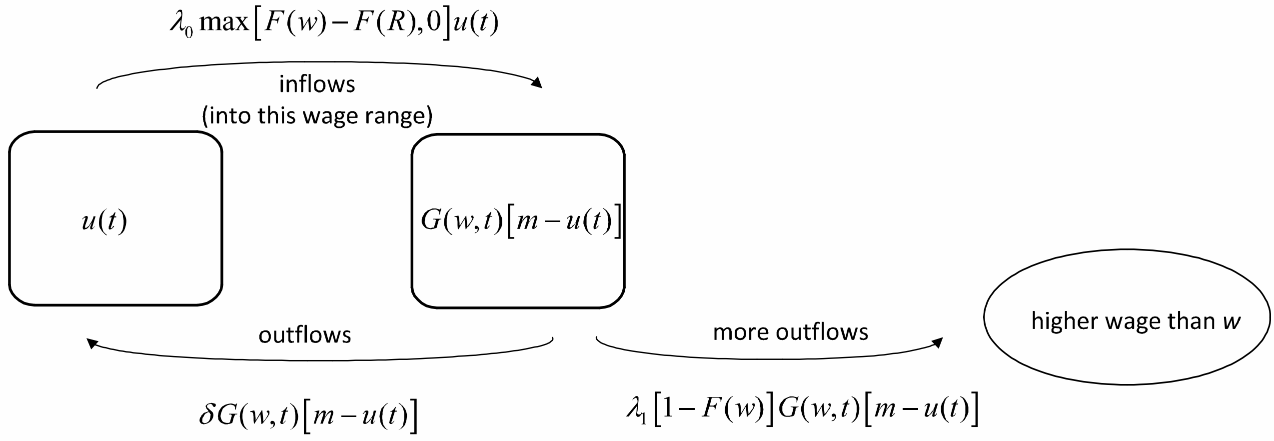


Figure 20 *Inflows into and outflows from the wage range below or equal to* w *(Example:* $G(w,t) = 40\%$*,* $m - u(t) = 50$ *million,* $G(w,t)[m-u(t)] = 20$ *million)*

To understand this equation, we split it into $w > R$ and $w \leq R$

- The case $w > R$
  - The number of workers receiving a wage $(R, w]$ changes according to

$$\begin{aligned}&\frac{d}{dt}G(w,t)[m-u(t)] \\ &= \lambda_0 [F(w) - F(R)] u(t) - [\delta + \lambda_1 [1 - F(w)]] G(w,t)[m - u(t)]\end{aligned}$$

  - The inflow consists of the number $u(t)$ of unemployed workers times the arrival rate λ_0 of job offers times the probability $[F(w) - F(R)]$ that the offer lies between R and w
  - The outflows are into unemployment or into higher paying jobs

- The case $w \leq R$

$$\frac{d}{dt}G(w,t)[m-u(t)] = -[\delta + \lambda_1 [1 - F(w)]] G(w,t)[m - u(t)]$$

  - We see that there are outflows only (as no one would accept jobs below R)
  - In the long-run, there are no workers earning a wage $w < R$

- The long-run equilibrium wage distribution (see exercise 3.6.1, question 6)

$$G(w) = \frac{[F(w) - F(R)] / [1 - F(R)]}{1 + \frac{\lambda_1}{\delta}[1 - F(w)]} \tag{3.5}$$

  - For $\lambda_1 = 0$ (no on-the-job offers), this is the truncated wage offer distribution
  - For a positive λ_1, the denominator is larger than 1 and the share of workers at wage w or lower becomes smaller: On the job searchers moves the probability mass to the right (see next figure)

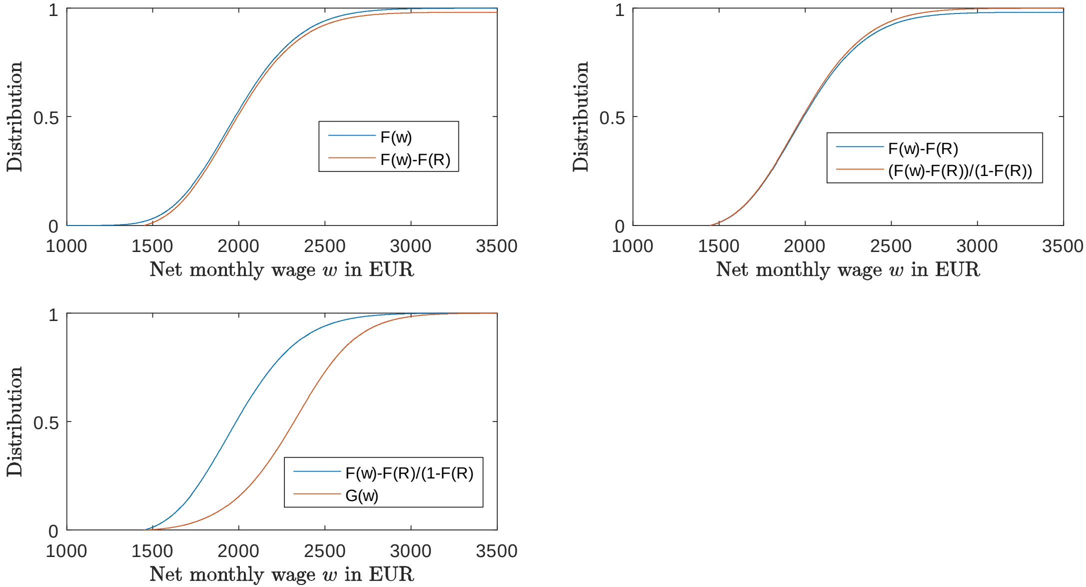


Figure 21 *Long-run equilibrium wage distribution* $G(w)$ *with exogenous wage offer distribution* $F(w)$

3.4.2 Endogenous wage distributions – demand side

- So far, we have a technically and economically fascinating result
  - We can characterise the long-run endogenous wage distribution in an economy with on the job search analytically
  - We obtain a result that "equal workers get unequal pay"
  - It is a pure question of luck whether someone is "rich" (i.e. earns a high wage) and someone else is not
- Two "buts"
  - It is an obvious statistical property of any random variable that there are distributions: When two people throw the same die, the probability that they have different numbers (i.e. earn different wages) is 5/6
  - The crucial question is therefore: can we imagine firm behaviour such that jobs with different wages are offered for identical workers?

- Optimal firm behaviour in the Burdett-Mortensen model
  - We first study optimal firm behaviour in a game consisting of two stages
  - This allows to understand the intuition in a simpler framework than in the full Burdett-Mortensen model
  - (this follows Mortensen, 2003, ch. 1.2.1)
- The basic structure
  - There are m employers and n workers
  - A match produces output p, unemployment benefits are $b < p$
  - In the first stage, each firm makes one wage offer communicated to a randomly drawn worker
  - This captures the idea that these offers are not costless (opening a vacancy, conducting interviews ...)
  - In the second stage, workers accept offers
  - When workers receive more than two offers, they take the highest-paying one
  - The probability that the firm gets a worker depends on (i) the wage offered and (ii) the wages offered by other firms $\Rightarrow$ this is a noncooperative game

- Probabilities
  - The number of offers a worker gets is (approximately) Poisson distributed
  - Why?
- [Digression] Binomial distribution (see e.g. Evans et al., 2000)
  - When one trial yields success with probability p and failure with $1-p$
  - when this trial is undertaken n times and
  - when X counts the number of successes
  - then X is distributed according to the binomial distribution

- [Digression - cont'd] Binomial distribution
  - Let X be binomially distributed
  - Then the probability of x successes is

$$\text{Prob}\,(X = x) = \begin{pmatrix} n \\ x \end{pmatrix} p^x q^{n-x}$$

    where

$$\begin{pmatrix} n \\ x \end{pmatrix} = \frac{n!}{x!\,(n-x)!}$$

    is the binomial coefficient and
  - where $x!$ (the "factorial of x", "x Fakultät" in German) is

$$x! = x \times [x-1] \times [x-2] \times ... \times 3 \times 2 \times 1$$

  - The tutorials shows some plots of the binomial probabilities
  - An example is an exam with n participants and probability of success p. The number of students that pass is (before marking) then binomially distributed. (After marking, the number is known, uncertainty is revealed.)

- How many offers does a worker receive?
  - When one firm makes one offer, it reaches a given worker with a probability $1/n$ (as there are n workers)
  - When m employers make one offer each, each worker has m chances to get an offer
  - The number of offers is therefore binomially distributed (see tutorial) – which can be approximated by a Poisson distribution with parameter $\lambda = m/n$ when n and m are large
  - A random variable X is Poisson distributed with parameter $\lambda > 0$ when the support are the non-negative integers, $X \in [0, 1, 2, ..., \infty[$ and the probability of $X = x$ is given by (see e.g. Evans et al., 2000)

$$\text{Prob}\,(X = x) = \lambda^x \frac{e^{-\lambda}}{x!}$$

  - The parameter $\lambda = m/n$
    * gives the expected number of offers per worker
    * rises in the number of firms and
    * falls in the number of workers

- Expected profits of a firm and wage choice
  - Imagine a firm considers offering a wage w
  - The probability that all other offers are below (or equal to) this offer is given by the wage offer distribution, i.e. by $F(w)$
  - Note that $F(w)$ can be seen as representing the rank of the wage w in the wage offer distribution
  - Assume (to be seen in a moment) that the probability of accepting an offer by a worker is a function of this rank and the expected number of offers

$$\text{Prob}\,(\text{acceptance of } w) = P\left(F\left(w\right),\lambda\right)$$

  - Then expected profits of a firm that offers w are given by

$$\pi = P\left(F\left(w\right),\lambda\right)\left(p-w\right)+\left(1-P\left(F\left(w\right),\lambda\right)\right)\times 0$$

  - Which wage should the firm optimally choose?

- Properties of optimal choices of firms
  - Firms do *not* choose the same wage
    - Imagine they did - then paying an ε more would lead to an acceptance of an offer by a worker with certainty
    - Expected profits of firms are higher when paying $w+\varepsilon$ as

$$p-(w+\varepsilon)>P(F(w),\lambda)(p-w)$$

as $P(F(w),\lambda)<1$ and as ε can be arbitrarily small
  - The same argument can be made for a *share* of firms
  - The same argument can be made for a deviation downwards if all firms offered $w=p$
  - This implies that there are no mass-points in the distribution (see next figure for illustration)

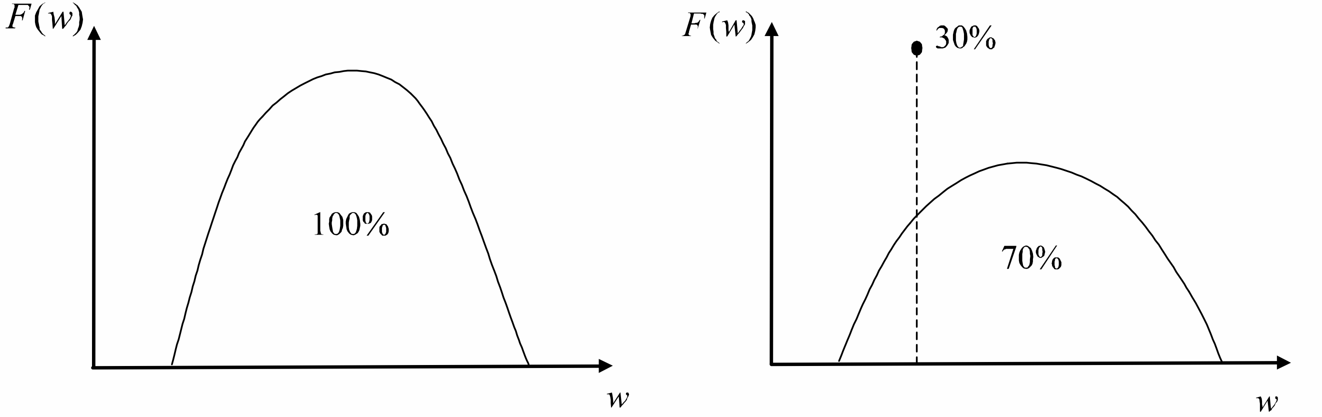


Figure 22 *Density without (left) and with masspoint*

- Endogenous wage offer distribution
  - It can also be shown (again, see Mortensen, 2003, ch. 1.2.1) that
    - there are no gaps in the wage offer distribution
    - the lower bound of the distribution is b and that
    - the upper bound is $\left(1-e^{-\lambda}\right)p+e^{-\lambda}b$ and thus strictly smaller than p
  - Finally, it can be shown that the endogenous wage offer distribution reads

$$F\left(w\right)=\frac{1}{\lambda}\log\left(\frac{p-b}{p-w}\right)$$

- Implication
  - Given these findings, firms offer different wages in equilibrium
  - Firms with a higher wage
    - have lower profits per worker
    - have more workers
    - These effects just cancel in the (expected) profit function of a firm such that $\pi(w_1) = \pi(w_2)$ for $w_1 \neq w_2$
  - Equal workers get unequal pay!

- Does this wage dispersion equilibrium hold for more standard structures?
  - Burdett and Mortensen (1998) obtain this finding in a dynamic framework (yet solved for a steady state)
  - Coles (2001) constructs a wage-dispersion equilibrium under different (more complicated?) assumptions
  - Postel-Vinay and Robin (2002) do not have full wage dispersion (with homogenous workers and firms)
  - Cahuc, Postel-Vinay and Robin (2006) do not have full wage dispersion (with homogenous workers and firms)
  - To be fair, they have a degenerate two-wage distribution
    - First-time jobs after unemployment pay one wage
    - Jobs obtained when already having a job pay another wage

3.5 Conclusion

- When thinking about wage distributions, take the effect of observable characteristics into account
  - individual productivity
  - individual choice (e.g. hours worked)
  - firm & regional characteristics
  - policy
- Is there more than observables?
  - As wage regressions (using all available worker and firm characteristics) do not explain all variation in wages, some source of (unobservable) luck seems appropriate
  - One can follow the Burdett-Mortensen setup or one can
  - Go for stochastic match quality
    * This can take the form of one initial draw (as seen in Pissarides, 1985, model)
    * One can also allow for repeated draws (as in models with endogenous separation rates following Mortensen and Pissarides, 1994)
  - In both cases we get unequal pay for equal workers

3.6 Exercises [towards end of lecture]

3.6.1 On-the-Job-Search (Burdett & Mortensen, 1998)

Assume there are a continuum of workers of a measure m and a continuum of employers of a measure 1. At any point in time, a worker is either employed or unemployed. Unemployed workers receive permanent wage offers according to a Poisson process at rate λ_0, and employed workers receive wage offers at rate λ_1. Wage offers are randomly drawn from F, the distribution of wage offers across employers. The productivity of a worker–employer match is p. The match is exogenously dissolved at rate δ. Unemployed workers enjoy a flow income of b, and employed workers obtain the wage w offered by their employer. V_0 is the worker's lifetime value of being unemployed. $V_1(w)$ is the worker's lifetime value of being employed at a wage of w. The time preference rate is denoted by r.

1. Discuss the Bellman equations for unemployed workers (3.1) and for employed worker (3.2).

2. Compare the Bellman equation for unemployed workers with the one in problem 2.4.3, i.e. write (3.1) without the max-operator.

3. Compare the pure-search reservation wage x in sect. 2.2.3 with the reservation wage R.

   (a) What is the difference between them, what is the same?

(b) Consider both the definition and the expressions in (2.2) and (3.3).

(c) Can the reservation wage R be lower than unemployment benefits? Would this make sense?

4. It is optimal for employed workers to switch to a new employer if the offered wage exceeds the current one. Similarly, unemployed workers' optimal behavior also exhibits the reservation wage property. Derive the unemployed workers' reservation wage, R.

5. For a given wage offer distribution, $F(\cdot)$, how does the reservation wage, R, depend on the offer arrival rates, λ_0 and λ_1, the job destruction rate, δ, and the level of unemployment benefits, b? Assume that $\lambda_0 > \lambda_1$.

6. Derive the steady sate mass of unemployed workers, u, and the long-run equilibrium wage distribution $G(w)$.

7. Derive the firm size, $l(w|R, F)$, as a function of the wage, w, conditional on the reservation wage, R, and the wage offer distribution, $F(\cdot)$.

8. Consider the steady state equilibrium that arises as the time preference rate, r, tends to zero. Derive the wage offer function, $F(\cdot)$, and the reservation wage, R, as functions of the model's primitive parameters.

3.6.2 The binomial distribution

To understand the two-stage model yielding a wage distribution, remind yourself of the properties of the binomial distribution.

4 Education and experience – Human capital

4.1 Learning objectives

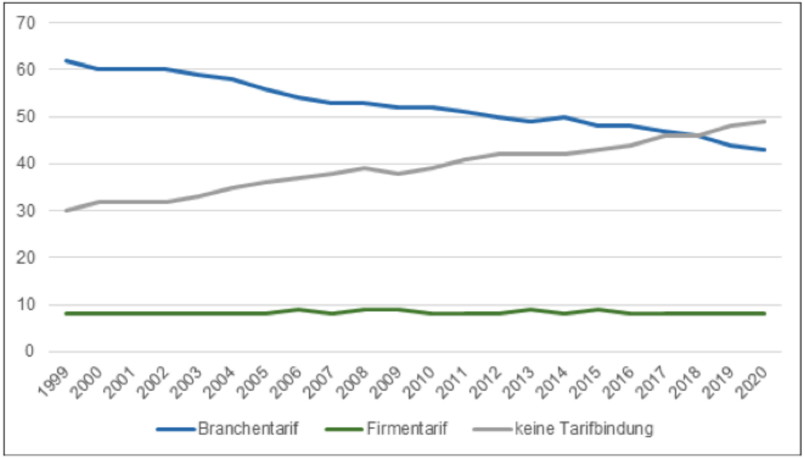
- Why does not everybody earn the same wage?
  - Do we talk about hourly wage?
  - Is it monthly wage we are interested?
  - Why wage distributions (and not income, wealth, consumption, happiness or utility)?
- Hourly wages differ as employees differ
  - What is the role of luck?
  - What is the role of choice?
  - What is the role of individual characteristics?
- What is the role of policy?

4.2 Basics of wage distributions

4.2.1 The institutional background

- A framework to think about determinants of wage distributions needs to be clear about the wage setting mechanism
- Empirically and institutionally speaking, how are wages being fixed?
  - market
  - individual bargaining
  - collective bargaining
  - by law (minimum wage)

Beschäftigte in tarifgebundenen Betrieben, 1999--2020, in Prozent



Quelle: IAB-Betriebspanel 1999–2020, hochgerechnete Werte.

Figure 23 *The share of wages set by the market ("keine Tarifbindung"), collective bargaining at the sectorial level ("Branchentarif") and by firm-level bargaining ("Firmentarif")*

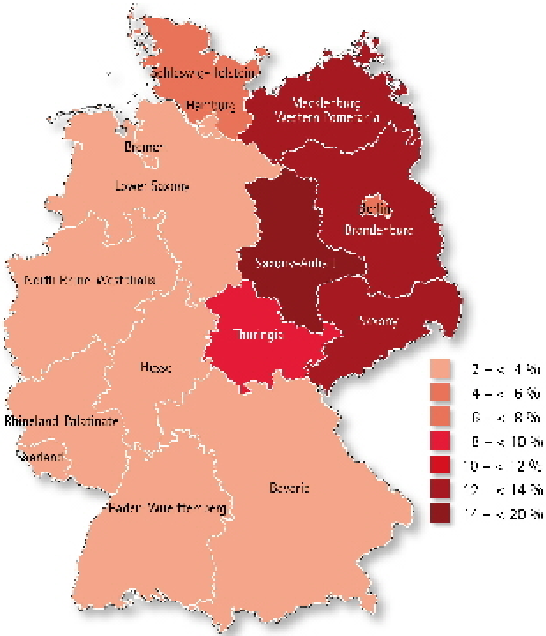


Figure 24 *Share of employees that earned less than 8.50 EUR/hour before 2015 (Source: Bellman et al., 2015)*

4.2.2 A simple but relevant structure

- The basics of wage distributions
- What are determinants for wage distributions in a simple example?
  - we assume the wage is given by the market clearing wage
  - we are interested in distributions of *net* monthly wages (as this is closer to individual well-being than gross wage)

- Setup
  - Technology $Y = Y(K, L)$ with standard properties (concave, constant returns to scale)
  - capital input K
  - labour input L in *efficiency* units $L = \Sigma_{i=1}^{N} h_i l_i$
    * h_i is hours worked per *month*
    * l_i is productivity of individual i
    * N is size of labour force (head count)
  - Profit function of firm $\pi = pY - wL$
    * p is price of output good and
    * w is total wage cost per *efficiency* unit
  - total wage cost w
    * gross wage plus
    * employers' contribution to social security system (pension system, unemployment insurance, health insurance plus some other)

- The wage distribution
  - Optimal labour input choice by firm: value marginal product per *efficiency* unit of labour equals unit wage cost

$$p\frac{\partial Y}{\partial L} = w$$

  - Given an average labour income tax τ (including social security contributions), net wage of household (income per *month*) is

$$w_i^{\text{net}} = (1 - \tau)\, h_i l_i w \tag{4.1}$$

  - There is a distribution of monthly nominal net wages because of distributions of
    * average taxes τ e.g. across countries (policy matters)
    * hours worked h_i (choice matters)
    * individual productivity l_i (luck and individual characteristics matter)
    * firm characteristics like productivity and capital stock (all in technology Y)
    * see Card et al. (2013) for evidence for Germany (see also Drechsel-Grau et al. 2022)

4.3 Conclusion

- Let us summarize
  - this chapter but also
  - the first part and
  - introductory chapter before we move on to part II
- Monthly labour income differs across individuals (at full time employment)
  - Differences are considerable in levels and an increase in inequality (in GSOEP data) can be observed
  - Where do these differences come from?
- Two fundamentally different answers can be given
  - Wages differ by chance
  - Wages differ because of differences in productivity (human capital)

- Wages differ by chance as ...
  - ... workers search for jobs and firms search for workers
  - ... firms find it optimal to differentiate their wage from the wages from competitors (despite identical workers and identical firms)
- Wages differ due to differences in individual productivity as ...
  - ... individuals differ in their human capital level (education, experience) and as
  - ... firms reward these differences when paying marginal productivities as real wages
- All of this are studies of pre-tax wages (or wages with an invariant tax rate)
- Let us imagine, pre-tax wage distributions are considered to be too unequal (by whatever measure of justice or equality)
  - What can be done?
  - Let us move on to next part

Part II

Redistribution

5 The basics of positive tax theory

5.1 Learning objectives

- What are taxes (marginal, average)?
- How do taxes affect individual labour income?
- How is income redistributed via the tax system?
- How does the tax system affect the monthly net wage distribution?
- [We (need to) ignore expenditure side of fiscal policy]

5.2 Some principles and two tax schedules

- Labour tax
  - The net wage w^{net} is a function of the gross wage w and of the tax rate $\tau(w)$
  - Think of the gross wage w as the annual gross labour income
  - The taxrate $\tau(w)$ is itself a function of the gross wage
  - The net wage is
$$w^{\text{net}} = (1 - \tau(w))\, w$$

- Average and marginal tax rates
  - Average tax rate
$$\frac{\text{total income tax paid}}{\text{total income (tax base)}} = \frac{\tau(w)\, w}{w} = \tau(w) \tag{5.1}$$
  - Marginal tax rate: how does total tax payments change when income rises?
$$\tau^{\text{marg}} \equiv \frac{d \text{ total income tax paid}}{d \text{ income}} = \frac{d\left[\tau(w)\, w\right]}{dw} = \tau'(w)\, w + \tau(w) \tag{5.2}$$

- Progressive tax system
  - A tax system is (by definition) progressive when the average tax rises in the tax base
  - For our setup, labour income taxation is progressive when

$$\frac{d\tau(w)}{dw} > 0$$

- Two tax schedule
  - Piecewise constant marginal tax rates ("US-style") – defined via *marginal* tax rates

$$\tau^{\text{marg}}(w) = \left\{ \begin{array}{c} \tau_1^{\text{marg}} \\ \tau_2^{\text{marg}} \\ \vdots \\ \tau_n^{\text{marg}} \end{array} \right\} \text{ for } \left\{ \begin{array}{c} w \leq w_1 \\ w_1 < w \leq w_2 \\ \vdots \\ w_{n-1} < w \end{array} \right\} \tag{5.3}$$

  - Continuous marginal tax rates ("German-style") – defined via *average* tax rate of

$$\tau(w) = \tau_0 + \tau_1 w \tag{5.4}$$

5.2.1 Properties of piecewise constant marginal tax rates

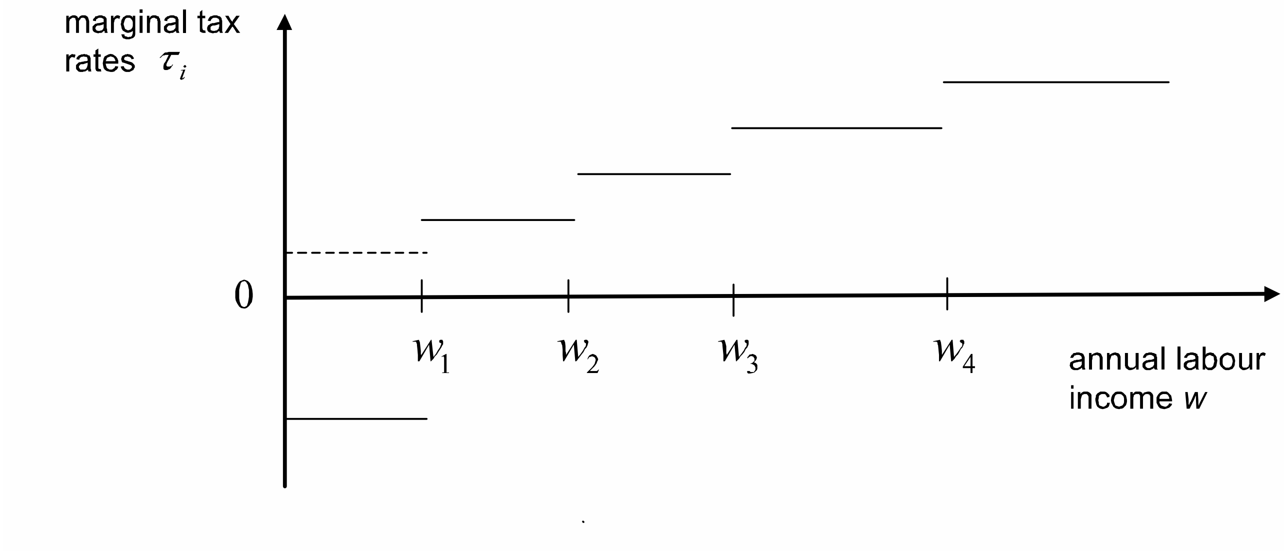


Figure 25 *Marginal tax for "US-style" tax system (5.3) with a negative or positive (or zero) marginal tax rate for* $w < w_1$

- Tax paid
  - Consider the example of a system with three tax brackets

$$T(w) = \left\{ \begin{array}{l} \tau_1^{\text{marg}} w \\ \tau_1^{\text{marg}} w_1 + \tau_2^{\text{marg}} [w - w_1] \\ \tau_1^{\text{marg}} w_1 + \tau_2^{\text{marg}} [w_2 - w_1] + \tau_3^{\text{marg}} [w - w_2] \end{array} \right\}$$

$$\text{for tax bracket} \left\{ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \right\} \text{ i.e. for } \left\{ \begin{array}{c} w \leq w_1 \\ w_1 < w \leq w_2 \\ w > w_2 \end{array} \right.$$

  - Obviously, this can be extended to more tax brackets

- Some properties (compare fig. 26)
  - If $\tau_1^{\text{marg}} < 0$, the tax paid falls in income w
  - There is a break-even income level $\hat{w}$ where $T(w) = 0$
  - Above $\hat{w}$, tax paid rises in income w

- Average tax rate

$$\tau(w) = \frac{T(w)}{w} = \left\{ \begin{array}{l} \tau_1^{\text{marg}} \\ \frac{\tau_1^{\text{marg}} w_1 + \tau_2^{\text{marg}} [w - w_1]}{w} \\ \frac{\tau_1^{\text{marg}} w_1 + \tau_2^{\text{marg}} [w_2 - w_1] + \tau_3^{\text{marg}} [w - w_2]}{w} \end{array} \right\} \text{ for tax bracket } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right.$$

  - Average tax differs from bracket to bracket
  - It is a constant in the first bracket, does not rise *between* brackets but rises in tax base w *within* higher brackets (see next figure)

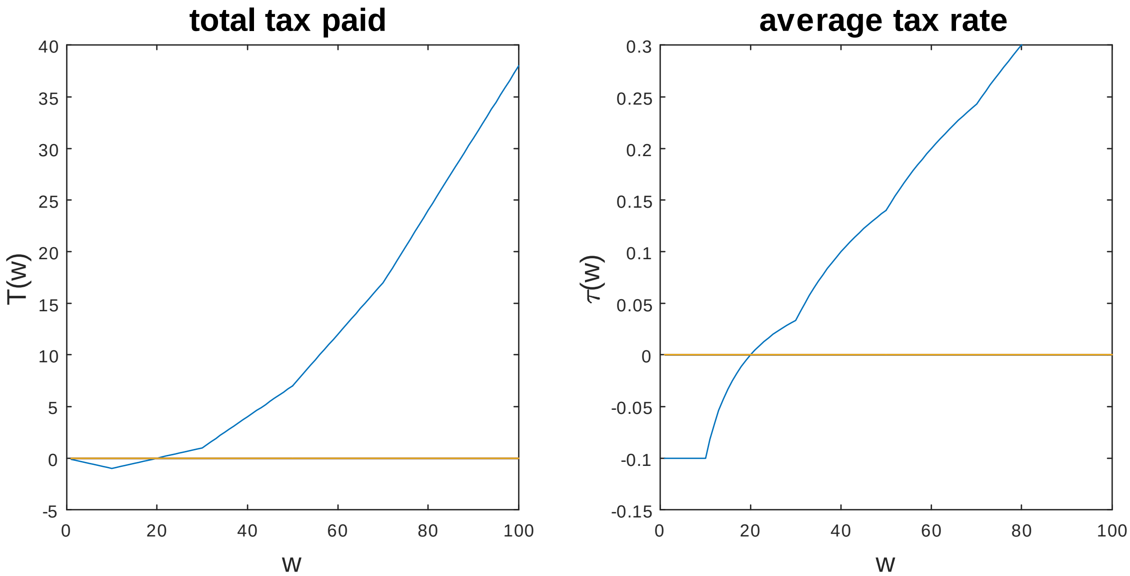


Figure 26 *Total and average tax paid by an individual with annual income w for "US-style" tax system (5.3) for 5 brackets as in fig. 25*

5.2.2 Properties of continuous marginal tax rates

- We now return to average tax from (5.4)

$$\tau(w) = \tau_0 + \tau_1 w \tag{5.5}$$

- It is tax system with continuous marginal tax rates
- It is progressive and allows negative tax rates when $\tau_0 < 0$ and $\tau_1 > 0$
- The *marginal* tax rate is (from (5.2))

$$\tau^{\text{marg}} = \tau'(w)\,w + \tau(w) = \tau_1 w + \tau_0 + \tau_1 w = \tau_0 + 2\tau_1 w \tag{5.6}$$

- See next figure for illustration

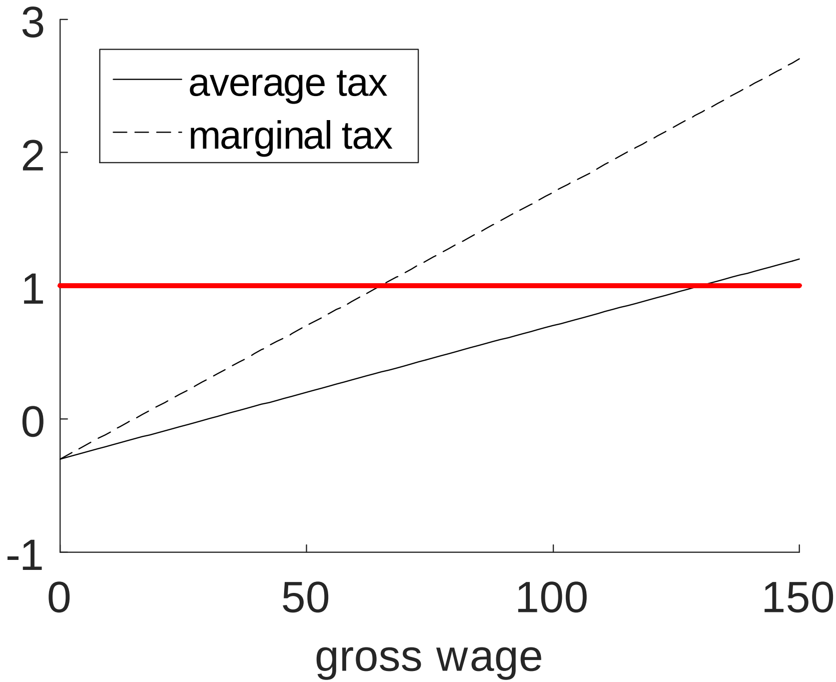


Figure 27 *Average and marginal tax rates ('German style')*
Source: **`average_marginal_tax.m`**, $\tau_0 = -0.3, \tau_1 = 0.01$

- Why we call this 'German style' tax system

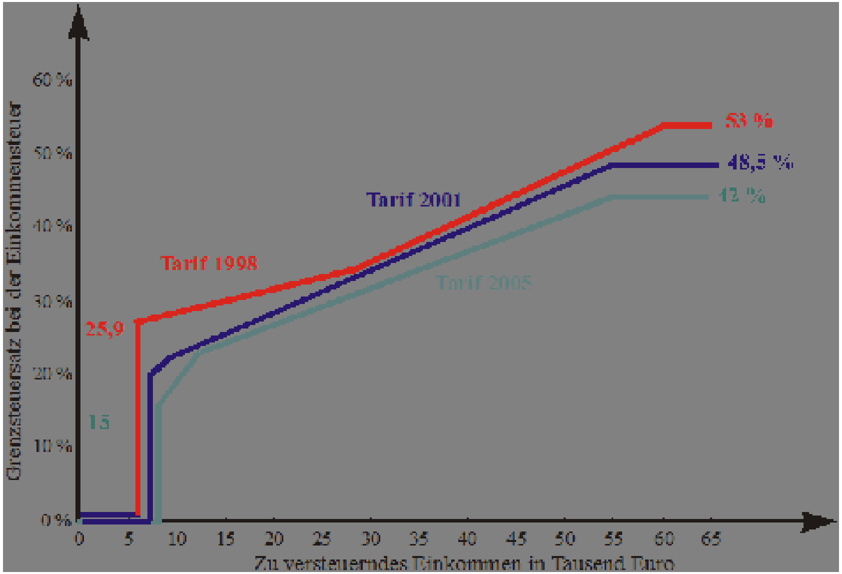


Figure 28 *Marginal tax rate as a function of gross income in Germany (1998 - 2005)*
Source: BMF (2000)

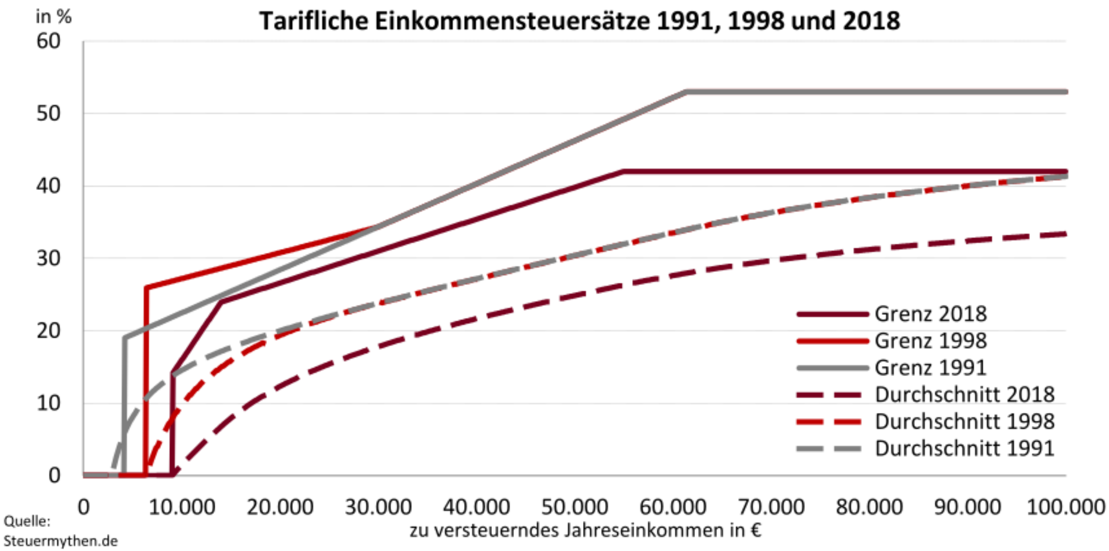


Figure 29 *Marginal tax rate as a function of gross income in Germany (1991-2018)*

(for the most recent update, see:
https://www.bmf-steuerrechner.de/2023_11_09_Tarifhistorie_Steuerrechner.pdf)

- Properties of the tax schedule (prohibitive wage)
  - Marginal tax rate is prohibitive (one does not want to earn a higher gross wage) if marginal tax rate equals 100% (or more)
  - The implied prohibitive wage can be computed from (5.6) as

$$\tau^{\text{marg}} \geq 1 \Leftrightarrow w \geq \frac{1-\tau_0}{2\tau_1} \equiv w^{\text{prohib}}$$

  - Using the tax rates from above ($\tau_0 = -0.3$ and $\tau_1 = 0.01$), we get $w^{\text{prohib}} = 65$ (see exercise 6.4.1)
  - The maximum net wage is therefore

$$w_{\max}^{\text{net}} = \left(1 - \tau\left(w^{\text{prohib}}\right)\right) w^{\text{prohib}}$$

  - See figure 30 for an illustration of w^{prohib}
  - Note that there is an alternative way to compute w^{prohib} that asks whether the net wage rise in the gross wage. Tutorial 6.4.1 tells us that

$$\frac{d}{dw} w^{\text{net}} > 0 \Leftrightarrow w < \frac{1-\tau_0}{2\tau_1}$$

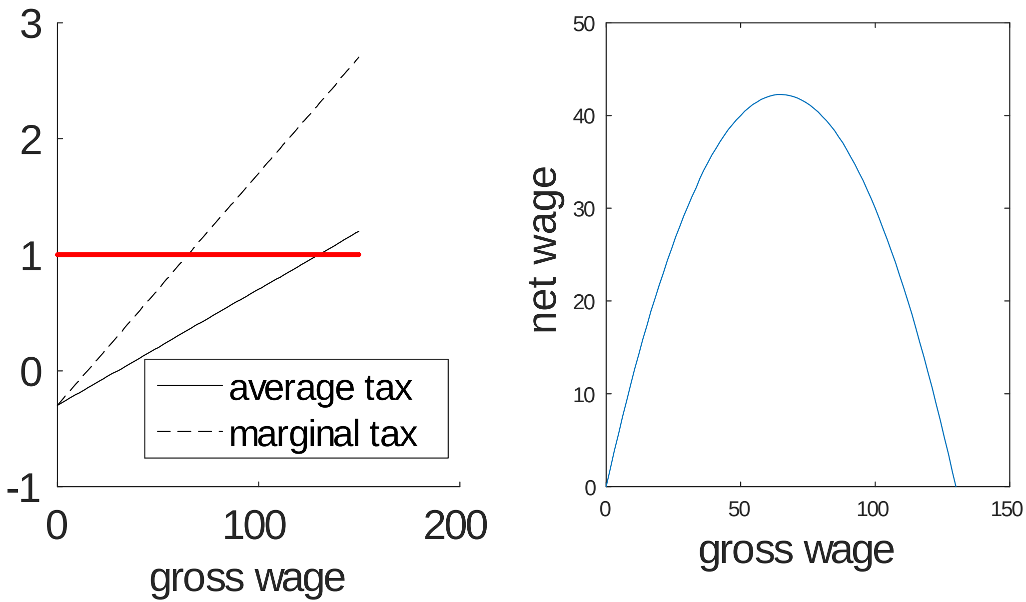


Figure 30 *Marginal tax rate and the net wage as a function of the gross wage (source:* `average_marginal_tax.m`*,* $\tau_0 = -0.3, \tau_1 = 0.01$*,* $w^{prohib} = 65$*, see BMF Tarifhistorie.pdf again for quadratic terms and constant top marginal tax rate as of 45%)*

- Properties of the tax schedule (wage subsidy)
  - Below which wage w^* do agents receive wage subsidies?
  - Identical question: Above which wage do agents pay taxes?
  - We get negative income taxes (wage subsidy) for

$$\tau(w) < 0 \Leftrightarrow \tau_0 + \tau_1 w < 0 \Leftrightarrow w < -\tau_0/\tau_1 \equiv w^* \tag{5.7}$$

  - Using again tax rate from above, the threshold wage is $w^* = 30$ (see exercise *6.4.1* question 5)
  - See the next figure for an illustration
    * Average tax paid is negative below $w^* = 30$
    * Net wage is *higher* than gross wage below w^*
    * Net wages are smaller than gross wages above w^*

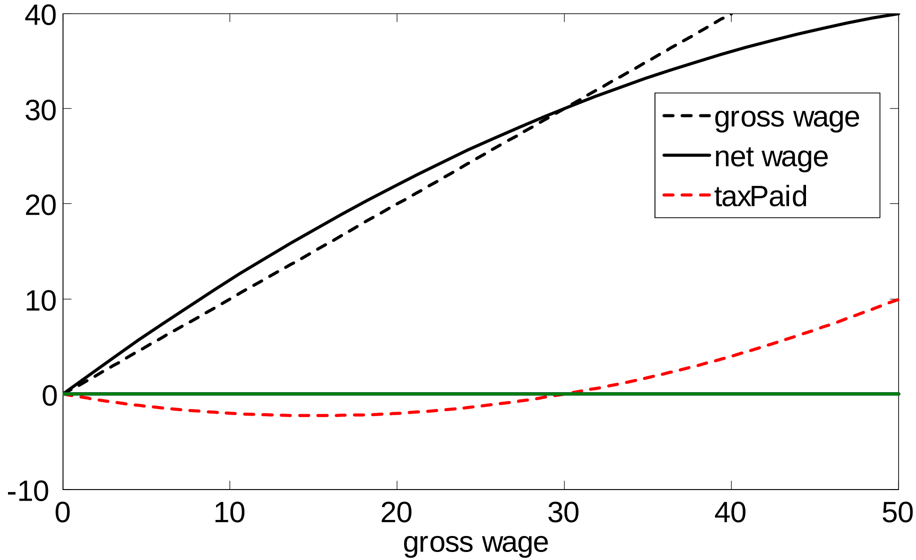


Figure 31 *The net wage and taxes paid as a function of the gross wage (source:* `average_marginal_tax.m`*, $\tau_0 = -0.3, \tau_1 = 0.01$, $w^* = 30$, $w^{prohib} = 65$)*

- Summary
  - We started from a certain exogenous gross wage
  - We introduced two tax schedules, one with a linear average tax and one with brackets with constant marginal tax rates
  - Both tax schedules are progressive, i.e. average tax rises in income
  - In both cases, we can choose parameters of tax schedule such that low income individuals receive a tax subsidy
  - Equality/ inequality of net wage distribution and poverty rates (among other characteristics of the net wage distribution) can be and are strongly influenced by policy
  - The question 'In which society do we want to live in?' finds an answer (at least to some extent) in 'which tax system do we want to implement?'

6 Taxation and inequality

We now return to two of the initial questions

- How is income redistributed via the tax system?
- How does the tax system affect the monthly net wage distribution?

How can we find answers?

- We need many individuals
- We need firms
- We need a government (whose budget balances wage subsidies below w^* with taxes)
- In short: we need a macroeconomic framework

6.1 The model and some technical background

6.1.1 The economy

- Input
  - Labour is the only factor of production
  - There is a discrete number N of workers indexed by i
  - Workers differ in their individual productivity l_i such that effective labour supply is

$$L = \Sigma_{i=1}^{N} l_i \tag{6.1}$$

  - A slightly more general version was seen before in sect. 4.2.2
  - Here we normalize hours worked per month to 1 (in order to keep the analysis as simple as possible)

- Distributional properties of aggregate employment
  - We want to relate individual characteristics to gross and net wages
  - We introduce a distribution for individual productivity l_i with mean $El_i = \mu_l$
  - When l_i is random, $L = \Sigma_{i=1}^N l_i$ in (6.1) is *random* as well
  - We can compute the *expected number* of effective labour supply based on (6.1)

$$EL = \Sigma_{i=1}^N El_i = \Sigma_{i=1}^N \mu_l = N\mu_l \tag{6.2}$$

  - With a large number of workers, the variance of L/N approaches zero (but not of L, see exercise 6.4.4) and we can write effective labour supply per worker as

$$\frac{L}{N} = \mu_l \tag{6.3}$$

  - [Many modelling frameworks assume that there is a continuum of workers (hence, not countable) whose size is measured by a measure N, which corresponds to number N in discrete case. For more discussion see exercise 6.4.4]

- Output Y
  - The production process uses labour only

$$Y = AL$$

    where A measures total factor productivity or non-individual labour productivity
  - Expected total output can be written as

$$EY = AEL = AN\mu_l \tag{6.4}$$

  - $\mu_l N$ is effective (productivity-adjusted) labour supply (or labour supply in efficiency units)
  - Output per worker is then again "deterministic" (the variance goes to zero) when the number of workers is large

$$\frac{Y}{N} = A\mu_l$$

- Distribution of individual productivity l
  - To be precise, we now pick one plausible example for the distribution of individual productivity
$$l \sim \text{LN}\left(\mu, \sigma\right) \tag{6.5}$$
  - Productivity l follows a log-normal density with scale parameter μ and shape parameter σ
  - (These parameters are *not* the mean and standard deviation of l)
  - Then, by definition of the lognormal distribution, the (natural) log of l is normally distributed with mean μ and standard deviation σ (and variance σ^2)
$$\ln l \sim N\left(\mu, \sigma\right)$$
  - The mean μ_l and variance σ_l^2 of l are given by (see e.g. Wälde, 2012, ch. 7.3.3)
$$\mu_l = e^{\mu+\frac{1}{2}\sigma^2}$$
$$\sigma_l^2 = e^{\left(2\mu+\sigma^2\right)}\left[e^{\sigma^2} - 1\right] \tag{6.6}$$
  - Mean μ_l and variance σ_l^2 rise in μ and σ

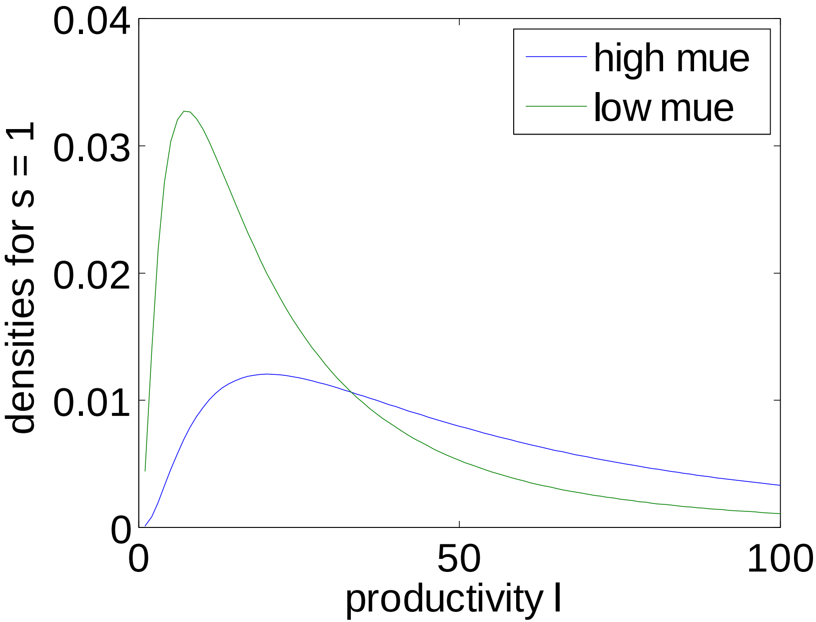


Figure 32 *Two examples for distributions of productivity: Two lognormal densities* ($f(l) = \frac{1}{\sqrt{2\pi}\sigma l} e^{-\frac{1}{2}\left(\frac{\ln l - \mu}{\sigma}\right)^2}$, *see (6.9) below) with high and low* μ *at identical* σ *(shown as s on vertical axis)*

- Perfect competition on all markets
  - Firms maximize expected profits using (6.4) and (6.2),

    $$E\pi = EpY - Ew^L L = pAN\mu_l - w^L N\mu_l,$$

    where p is price of final good and w^L is price per efficiency unit of labour (i.e. not the wage rate)
  - Optimal choice of number N of workers implies

    $$\frac{\partial E\pi}{\partial N} = pA\mu_l - w^L\mu_l = 0 \Leftrightarrow w^L = pA$$

    i.e. the nominal wage (per efficiency unit) equals the value marginal productivity of one efficiency unit of labour

- Labour income
  - The (gross) wage of a worker i with productivity l_i is then the wage per efficiency unit times individual productivity l_i

$$w_i = w^L l_i = pAl_i \tag{6.7}$$

  - Given an average tax rate schedule $\tau(w)$, the net wage is

$$w_i^{\text{net}} = [1 - \tau(w_i)]\, w_i \tag{6.8}$$

  - Question: what is the distribution of net wages given the tax system and the distribution of gross wages?

6.1.2 A bit of statistics: The density of a function of a random variable

- Consider a transformation of the type $y = y(x)$ where the random variable X has a density $f(x)$
- What is the density of Y?
- The answer comes from the theorem below
- Why all of this? To understand the link between the distribution of individual productivity, the gross wage and the net wage

Theorem 1 *Let X be a random variable with density $f(x)$ and range $[a,b]$ which can be $]-\infty,+\infty[$. Let Y be defined by the monotonically increasing function $y = y(x)$. Then the density $g(y)$ is given by $g(y) = f(x(y)) \frac{dx}{dy}$ on the range $[y(a), y(b)]$.*

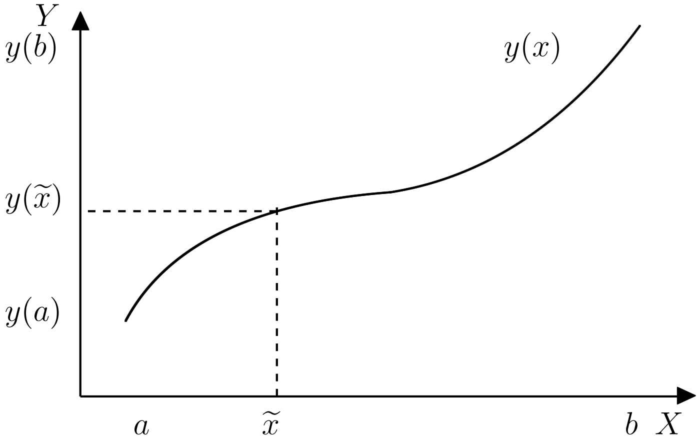


Figure 33 *Transforming a random variable (Wälde, 2012, theorem 7.3.2)*

- The figure plots the RV X on the horizontal and the RV Y on the vertical axis
- A monotonically increasing function $y(x)$ represents the transformation of realizations x into y

- Proof of the theorem
  - Transformation of the range is immediately clear from the figure: When X is bounded between a and b, Y must be bounded between $y(a)$ and $y(b)$
  - The proof for the density of Y requires a few more steps. The probability that y is smaller than some $y(\tilde{x})$ is identical to the probability that X is smaller than this $\tilde{x}$ (due to monotonicity of the function $y(x)$)
  - Hence, the distribution function (cumulative density function) of Y is given by $G(y) = F(x)$ where $y = y(x)$ or, equivalently, $x = x(y)$
  - The derivative then gives the density function, $g(y) \equiv \frac{d}{dy} G(y) = \frac{d}{dy} F(x(y)) = f(x(y)) \frac{dx}{dy}$.

6.1.3 Distribution of the gross wage

- Let us now return to our application
- The starting point is the log-normal density from (6.5) of individual productivity $l > 0$ with parameters μ and σ (which is not mean and variance of l, see (6.6))
- This density reads

$$f(l) = \frac{1}{\sqrt{2\pi}\sigma l} e^{-\frac{1}{2}\left(\frac{\ln l-\mu}{\sigma}\right)^2} \tag{6.9}$$

  with a mean and variance as given in (6.6)

- We now need to understand two aspects
  - Given the density for productivity, what is the density for the gross wage?
  - Given the density for the gross wage, what is the density for the net wage?

- Before we look at the density for the gross wage pAl, let us look at mean and variance
  - The mean of the gross wage is

$$Ew = E[pAl] = pAE[l] = pA\mu_l \tag{6.10}$$

  - Its variance is

$$\text{var}(w) = (pA)^2\, var(l) = (pA)^2\, \sigma_l^2 \tag{6.11}$$

  - Both the mean and the variance of the gross wage are larger (as long as pA is larger than 1) than mean and variance of labour productivity
  - To obtain a scale-free measure of inequality, we can look at the coefficient of variation (CoV)

$$\frac{\sqrt{\text{var}(w)}}{Ew} = \frac{\sqrt{(pA)^2\, \sigma_l^2}}{pA\mu_l} = \frac{\sigma_l}{\mu_l}$$

  - Inequality of the gross wage is identical to inequality of individual productivity when measured by CoV (as the latter is scale independent)

- Let us now construct the density of the gross wage pAl
  - Using theorem 1, we can compute the density of the gross wage as (see exercise 6.4.3)

$$f(w) = \frac{1}{\sqrt{2\pi}\sigma_w w} e^{-\frac{1}{2}\left(\frac{\ln w - \mu_w}{\sigma_w}\right)^2} \tag{6.12}$$

    where

$$\mu_w = \mu + \ln(pA)$$
$$\sigma_w = \sigma$$

  - Apparently, only the scale parameter is affected by a multiplicative transformation, but not the variance parameter
- What we found
  - gross wage is log-normally distributed
  - known from (6.12) and from the fact that gross wage in (6.7) is the product of a constant and a LN variable
  - scale and variance parameter shown above
  - mean and variance visible on previous slide

6.2 The effect of taxation

6.2.1 Distribution of the net wage

- Next question: How is the net wage $w^{\text{net}} = [1 - \tau(w)]\, w$ from (6.8) distributed?
- Application of theorem 1 to our case
  - Our random variable X here is the gross wage w with density $f(w)$ from (6.12)
  - The transformation $y = y(x)$ is the net wage $w^{\text{net}} = [1 - \tau(w)]\, w$ from (6.8) with the tax from (5.4)
  - Hence we know $G(w^{\text{net}}) = F(w)$ as long as $w^{\text{net}} = w^{\text{net}}(w)$ is monotonic, i.e. for $w < w^{\text{prohib}}$
  - For $w < w^{\text{prohib}}$, we can compute the inverse function (see appendix 13 for a reminder), i.e. write the gross wage as a function of the net wage, $w = w(w^{\text{net}})$
  - The density is then

$$g\left(w^{\text{net}}\right) = f\left(w\left(w^{\text{net}}\right)\right) \frac{dw\left(w^{\text{net}}\right)}{dw^{\text{net}}} \tag{6.13}$$

- What is the inverse function $w\left(w^{\text{net}}\right)$?
  - We start from $w^{\text{net}} = [1 - \tau(w)]\, w$ in (6.8) with the average tax rate $\tau(w) = \tau_0 + \tau_1 w$ from (5.4) to get (see the right panel of figure 30)

$$\begin{aligned} w^{\text{net}} &= [1 - \tau_0 - \tau_1 w]\, w \\ &= (1 - \tau_0)\, w - \tau_1 w^2 \end{aligned}$$

  - This is a quadratic equation in the gross wage w which we can solve (see exercise 6.4.2, question 1) as

$$w_{1,2} = \frac{1 - \tau_0 \pm \sqrt{(1 - \tau_0)^2 - 4\tau_1 w^{\text{net}}}}{2\tau_1}$$

  - The economically meaningful solution (see exercise 6.4.2, question 1) is

$$w\left(w^{\text{net}}\right) = \frac{1 - \tau_0 - \sqrt{(1 - \tau_0)^2 - 4\tau_1 w^{\text{net}}}}{2\tau_1} \tag{6.14}$$

- What is its derivative $dw\left(w^{\text{net}}\right)/dw^{\text{net}}$?
  - We start from the expression for $w\left(w^{\text{net}}\right)$ from (6.14)
  - Computing the derivative gives (see exercise 6.4.2, question 2 and remember that $\sqrt{x} = x^{1/2}$)

$$\frac{dw}{dw^{\text{net}}} = \frac{1}{\sqrt{\left(1-\tau_0\right)^2 - 4\tau_1 w^{\text{net}}}} \tag{6.15}$$

- The density of the net wage
  - The net wage's density from (6.13) is (reminder)

$$g\left(w^{\text{net}}\right)=f\left(w\left(w^{\text{net}}\right)\right)\frac{dw\left(w^{\text{net}}\right)}{dw^{\text{net}}}$$

  - Plugging in the density of the gross wage from (6.12), the gross wage as a function of the net wage from (6.14) and the derivative from (6.15), we get

$$g\left(w^{\text{net}}\right)=\frac{1}{\sqrt{2\pi}\sigma_w w\left(w^{\text{net}}\right)}e^{\left(-\frac{\left(\ln w\left(w^{\text{net}}\right)-\mu_w\right)^2}{2\sigma_w^2}\right)}\frac{1}{\sqrt{\left(1-\tau_0\right)^2-4\tau_1 w^{\text{net}}}}$$

  - The expression can somewhat be condensed to

$$g\left(w^{\text{net}}\right)=\frac{1}{\sqrt{2\pi}\sigma_w \tilde{w}\left(w^{\text{net}}\right)}e^{\left(-\frac{\left(\ln w\left(w^{\text{net}}\right)-\mu_w\right)^2}{2\sigma_w^2}\right)} \tag{6.16}$$

    where

$$\tilde{w}\left(w^{\text{net}}\right)=w\left(w^{\text{net}}\right)\sqrt{\left(1-\tau_0\right)^2-4\tau_1 w^{\text{net}}},$$

$$w\left(w^{\text{net}}\right)=\frac{1-\tau_0-\sqrt{\left(1-\tau_0\right)^2-4\tau_1 w^{\text{net}}}}{2\tau_1}$$

6.2.2 Taxation and the distribution of net wages

- Why have we done all of this?
  - Remember the objective of this lecture: How can we tackle inequality?
  - Alternatively, look at section 1.4.2 on various ideas of equality
  - Equation (6.16) on the density of the net wage distribution shows (in general equilibrium but without endogenous labour supply) how tax system affects net-wage distribution
  - Now vary τ_0 and τ_1 (i.e. the progressiveness of the tax system) to see how policy makers can change inequality and create equality
- Consider the following two figures (explanations on slides thereafter)

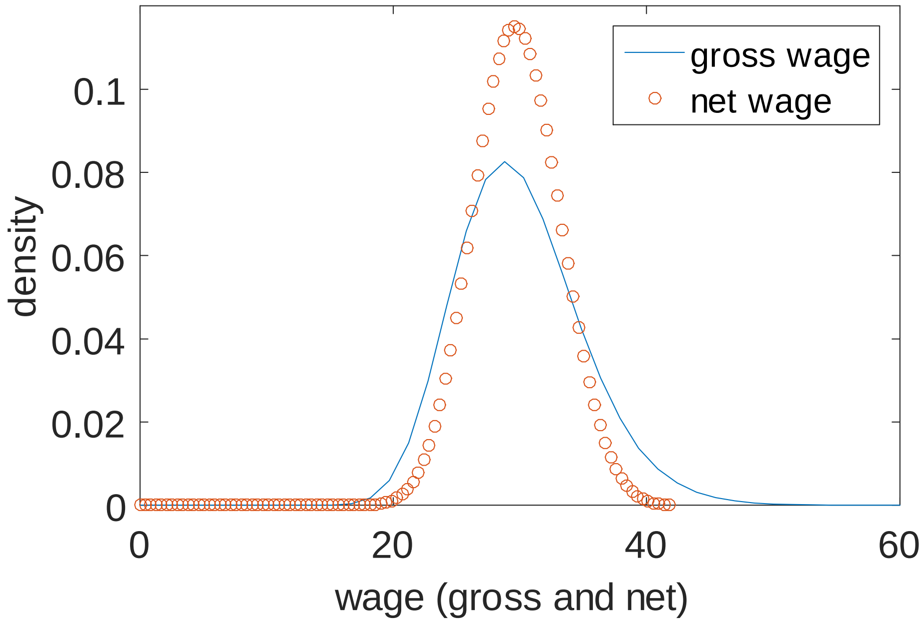


Figure 34 *The densities for the gross and net wage*
source: ***average_marginal_tax.m,*** $\tau_0 = -0.3, \tau_1 = 0.01$, $w^* = 30$, $w^{prohib} = 65$

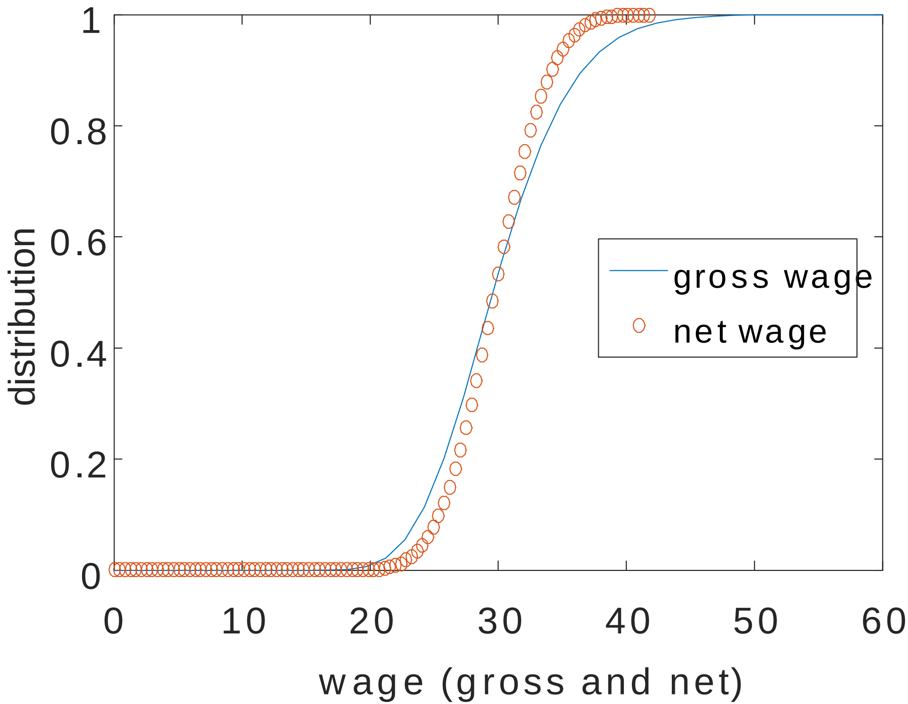


Figure 35 *The distribution (cumulative densities) for the gross and the net wage*

- Figure on densities
  - The density for the net wage is more compressed than the density for the gross wage
  - This is the effect of redistributive taxation with wage subsidies below w^*
  - This can better be seen in the figure on the distribution
- Figure on distribution
  - There are just as many individuals with a net wage below w^* from (5.7) than there are individuals with a gross wage below w^* (compare $w^* = 30$ in fig. 31)
    - The tax system does not push the net wage of anyone above (or below) w^*
    - The tax system does increase the net wage of anyone below w^*...
      - Without tax system 20% of individuals would earn a wage of 25 or less
      - With a tax system (red net wage circles), only around 10% earn a wage of 25 or less (the share of poor falls)
    - ... and reduces net wage of anyone above w^* ('Robin Hood system')
      - Without a tax system, 90% of individuals earn a wage of around 35 or less
      - With a tax system, 90% of individuals earn a wage of around 33 or less (the share of rich falls)

6.2.3 Can government finance all of this?

- Tax revenue from an individual of productivity l is given by

$$T\left(w\left(l\right)\right)=\tau\left(w\left(l\right)\right)w\left(l\right)$$

- The average tax rate (for an individual with gross wage $w\left(l\right)$ and productivity l) is $\tau\left(w\left(l\right)\right)$ as defined in (5.1) and the density of productivity is $f\left(l\right)$ from (6.9)

- The government's budget is balanced if

$$\int_0^\infty T\left(w\left(l\right)\right)f\left(l\right)dl=0,$$

  i.e. if the sum over all tax revenues is zero

- The budget can be written as

$$\int_0^{l^*} T(w(l)) f(l)\, dl + \int_{l^*}^{\infty} T(w(l)) f(l)\, dl = 0,$$

where l^* is the productivity level that implies a gross wage of w^*, i.e. where l^* satisfies

$$w^* = pAl^*$$

from (6.7)

- The first term $\int_0^{l^*} T(w(l)) f(l)\, dl$ is negative and amounts to all wage subsidies
- The second term is positive and amounts to tax income
- Balanced budget to be confirmed numerically: Adjust τ_0 and τ_1 (progressivity of tax system) to achieve a balance

6.2.4 The effect of the tax system on measures of inequality

One can proceed and compute the effect of the tax system on

- Variance / standard deviation
- Coefficient of variation
- Wage ratios like w_{p1}/w_{p2}
- Poverty rates
- Government budget
- and the like

Nice seminar paper / master thesis (for those who want to learn matlab/ Python/ Julia)

6.3 Conclusion

- We saw what marginal and average taxes are and how they affect individual labour income
- How is income redistributed via the tax system?
  - By using a progressive tax system with a negative coefficient τ_0, income tax below a threshold w^* is negative
  - In other words, this allows for wage subsidies in an otherwise perfectly competitive economy
- How does the tax system affect the monthly net wage distribution?
  - The number of individuals with low labour income can be reduced
  - The degree of redistribution depends on the degree of progressivity

- Where to go from there
  - Any tax system has implications for individual behaviour (e.g. labour supply decisions)
  - This should be modelled (and was neglected above)
  - One needs to be aware of the difference between positive and normative tax analyses
    - Positive analyses ask what the implications of real-world tax systems are
    - Normative analyses would ask about optimal tax system (given some objective function like social welfare function)
    - Distinction between
      - full-information analyses in the Ramsey tradition or
      - analyses taking informational asymmetries (Mirrlees tradition) into account
  - A lot remains to be done ... (Master thesis?)

6.4 Exercises

6.4.1 Prohibitive tax rates and wages and negative income taxes

1. Compute the prohibitive wage as illustrated in figure 30 in the lecture.

2. What is the highest net wage that can be earned in this economy?

3. Is this the hourly wage or the annual wage?

4. Compute the prohibitive wage by starting from the net wage.

5. Compute the wage illustrated in figure 31 below which a wage subsidy is paid.

6.4.2 Gross and net wages

Assume that the tax rate, $\tau(w)$, is increasing in the gross wage, w. Specifically, let the tax scheme be of the form $\tau(w) = \tau_0 + \tau_1 w$. Therefore, the net wage, w^{net}, is given by $[1 - \tau_0 - \tau_1 w]\, w$.

1. Derive the gross wage, w, as a function of the net wage, w^{net}.

2. Compute the derivative of gross wages with respect to net wages.

6.4.3 The density of gross and net wages

1. Show that the density of the gross wage pAl is given by the expression in (6.12).

   (a) Show this by using theorem 1.

   (b) Show this by noting that l is log-normally distributed and that the mean and variance of pAl are given in (6.10) and (6.11).

2. Compute the density of the net wage.

6.4.4 Measuring the number of workers

[very technical – only as background and not for the exam – probably not covered in tutorial]

1. How can the number of hours worked be measured when there is

   (a) a discrete number of workers?

   (b) a continuous number of workers?

   (c) We described aggregate labour supply by $L = \Sigma_{i=1}^{N} l_i$. Let us now assume there is a continuum of workers with mass N. Aggregate labour supply is now

$$L = \int_0^N l(i)\, di \tag{6.17}$$

where $l(i)$ is productivity of individual i and we integrate over individuals. Show the assumptions behind this expression.

2. How can one measure total productivity and output of workers?

3. The randomness of aggregate employment

   (a) Compute the variance of employment L as defined in (6.1).

   (b) Is output a deterministic measure or does it have a positive variance?

   (c) What about output per capita?

Johannes Gutenberg University Mainz
Master in International Economics and Public Policy

Labour market theory: Wage inequality, redistribution and international trade

Summer Term

Klaus Wälde (lecture) and Hoang Van Khieu (tutorial)
www.macro.economics.uni-mainz.de
February 5, 2026

Part III

Trade and labour markets

7 The literature on trade and labour markets

- International trade and all aspects related to globalisation (migration, capital flows, pandemics ...) are often associated with distributional effects
- There are winners and losers from globalisation
- At the same time, globalisation leads to welfare gains of (almost) all involved countries

7.1 Learning objectives

- What are the reasons for why there are winners and losers from trade?
- Are overall gains from trade large enough to compensate losers?
- Can we construct Pareto-improving gains from trade?

7.2 Distributional effects of trade

- Stolper-Samuelson theorem
  - Stolper and Samuelson derived a theorem for the Heckscher-Ohlin model which is the classic for distributional effects of international trade
  - Heckscher-Ohlin model also shows that (in the absence of market distortions) there are overall gains from trade
  - Society gains, some individuals gain, some individuals lose
    * relative factor prices

$$\hat{p}_X > \hat{p}_Y \Rightarrow \hat{w}^K > \hat{w}^L \Leftrightarrow k_X > k_Y$$

    * absolute factor prices

$$\hat{w}^K > \hat{p}_X > \hat{p}_Y > \hat{w}^L \Leftrightarrow k_X > k_Y$$

  - (see lecture Philipp Harms or many textbooks or an earlier lecture)

- North-South distributional findings – wage inequality rising in all countries
  - Stolper-Samuelson theorem predicts that abundant factor gains in the North (say capital or human capital)
  - The same factor loses in the South (as it is not abundant there)
  - Empirical findings suggest that skilled individuals gain all over the world (e.g. Zhu and Trefler, 2005)
  - Zhu and Trefler (2005) explain this within a combination of Dornbusch-Fischer-Samuelson models (1977, 1980) with (exogenous) catching up of the South
- Trade and competition
  - A rise in wage inequality can also be the outcome of more competition via international trade
  - In a setup with Cournot competition, firms are forced to reduce overheads under trade
  - This leads to reallocation of factors of production and wage inequality rises (Wälde and Weiß, 2007)

- International trade, unemployment and inequality
  - Helpman, Itskhoki and Redding (2010)
  - Helpman and Itskhoki (2010)
  - combines matching with trade – different lecture
- Offshoring and labour markets
  - Grossman and Rossi-Hansberg (2008) provide a baseline model for offshoring
  - Hummels, Munch and Xiang (2018) survey i.a. the distributional effects of offshoring
  - See Felbermayr et al. (2018) for a comparison of the effects of labour market and product market reforms (and the link to international trade) on wage inequality

8 A simple (autarky) model with heterogeneous agents

8.1 Learning objectives

- Understand a simple mechanism how international trade creates (real) winners and losers
  - More demand for a product and skills through trade increases wage
  - More competition through supply of same good decreases wage
  - Basic trade-off: which effect is stronger?
- How Pareto-improving gains from trade can be constructed
  - Understand how fiscal policy can (in principle) construct appropriate tax schedule
  - Understand real-world challenges

- Relation to literature with a continuum of goods
  - Dornbusch-Fischer-Samuelson model (1977) – Ricardian flavour
    - allow for international differences in labour productivity
    - employ homogenous labour as only factor of production, mobile across sectors
  - Dornbusch-Fischer-Samuelson model (1980) – Heckscher-Ohlin flavour
    - employ two factors of production, capital and labour, mobile across sectors
    - continuum of goods that differ in capital intensity
- Our model (details to be seen)
  - has n goods (discrete amount)
  - has n types of labour (discrete amount)
  - labour is good-specific (short-term feature, i.e. 3-5 years)
  - international differences in labour productivity

- Dixit-Stiglitz (1977) structure for preferences
  - Imperfect substitutes with typical demand functions
  - no market entry costs
  - We assume perfect competition on each market
  - No mark-up pricing (not essential)

8.2 The setup

- Endowment
  - Each individual of type $j \in \{1, 2, ..., n\}$ supplies one unit of human capital of this type
  - The economy is endowed with $\tilde{h}_j$ workers (headcount) per type j
  - There are N workers (headcount), i.e. $\Sigma_{j=1}^{n} \tilde{h}_j = N$
  - We can think of this as a discrete (non-stochastic) "distribution" of human capital
- Technology
  - There is a discrete number n of varieties with index i
  - Variety i is produced by a number $\tilde{h}_i$ of specialized workers supplying human capital of type i
  - The production takes place with a productivity a_i

$$y_i = a_i \tilde{h}_i \equiv h_i \tag{8.1}$$

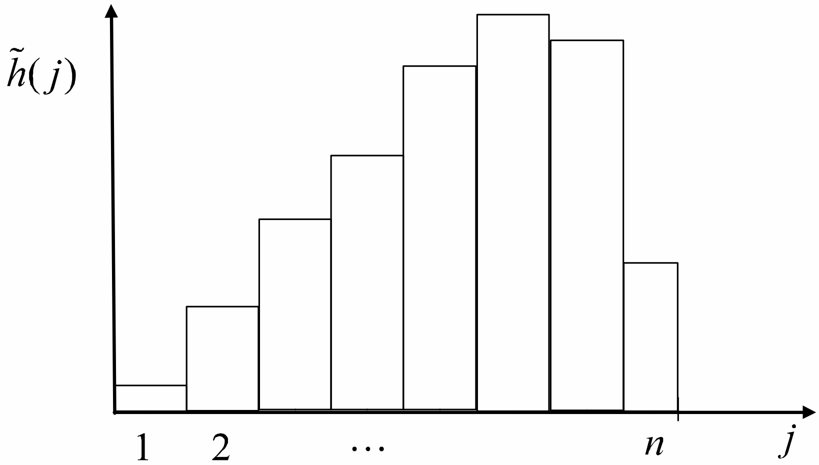


Figure 36 *Endowment with human capital of types j (one arbitrary example illustrated by a histogram)*

- Histograms for educational background in Germany

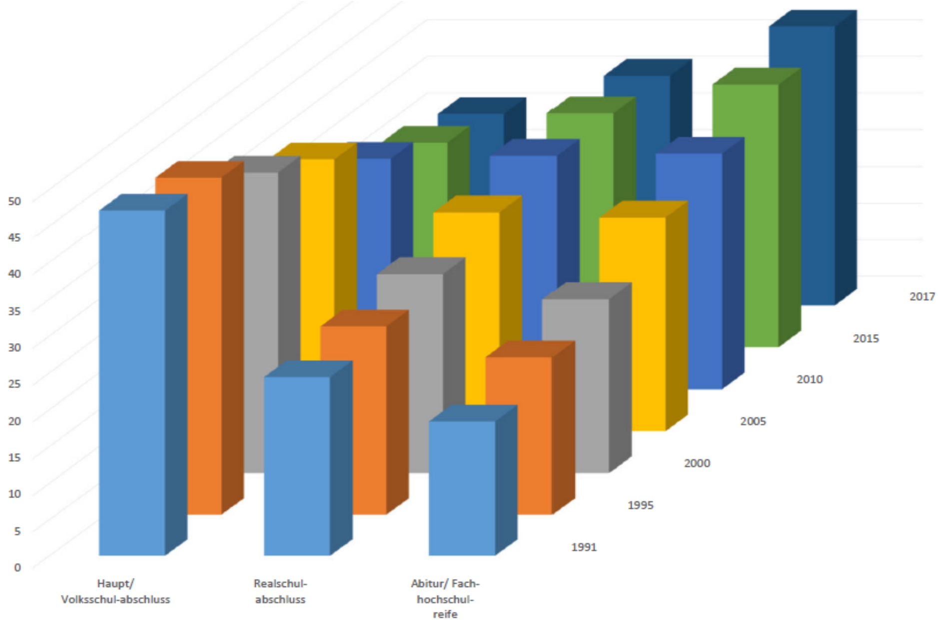


Figure 37 *Highest educational degree in Germany (men, age 15 to 65, Hauptschule - Realschule - Abitur, 1991 to 2017. Source: WSI 2019, Mikrozensus - qualitatively similar figure for women)*

- Households' preferences and budget constraint
  - Discrete Dixit-Stiglitz structure (similar to 1st term Trade or 1st term Macro or 2nd term Advances Trade)

$$u(c_j) = \left(\Sigma_{i=1}^{n} c_{ij}^{\theta}\right)^{1/\theta}, \qquad \theta < 1 \tag{8.2}$$

    where c_{ij} is consumption of variety i by household j and $c_j = \{c_{1j}, c_{2j}, ..., c_{nj}\}$ is a vector
  - A household j equates labour income with total expenditure

$$w_j = \Sigma_{i=1}^{n} p_i c_{ij}$$

    where w_j is the wage for one unit of human capital of type j

- As each worker supplies one unit of labour, w_j is also the, say, monthly income
- Free entry of firms
  - There is free entry of firms into each market at *zero* cost
  - [Big difference to literature on monopolistic competition]
  - We therefore assume that each single variety is produced by a large number of firms
  - Firms act in a perfectly competitive way

8.3 Optimal behaviour

- Demand functions (one individual or household)
  - Demand by households j for variety i (see exercise)

$$c_{ij} = \left(\frac{p_i}{P}\right)^{-\varepsilon} \frac{w_j}{P} \tag{8.3}$$

where the price index P reads

$$P = \left(\Sigma_{i=1}^{n} p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{8.4}$$

and the price elasticity of demand is $\varepsilon = 1/(1-\theta)$ and positive

- Are properties of demand c_{ij} familiar?
  - Yes, from advanced macro, trade and other
  - Price elasticity of demand is $-\varepsilon$, price index P matters, as does real income w_j/P
  - Elasticity ε can be smaller than one as (θ can be negative as) firms are competitive (not so familiar)

- Aggregate demand for variety i ...
  - ... follows from aggregating (8.3) over households j,
  - employing individual demand c_{ij} and number $\tilde{h}_j$ of households

$$c_i = \Sigma_{j=1}^{n} c_{ij} \tilde{h}_j = \left(\frac{p_i}{P} \right)^{-\varepsilon} \frac{W}{P} \tag{8.5}$$

  - Total nominal income in this economy is denoted by

$$W = \Sigma_{j=1}^{n} w_j \tilde{h}_j \tag{8.6}$$

- Firms
  - maximize profits $p_i y_i - w_i \tilde{h}_i$ by
  - choosing $\tilde{h}_i$ and thereby
  - equate marginal revenue with marginal costs

$$p_i = \frac{w_i}{a_i} \tag{8.7}$$

8.4 Autarky equilibrium

- Production
  - Households provide $\tilde{h}_j$ units of type j human capital (reminder)
  - Human capital employed in the production of variety i is therefore

$$\tilde{h}_i = \tilde{h}_j \text{ for } j = i \tag{8.8}$$

  - Output is determined by factor supply and, replacing j by i

$$y_i = a_i \tilde{h}_i = h_i \tag{8.9}$$

- In other words, output is basically exogenous in this economy
  - Factor supply is constant
  - There is full employment
  - Factors do not move across sectors

- Goods market equilibrium
  - Equality of supply and demand on the goods market requires

$$y_i = \left(\frac{p_i}{P}\right)^{-\varepsilon} \frac{W}{P}$$

  - Employing output from (8.9) and employing h_i from (8.8) for notational convenience allows to solve for real prices of varieties i

$$\left(\frac{p_i}{P}\right)^{-\varepsilon} = \frac{h_i}{W/P} \Leftrightarrow \frac{p_i}{P} = \left(\frac{W/P}{h_i}\right)^{1/\varepsilon} \tag{8.10}$$

- How can we understand this expression?
  - Expression is central to understand wage effects of international trade
  - (wage is goods price times productivity)
  - The real price for variety i
    * rises in total real demand W/P and
    * falls in supply $y_i = h_i$

- Equilibrium wages
  - Remember equality of productivity-adjusted wages and prices from (8.7)
  - Equilibrium real wages are then from (8.10) given by

$$\frac{w_i}{P} = a_i \left[\frac{W/P}{h_i} \right]^{1/\varepsilon} \tag{8.11}$$

  - Real labour income is (see exercise 8.5.2 that also proves Walras' law)

$$\frac{W}{P} = \left(\Sigma_{j=1}^{n} h_j^{(\varepsilon-1)/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{8.12}$$

  - The interpretation is similar to the interpretation for the price of variety i in (8.10)
    * Demand (i.e. high real labour income via productive workers) increases the wage, supply of skills decreases it
    * Real labour income W/P from (8.12) rises e.g. when the entire density moves to the right (shift the support by e.g. adding 5 units of productivity to each j - see figure 36)
    * Note that real wage is independent of country size N: multiplying number of workers $\tilde{h}_i$ by a common factor does not affect w_i/P

- Wage distribution in autarky
  - Note that (8.11) with (8.12) determine the autarky wage distribution
  - Merging the equations, the wage distribution reads

$$\frac{w_i}{P} = a_i \left[\frac{\left(\Sigma_{j=1}^{n} h_j^{(\varepsilon-1)/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{h_i} \right]^{1/\varepsilon} \tag{8.13}$$

  - Obviously, the crucial determinants are the productivities a_i and factor endowments h_j
  - Preferences matter via the elasticity of substitution ε
  - We will now study how trade affects this distribution of wages

8.5 Exercises

8.5.1 Demand functions

Consider a type-j household represented by Dixit-Stiglitz preferences over n varieties of goods

$$U = \left(\sum_{i=1}^{n} c_{ij}^{\theta} \right)^{1/\theta} \tag{8.14}$$

where θ represents tastes for different varieties with $\theta < 1$. The budget constraint is given by

$$w_j = \sum_{i=1}^{n} p_i c_{ij}. \tag{8.15}$$

1. Provide an interpretation to the budget constraint (8.15).

2. Derive the demand for each variety i.

8.5.2 The real wage and Walras' law under autarky

1. Derive real labour income $$\frac{W}{P} = \left(\Sigma_{i=1}^{n} h_i^{(\varepsilon-1)/\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

using the price index

$$P = \left(\Sigma_{i=1}^{n} p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)}.$$

2. Derive real labour income

$$\frac{W}{P} = \left(\Sigma_{j=1}^{n} h_j^{(\varepsilon-1)/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

using the formula for total nominal income

$$W = \Sigma_{j=1}^{n} w_j \tilde{h}_j$$

and the formula for real wages

$$\frac{w_i}{P} = a_i \left[\frac{W/P}{h_i}\right]^{1/\varepsilon}.$$

3. Argue that the Walras' law has been proven.

9 Opening up to trade

9.1 Country characteristics and opening up to trade

- Assume there is a world with 2 countries, country A and country B
- Let there be $\tilde{h}_j^A$ workers of type j in country A and $\tilde{h}_j^B$ in country B
- Technologies a_i^k can differ across countries $k \in \{A, B\}$
- Country A has a more skilled labour force than country B (see next figure)

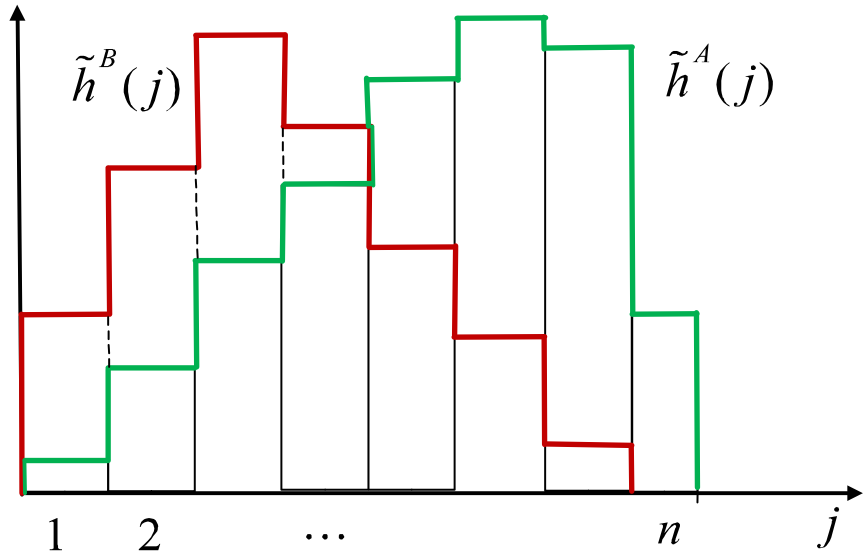


Figure 38 *Endowment histograms with human capital in country A and country B*

- Opening up the country means international trade in all varieties
  - Prices under trade p_i of goods i are the same in all countries
  - There is a perfect overlap (compare Tang and Wälde, 2001) of varieties
- Given exogeneity of output ...
  - trade will not imply any specialization effects in any Ricardian, Heckscher-Ohlin or any other sense
  - model focuses on the competitive effects within sectors
  - All countries produce all varieties

9.2 The wage effects of international trade

- There is a simple result and a more involved result
  - Simple result: equal goods prices imply equality of productivity-adjusted wages

$$p_i^A = p_i^B \Rightarrow \frac{w_i^A}{a_i^A} = \frac{w_i^B}{a_i^B} \tag{9.1}$$

  - Follows from the identity of goods prices p_i^k and first-order condition of firms in (8.7)
  - The more involved result asks how trade wages differ relative to their autarky level
  - This is what we study now ...

- World output of variety i replaces national output (8.9) in autarky by

$$y_i = h_i^A + h_i^B$$

- World output depends on, recalling (8.9), productivity and skill stocks across countries

$$y_i = a_i^A \tilde{h}_i^A + a_i^B \tilde{h}_i^B$$

  making output again perfectly exogenous

- Individual demand (8.3) is unchanged but aggregated world demand reads

$$c_i^W = \Sigma_{j=1}^n \left\{ c_{ij}^A \tilde{h}_j^A + c_{ij}^B \tilde{h}_j^B \right\} = \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A}{\tilde{P}} + \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^B}{\tilde{P}}$$
$$= \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A + W^B}{\tilde{P}} \quad (9.2)$$

where, as in (8.6), labour income in country k is

$$W^k = \Sigma_{j=1}^n w_j^k \tilde{h}_j^k \quad (9.3)$$

- The price index under international trade is denoted by $\tilde{P}$
  - It has the same functional form as under autarky in (8.4)
  - The number n of varieties is unchanged but ...
  - ... goods prices p_i differ under trade

- Equality of supply and demand on the goods market

$$h_i^A + h_i^B = \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A + W^B}{\tilde{P}} \tag{9.4}$$

allows to pin down trade effects on

  - prices, the real wage sum (see app. 10.5.1)

$$\frac{W^A + W^B}{\tilde{P}} = \left(\Sigma_{j=1}^n \left(h_j^A + h_j^B\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{9.5}$$

  - and on wages (see next slide and app. 10.5.1 as well)

- Trade influences wages through two channels
  - Wages under trade

$$\frac{w_i^k}{\tilde{P}} = a_i^k \left(\frac{\left(\Sigma_{j=1}^{n} \left(h_j^A + h_j^B\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}{h_i^A + h_i^B} \right)^{1/\varepsilon} \tag{9.6}$$

  - demand effect (numerator $\left(W^A + W^B\right)/\tilde{P}$) increases wage of type i
    * this effect is identical for all skill groups i
    * all skills i benefit from trade through this channel for $h_j^B > 0$
  - competition effect (denominator $h_i^A + h_i^B$) decreases wage of skill group i
    * this effect depends on number of workers abroad that have the same type i
    * all workers lose through this channel for $h_i^B > 0$

9.3 Winners and losers from trade

9.3.1 Is the real wage under trade higher than the real wage under autarky?

Proposition 1 *The real wage for skill group i in country A rises due to international trade $\Leftrightarrow$ The demand effect exceeds the competition effect*

$$\left.\frac{w_i^A}{\tilde{P}}\right|_{trade} > \left.\frac{w_i^A}{P}\right|_{autarky} \Leftrightarrow \left(\frac{\Sigma_{j=1}^n \left(h_j^A + h_j^B\right)^{(\varepsilon-1)/\varepsilon}}{\Sigma_{j=1}^n h_j^{A(\varepsilon-1)/\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)} > \frac{h_i^A + h_i^B}{h_i^A}$$

- See tutorial for formal proof
- Intuitive understanding is clear from
  - autarky price equation (8.10)
  - autarky wage equation (8.11) and
  - trade wage equation (9.6) and its interpretation we just saw

9.3.2 Do real wages rise when countries are identical in their skill distribution?

Proposition 2 *When countries have identical skill distributions,* $\tilde{h}_i^A = \tilde{h}_i^B \equiv \tilde{h}_i$*, international trade does not affect wages*

$$\left.\frac{w_i}{\tilde{P}}\right|_{trade} = \left.\frac{w_i}{P}\right|_{autarky}$$

- Why do real wages remain unchanged when identical countries start to trade?
  - There are twice as many workers within each skill group and therefore twice as much output (negative competition effect) but also twice as much demand (positive effect)
  - Effects just cancel
  - (Real) prices and thereby wages do not change
  - (again see tutorial for formal proof)

9.3.3 Do real wages rise when one country is n times larger?

Proposition 3 *When country B is n times larger than country A with respect to each skill-group, $\tilde{h}_i^B = n\tilde{h}_i^A$, international trade does not affect wages*

$$\left.\frac{w_i}{\tilde{P}}\right|_{trade} = \left.\frac{w_i}{P}\right|_{autarky}$$

- Why do real wages remain unchanged when identical countries start to trade?
  - Simple generalization of earlier result
  - There are $(n+1)$-times as many workers within each skill group, $(n+1)$-times as much output but also $(n+1)$-times as much demand
  - (Real) prices and thereby wages do not change
  - (Tutorial has the formal details – see next figures for an illustration)

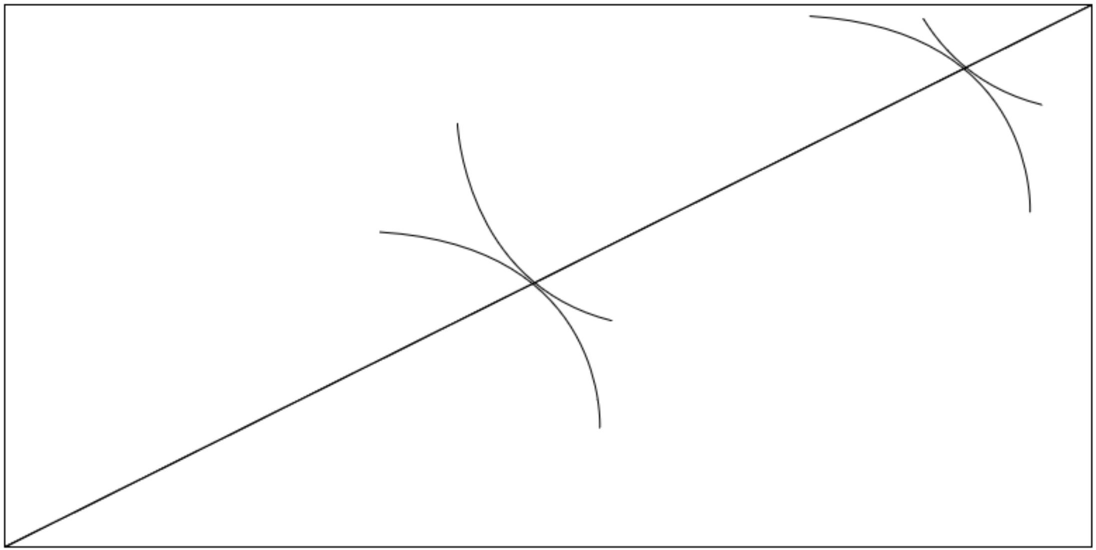


Figure 39 *Identical production and no gains from trade between countries of identical structure*

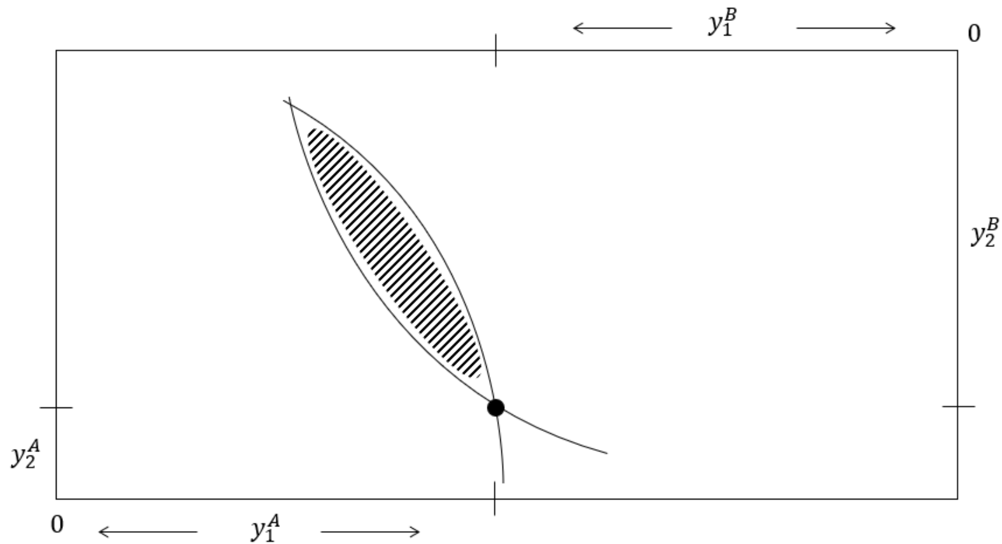


Figure 40 *Potential gains from trade between countries of different structure*

9.3.4 Do real wages rise when there is no comparable industry abroad?

- Imagine a country allows for trade that has a skill group which is lacking abroad, $h_i^B = 0$
- Silicon Valley in USA or low-cost industries in developing countries
- Who gains and who loses?
- Same rationale as before
  - Comparison of demand and competition effect
  - As there is no competition effect, workers in country A gain

Proposition 4 *When a country trades with another country and offers a good i which is not produced abroad, $h_i^B = 0$, wages of workers in industry i rise*

$$\left.\frac{w_i^A}{\tilde{P}}\right|_{trade} > \left.\frac{w_i^A}{P}\right|_{autarky}$$

9.4 Trade flows under international trade

- Trade patterns
  - By definition, exports amount to the difference between domestic production and domestic consumption
$$EX_i^A = a_i^A \tilde{h}_i^A - \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A}{\tilde{P}}$$
  - The production structure does not change under trade (labour is sector specific)
  - If there is trade, it must result from consumption reactions
  - Domestic consumption of variety i falls, leading to exports of i, if c_i^{trade} from (9.2) (where only the domestic part is used) is smaller than c_i^{aut}

$$c_i^{\text{trade}} < c_i^{\text{aut}}$$

Proposition 5 *A country exports variety* $i \Leftrightarrow$

$$EX_i > 0 \Leftrightarrow \frac{\sum_{i=1}^{n} \frac{a_i^k \tilde{h}_i^k}{\left(h_i^A + h_i^B\right)^{1/\varepsilon}}}{\sum_{j=1}^{n} \left(h_j^A + h_j^B\right)^{(\varepsilon-1)/\varepsilon}} < \frac{h_i}{h_i^A + h_i^B}$$

- Interpretation
  - An interpretation would start from $c_i^{\text{trade}} = \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A}{\tilde{P}}$
  - Consumption falls relative to autarky if the real price $\frac{p_i}{\tilde{P}}$ or real income $\frac{W^A}{\tilde{P}}$ or both fall (relative to autarky) - or if one falls more than the other rises
  - to be thought about

9.5 A quantitative example

- As a preparatory step to construct Pareto-improving gains from trade, we compute the quantitative properties of our trade scenario
- For this example, we assume that the number of factors and goods amounts to $n = 3$
- Assume country B is relatively richer in unskilled labour than country A (Heckscher-Ohlin flavour – think of Deardorff, 1980)

$$\frac{\tilde{h}_1^B}{\tilde{h}_2^B + \tilde{h}_3^B} > \frac{\tilde{h}_1^A}{\tilde{h}_2^A + \tilde{h}_3^A} \tag{9.7}$$

- Technological differences (Ricardian aspect) are neglected
  - as they do not play a role for gains or losses from trade
  - we set $a_i^k = 1$ such that $h_i^k = \tilde{h}_i^k$
- See the next figure for an illustration of factor endowments

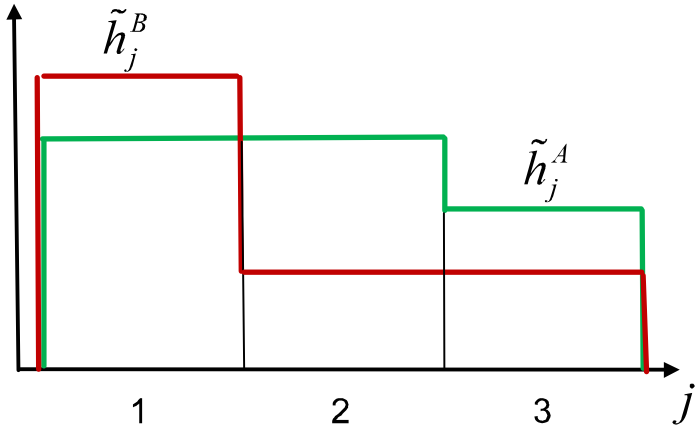


Figure 41 *The number* $\tilde{h}_j^k$ *of workers with educational backgrounds* $j \in \{1, 2, 3\}$ *in country A and B – Think of the number of individuals with (1) some educational degree, with (2) highschool leaving exam and (3) university degree*

- Assume the following parameter values

$\tilde{h}_1^A$	$\tilde{h}_2^A$	$\tilde{h}_3^A$	$\tilde{h}_1^B$	$\tilde{h}_2^B$	$\tilde{h}_3^B$	a_i^k	θ
15	15	10	30	5	5	1	.5

Table 2 *Number of workers for 3 levels of education (in millions) and further parameters*

- Who gains from trade in this example?
  - We employ the parameter values from table 2
  - Resulting real wages and real wage sums are in table 3

	autarky				trade			
wages from (8.11)	w_1^k	w_2^k	w_3^k	sum from (8.12)	w_1^k	w_2^k	w_3^k	sum from (9.5)
country A	2.8	2.8	3.4	119	2.2	3.4	3.9	123
country B	1.8	4.4	4.4	99	as in country A			104

Table 3 *Real wages and real wages sums in countries A and B in autarky and under trade*

- Autarky-abundant factors gain
  - Skill group 1 ("low skilled") is abundant in country B
  - It enters a world where it is scarce (country A is relatively rich in skills by (9.7))
  - Demand rises more than supply and wage goes up
  - Same argument for skill groups 2 and 3 in country A
  - The high-skilled in A gain as they become more scarce under trade
- Autarky-scarce factors lose
  - High-skilled in country B (skill types 2 and 3) face a lot of competition from A
  - Extra demand is not enough
  - They lose
  - Same argument for low-skilled in country A (skill type 1)

- Reminder of Heckscher-Ohlin model and Stolper-Samuelson result
  - Stolper-Samuelson theorem for small open economy: A relative increase in the price of the capital intensive good leads to a real increase in the factor reward for capital and a real drop in the factor reward for labour

$$\frac{dw^K}{w^K} > \frac{dp^X}{p^X} > \frac{dp^Y}{p^Y} > \frac{dw^L}{w^L} \Leftrightarrow \frac{K_X}{L_X} > \frac{K_Y}{L_Y}$$

  - Stolper-Samuelson theorem for two-country world: The relative price p_X/p_Y of the capital intensive good X rises under trade for the capital rich country and falls for the capital poor country. The real factor reward for capital rises in the capital rich country and falls in the capital poor country.

- Comparison to Stolper-Samuelson
  - Here we do not have factor intensities (or factor intensities of infinity or zero)
  - We observe a rise in the price of goods if demand exceeds competition effect
  - Alternative way of putting it: price of good rises if good becomes scarcer relative to other goods
  - The factor producing this good (employing this factor "very intensively") gains
  - Similar spirit (but applicable to many factors)

10 Redistribution and Pareto-improving gains from trade

10.1 The idea

10.1.1 The basic argument

- There are economy-wide gains from trade
- There are distributional effects of trade
- Redistribution can make sure that there are Pareto-improving gains from trade
- [Pareto-improvement: at least one individual gains and no individual loses (in absolute terms)]
- First analyses include Grandmont and McFadden (1972), Kemp and Wan (1972), Chipman and Moore (1972), more current work is by Willmann (2004), see the latter also for further references
- Antras et al (2017) look at tax system that redistributes gains from trade and study welfare implications

10.1.2 The idea in our model

- Table 3 shows that there are economy-wide gains from trade
  - Real labour income in country A rises due to trade
  - So does real labour income in country B
  - All consumers face the same set of consumption goods under trade as under autarky
  - Aggregate utility rises in both countries
- Table 3 also shows distributional effects
  - Some factors of production lose, some gain (in real terms)
  - True for country A and B
- Pareto-improving gains from trade?
  - to be found out
  - see next slides ...

10.2 The implementation

10.2.1 Average gains from trade

- The real wage sum in autarky can be written as in (8.12) or as

$$\frac{W}{P} = \Sigma_{j=1}^{n} \frac{w_j}{P} \tilde{h}_j$$

- The average increase can be seen from comparing wage sums in table 3
- Denote this average increase by γ, i.e.

$$\left.\frac{W}{P}\right|_{\text{trade}} = (1+\gamma) \left.\frac{W}{P}\right|_{\text{autarky}} \tag{10.1}$$

10.2.2 Constructing Pareto-improving gains from trade

- Applying the gains from trade factor to individual wages, we get "desired" wages

$$\left.\frac{w_j}{P}\right|_{\text{desired}} = (1+\gamma) \left.\frac{w_j}{P}\right|_{\text{autarky}}$$

- We compute a tax (negative or positive) for each wage such that each worker gets the desired wage

$$\frac{w_j^k}{\tilde{P}} \left(1 - \tau_j^k\right) = \left.\frac{w_j}{P}\right|_{\text{desired}} \tag{10.2}$$

  - The left-hand side starts from the real wage under trade and multiplies it by a tax rate specific to this skill-group j in country k
  - The resulting after-tax wage is to be equal to the desired wage
  - This allows to compute the skill-country tax rates

$$\tau_j^k = 1 - \frac{\left.\frac{w_j}{P}\right|_{\text{desired}}}{\frac{w_j^k}{\tilde{P}}} = \frac{\frac{w_j^k}{\tilde{P}} - \left.\frac{w_j}{P}\right|_{\text{desired}}}{\frac{w_j^k}{\tilde{P}}} \tag{10.3}$$

    which can be positive (for those that gain before tax) and negative (for those that lose)

- This tax scheme yields more than Pareto-improving gains from trade

10.2.3 The tax scheme

- Table 3 again

wages from (8.11)	autarky w_1^k	w_2^k	w_3^k	sum from (8.12)	trade w_1^k	w_2^k	w_3^k	sum from (9.5)
country A	2.8	2.8	3.4	119	2.2	3.4	3.9	123
country B	1.8	4.4	4.4	99	as in country A			104

Table 4 (identical to table 3) *Real wages and real wages sums in countries A and B in autarky and under trade*

- Pareto-improving gains from trade

	gains before tax w_1^k	w_2^k	w_3^k	aggregate gains γ from (10.1)	tax scheme w_1^k	w_2^k	w_3^k
country A	-20.3%	19.5%	12.7%	3.38%	-29.8%	13.5%	8.2%
country B	23.5%	-24.4%	-12.6%	4.64%	15.8%	-37.6%	-19.1%

Table 5 *Individual gains from trade before tax and tax schemes for Pareto-improving gains from trade*

- Result (in words)
  - Gains from trade imply that real labour income in country A increases by $\gamma^A = 3.38\%$ and $\gamma^B = 4.64\%$
  - These are aggregate effects as described in (10.1)
  - Individuals experience gains or losses from trade as computed in columns "gains before tax" in table 5
  - Skill groups in countries A and B pay a tax (positive sign) or receive a subsidy (negative sign) as computed in columns "tax scheme", following (10.3)
  - Individuals that experience a loss (say -20.3% for skill group 1 in country A) before the tax scheme receive a subsidy, i.e. they pay a negative tax amounting to -29.8%
  - Individuals that gain (groups 2 and 3 in country A and group 1 in country B) pay a tax
  - With this tax system, all individuals in country A experience a real increase of their labour income of γ^A (and all individuals in B experience an increase of γ^B)
  - This implies Pareto-improving gains from trade

10.3 Real-world issues

Can this idea be implemented in reality?

10.3.1 Information

- The starting point is the information on wages of skill groups
- Imagine they change: how do we know why they change?
- Wage changes can be caused (inter alia) by
  - new technologies of own firm or of competitors
  - supply effects of the factor of production under consideration
  - government or central bank policy changes
  - individual choices concerning labour supply (observable or via effort)
- There is a literature that looks at this
  - Autor et al. (2014) study the effects of trade exposure for US workers from 1992 to 2007
  - Autor et al. (2013) look at the effect of imports from China on the US labour market

10.3.2 Behavioural reactions

- We assumed perfectly inelastic labour supply
- T(rade-t)axes have behavioural consequences – e.g. via labour supply
- See Antras et al. (2017) for a more elaborate analysis

10.3.3 What is implemented in reality

- At general level
  - Progressive tax system: real wage changes are gross, net changes are not that strong
  - Unemployment insurance system: individual gets UI payments in case of job loss
  - Social welfare payments ("Hartz IV" payments, now "Bürgergeld" in the German system)
  - Severance payments, "Auffanggesellschaften" in case of firm bankruptcy
- At trade level
  - e.g. Trade adjustment assistance programme in the US (TAA)
  - see https://www.dol.gov/general/topic/training/tradeact
  - see Kondo (2018) for an analysis of trade-induced displacements

10.4 Conclusion

- We studied a model (first in autarky) that allowed us to understand
  - wage distributions in autarky
  - link between demand and competition effects and wages of skill groups
  - More workers of type i reduce wage of type i (competition effect)
  - More workers of types other than i increase wage of type i (demand effect)
- A two-country world analysis showed that
  - international trade leads to real gains and losses of workers
  - driving forces are changes in endowment of world as compared to autarky endowments
  - workers of type i abroad decrease the real wage of workers of type i domestically (competition effect)
  - more workers of other types increase the real wage (demand effect)
  - a country exports a good for which the demand effect overcompensates the competition effect

- Pareto-improving gains from trade are possible
  - A quantitative example computed
    * aggregate gains and
    * individual gains and losses
  - A tax and subsidy scheme was computed that implied Pareto-improving gains from trade
  - Challenges for implementing such schemes as policy measures were discussed
  - Real world examples of related schemes (TAA) were given

10.5 Exercises

10.5.1 The real wage under trade

1. Derive the real wage sum

$$\frac{W^A + W^B}{\tilde{P}} = \left(\Sigma_{j=1}^{n} \left(h_j^A + h_j^B\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{10.4}$$

employing the market clearing condition

$$h_i^A + h_i^B = \left(\frac{p_i}{\tilde{P}}\right)^{-\varepsilon} \frac{W^A + W^B}{\tilde{P}}.$$

2. Given the real wage sum 10.4), show that wages under trade are given by

$$\frac{w_i^k}{\tilde{P}} = a_i^k \left(\frac{\left(\Sigma_{j=1}^{n} \left(h_j^A + h_j^B\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{h_i^A + h_i^B}\right)^{1/\varepsilon}.$$

10.5.2 Proofs of trade propositions

1. Provide formal proofs to propositions 1, 2, 3, 4, and 5.

11 Linking theoretical models to data

- Theory gives answers
- More often than not, the answer is “it depends”
- Often we also want quantitative anwers – how *large* is an effect?
- To get anwers beyond “it depends” we need
  - numerical solution to a model
  - calibration of the model

11.1 Numerical solutions

- Numerical solution requires analytical solution
  - we need a well-defined system
  - do we have as many equations as we have (endogenous) variables?
  - does a (unique) solution to our equation system exist?
  - Is the numerical complexity of our problem sufficiently low?
    * one can study 'numerical analysis' and there are specialized courses
    * many of our problems are simple from a numerical perspective
- Example of Pareto-improving tax rates
  - very simple numerical problem
  - we need "reasonable" values for parameter
  - let us see how it works (here and tutorial)

11.2 Calibration

- How *large* is an effect?
  - now we need not just "reasonable" values for parameter
  - we want our model to match certain targets
- What is a target?
  - A model is built to explain something - let's say GDP or trade flows or changes in distributions
  - Afterwards we are often interested in effect of some event (opening up to trade, changes in tax rates, etc.)
  - When we want a model to match a target, we choose parameters such that quantitative model solution corresponds to empirical value

11.2.1 Example GDP in Germany

- Consider Cobb-Douglas production function

$$Y = AK^{\alpha}L^{1-\alpha} \tag{11.1}$$

- We want to understand the effect of a 3% increase in TFP A on GDP in Germany
- target: GDP Y
- Available data (2019)
  - GDP $Y = 3436 \times 10\hat{}9$ Euro
  - capital stock $K = 20.8 \times 10\hat{}12$ Euro
  - employment $L = 43 \times 10\hat{}6$ workers
  - output elasticity $\alpha = 0.3$
  - for capital stock, see in English or in German
  - for GDP, see here

- How large is TFP A (the 'Solow-residual')?
  - compute this in –> matlab
  - having done this, we calibrated our "model" (11.1) to match GDP in Germany by adjusting parameter A
- Let us now compute the effect of a 3% increase in A on GDP (or of the capital stock or other) –> matlab

11.2.2 Quantifying gains from trade

- Idea
  - Wage sum in country A and B from (8.12)

$$\frac{W}{P} = \left(\Sigma_{j=1}^{n} h_j^{(\varepsilon-1)/\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{11.2}$$

  - We want to match our target 'wage sum' for country A (say Germany) and for country B (say Poland)
  - How many (independent) equations and how many parameters do we have in autarky?
  - Write (11.2) as

$$\frac{W}{P} = \left(\Sigma_{j=1}^{n} \left(a_j \tilde{h}_j\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{11.3}$$

  - Answer: 2 and 6 for $n = 3$ but why?

- How to meaningfully fix parameters
  - Take wage sum per skill group i as target
  - From (8.13), we obtain

$$\frac{w_i}{P} = a_i \left[\frac{\left(\Sigma_{j=1}^{n} h_j^{(\varepsilon-1)/\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{h_i} \right]^{1/\varepsilon} \Leftrightarrow \frac{w_i \tilde{h}_i}{P} = a_i \tilde{h}_i \left[\frac{\left(\Sigma_{j=1}^{n} \left(a_j \tilde{h}_j \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{a_i \tilde{h}_i} \right]^{1/\varepsilon}$$

$$\Leftrightarrow \frac{w_i \tilde{h}_i}{P} = \left(a_i \tilde{h}_i \right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\Sigma_{j=1}^{n} \left(a_j \tilde{h}_j \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} \qquad (11.4)$$

  - The left-hand side $\frac{w_i \tilde{h}_i}{P}$ is the wage sum of skill group i, the right hand side has how many parameters that can (and need to) be computed? (3 plus ε, but where are they?)
  - How many equations do we have for $n = 3$?

- Finding data for population size and education structure
  - Workforce see information at Eurostat page, table 1
    - directly available as Excel file
  - Educational attainment see another Eurostat page and then → Distribution_of_the_EU_population_by_educational_attainment_in_2023
  - EU shares of income components also at Eurostat
    - Wage sum Germany is also at destatis
  - GDP is at ECB data portal
  - Wage sum per education group? Microdata needed (we need to guess for this lecture)

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- This was 2024!
- In 2025, we have chatGPT

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  - Workforce see information at Eurostat page, table 1
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  - Educational attainment see another Eurostat page and then
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  - EU shares of income components also at Eurostat
    - Wage sum Germany is also at destatis
  - GDP is at ECB data portal

- Wage sum per education group
  - estimate wage sums per education group by relative wages and absolute size
  - we get the values below (in Billion, i.e. 10^9, Euro)
  - (works for lecture, does not work for seminar paper or master thesis or beyond)

wage sums	Germany	Poland
$\frac{w_1 \tilde{h}_1}{P}$	420	174
$\frac{w_2 \tilde{h}_2}{P}$	360	178
$\frac{w_3 \tilde{h}_3}{P}$	1104	18

- The output of chatGPT...

🧮 Step 4: Calculate Wage Sums

Group	Workers (M)	Avg Wage (€)	Wage Sum (€B)
🎓 University degree	7.0	60,000	420
⌛ Abitur only	9.0	40,000	360
🚫 Neither Abitur nor degree	26.1	42,890*	1,104
Total	**42.1**	—	**1,884**

* The €42,890 average was adjusted so that the total matches €1.884 trillion exactly.

3. Wage Sum by Group (in EUR)

Education Level	Workers (M)	Avg Wage (EUR)	Sum (bn EUR)
🎓 Tertiary (degree)	6.25	27 800	**174**
⌛ Upper secondary only	10.85	16 370	**178**
🚫 Below upper secondary	1.29	13 880	**18**
Total	18.39	—	**369**

These add up to **~369 bn EUR**, matching the national total.

- ... and obvious issues
  - different empirical categories for skill groups
  - rough adjustment to match aggregate sums
  - (and more, e.g. self-employed & high-income employees are missing due to administrative data)

- Back to calibration
  - We return to (11.4)

$$\frac{w_i \tilde{h}_i}{P} = \left(a_i \tilde{h}_i\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\Sigma_{j=1}^{n} \left(a_j \tilde{h}_j\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} \tag{11.5}$$

  - In autarky, 3 equations for each country (as $n = 3$) and 4 parameters (a_j and ε)
  - Set ε to 0.8 (do robustness checks afterwards by varying ε)
  - Then we have 3 equations in 3 unknowns (and the same for the other country)
  - We observe $\frac{w_i \tilde{h}_i}{P}$ and $\tilde{h}_j$ and let matlab fix the a_j
- How to compute a_j for each country in matlab?
  - employ fsolve - solves system of nonlinear equations
  - rewrite equations by dividing – $\left(\Sigma_{j=1}^{n} \left(a_j \tilde{h}_j\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}$ is independent of i
  - check what works better numerically

–> fantastic (in the sense of "a lot can be learned") term paper

- Quantitative gains from trade
  - compute gains as before in tables 3 and 5
  - how large are they?

–> check it out in matlab

Part IV
Summary

12 Conclusion and long-term redistribution

12.1 What have we learned in this lecture?

- We studied immediate measures to move towards more equal distribution of current (labour) income
- This is a useful but short-run and only partial tool
- Other more fundamental and more long-run measures are also needed

12.2 What is missing?

- Labour income is important and central for most households in determining their material well-being
- Labour income is the result of (luck and) human capital
- The latter is endogenous, i.e. it depends on other determinants
- Tax system can correct effects of distribution of human capital but it remains redistribution "after the fact"
- Further measures to make human capital distribution more equal are needed
- (Topic for further lectures or seminar papers or master theses)

12.2.1 Redistributing wealth

- Financial background of children crucial for equality of chances
- Should we tax wealth?
  - Big question
  - Big literature
- We could also tax bequests
  - High allowance per child (say, 1 million Euro for Germany)
  - Bequests beyond this level goes to pool
  - Pool is redistributed to all inheritors (within say a year)
  - Wealth should be fairly equally distributed after 1 generation or so
  - [to be worked out – research needed]

12.2.2 Equality in education

- Access to education and educational achievements should be more equally distributed
- Standard issue in many countries (including Germany) that most university students have parents with university degree
- Basis for educational success is laid very early in life (Heckman, 2006)
  - One might think about compulsory nursery school/ kindergarten
  - Big trade-off between liberty (of parents) and equality (of children)
  - See e.g. https://www.oekonomenstimme.org/artikel/2018/12/die-reform-des-sozialstaates-zu-ende-gedacht-kindergartenpflicht/
- Other measures useful as well
  - Vouchers
  - Household support for single parents
  - (see Hahn, 2020, for current research)
- Also topic of another lecture or further research


The End

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Part V
Appendix

13 A reminder of inverse functions

- Notation

Consider a function $f(x)$ with argument x. An inverse function is denoted by f^{-1}. This does NOT mean “to the power of minus one”.

- Definition

An inverse function is defined by $f^{-1}(f(x)) = x$. In words: when the inverse function of function f is applied to function f, the argument x of function f results.

- Example 1

Consider a function

$$y = f(x) = a + bx \tag{13.1}$$

where a and $b \neq 0$ are constants. For any value x, there is a unique value y.

Now we can solve this expression for x,

$$x = \frac{y - a}{b} \equiv f^{-1}(y) . \tag{13.2}$$

For any value of y, there is a unique x. We claim that this resulting expression is the inverse function of f.

We prove this by starting from the left-hand side of the definition of the inverse function,

$$f^{-1}(f(x)) = f^{-1}(a + bx) , \tag{13.3}$$

and substituting $f(x)$ from (13.1). Now we employ our claim of the inverse function from (13.2), noting that the argument y in (13.2) is $a + bx$ as we just got in (13.3). We get

$$f^{-1}(a + bx) = \frac{a + bx - a}{b}.$$

Simple rearrangement yields

$$f^{-1}(a + bx) = x.$$

Hence, $f^{-1}(f(x)) = x$ and we have proven that (13.2) is indeed the inverse function of (13.1).

- Example 2

Consider another function

$$y = g(x) = \ln x.$$

For any value x, there is a unique value y.

Now we can solve this expression for x,

$$x = e^y \equiv g^{-1}(y). \tag{13.4}$$

As for (13.2), for any y, there is a unique x. We claim that e^y is the inverse function of $g(x)$.

As before,

$$g^{-1}(g(x)) = g^{-1}(\ln x).$$

Employing the (claim for the) inverse function, we get

$$g^{-1}(g(x)) = e^{\ln x} = x.$$

Hence, we have proven that (13.4) is the inverse function of $g(x)$.

- Illustration for example 2